Finding Extraneous Solutions
ID: 8109

Time required
45 minutes

Activity Overview
Students will solve different types of equations step-by-step graphically. They will discover that some of the equations have an extraneous solution and they will investigate at which step in solving the equation that these "extra" solutions appear.

Concepts
- *Solving equations graphically*
- *Solving equations algebraically*

Teacher Preparation
This investigation offers an extension after students have learned how to solve equations, or may serve as an introduction to solving more difficult equations.

- *This activity could be used in Algebra 2 as an extension after students have solved different types of equations. This activity could also be used in Precalculus as review of the different types of equations that students solved in Algebra 2.*

- *Begin by reviewing with students how to solve equations graphically by treating each side of an equation as a separate function—the solution to the equation is the functions' point of intersection.*

- *The screenshots on pages 44–47 demonstrate expected student results. Refer to the screenshots on page 48 for a preview of the student .tns file.*

- ***To download the .tns file and student worksheet, go to http://education.ti.com/exchange and enter "8109" in the search box.***

Classroom Management
- *This activity is intended to be **teacher-led**. You may use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds, although the majority of the ideas and concepts are only presented in **this** document; be sure to cover all the material necessary for students' total comprehension.*

- *The student worksheet is intended to guide students through the main ideas of the activity, while providing more detailed instruction on how they are to perform specific actions using the tools of the TI-Nspire handhelds. It also serves as a place for students to record their answers. Alternatively, you may wish to have the class record their answers on a separate sheet of paper, or just use the questions posed to engage a class discussion.*

- *Information for an optional extension is provided at the end of this activity; information for students is provided on the student worksheet and in Problem 3 of the student TI-Nspire file. Should you not wish students to complete the extension, you may delete Problem 3 of the .tns file and have students disregard that portion of the student worksheet.*

TI-Nspire™ Applications
Calculator, Graphs & Geometry, Lists & Spreadsheet, Notes

TI-*nspire*™

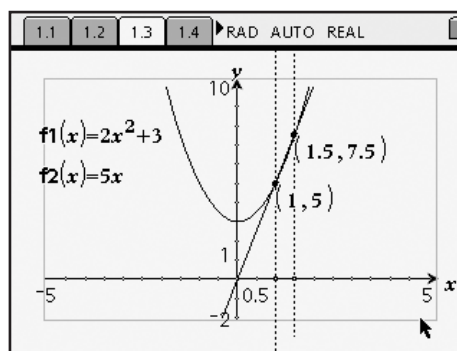**Problem 1 – Solving a quadratic equation**

Students will work through the solving of the equation $2x^2 + 3 = 5x$ graphically and step-by-step. They simply need to change the equations in **f1(x)** and **f2(x)** to represent the left- and right-hand sides, respectively, of the equation in each step. The functions can be altered by double-clicking in the text box on the *Graphs & Geometry* page. Students will find that the *x*-coordinates of the intersection points remain the same each time the equation is manipulated graphically.

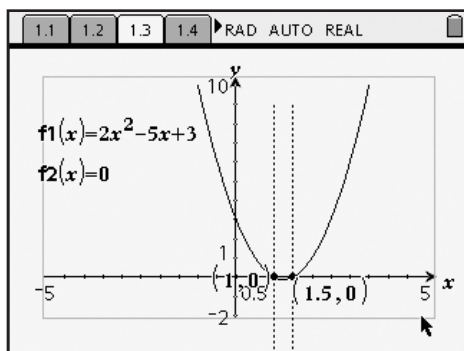| 1.1 | 1.2 | 1.3 | 1.4 | ▶RAD AUTO REAL | |

Solve graphically: $2x^2 + 3 = 5x$

Step 1: $2x^2 + 3 = 5x$

Step 2: $2x^2 - 5x + 3 = 0$

Step 3: $(2x - 3)(x - 1) = 0$

Step 4: $x = \dfrac{3}{2}$ and $x = -1$

You may wish to work though this first problem with students if they are unfamiliar with solving equations graphically. Students will be moving back and forth between pages 1.3 and 1.4 to access the equations and then the graph. Encourage them to write the solution(s)—i.e., the *x*-coordinates of the point(s) of intersection of functions **f1** and **f2**—on their worksheet.
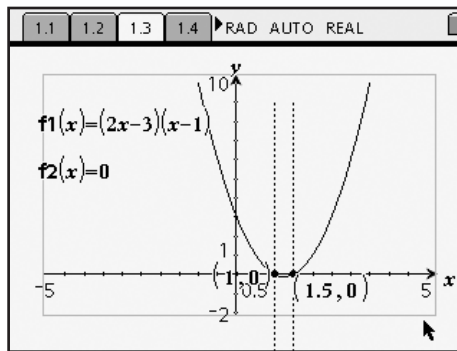
The graphs of the functions at each step in the solution are shown at right and below.

| 1.1 | 1.2 | 1.3 | 1.4 | ▶RAD AUTO REAL | |

$f1(x) = 2x^2 + 3$

$f2(x) = 5x$

$(1.5, 7.5)$

$(1, 5)$

Step 1

| 1.1 | 1.2 | 1.3 | 1.4 | ▶RAD AUTO REAL | |

$f1(x) = 2x^2 - 5x + 3$

$f2(x) = 0$

$(1.5, 0)$

Step 2

| 1.1 | 1.2 | 1.3 | 1.4 | ▶RAD AUTO REAL | |

$f1(x) = (2x - 3)(x - 1)$

$f2(x) = 0$

$(1.5, 0)$

Step 3

On page 1.5, students can check their solutions algebraically and by using a table of values. To use the table, they will need to return to the *Graphs & Geometry* page and re-enter the Step 1 equations as **f1(x)** and **f2(x)**. For the *Calculator* application, the first solution is checked for them—entering "**f1(1)=f2(1)**" returns **true**. Have students check the second solution, 1.5, by entering "**f1(1.5)=f2(1.5)**" and pressing (enter). If the result is **true**, then the value is, in fact, a solution.

Students should conclude that $x = 1$ and $x = 1.5$ are both solutions to the equation $2x^2 + 3 = 5x$.

Problem 2 – Solving a radical equation

In Problem 2, students will approach the equation $\sqrt{x + 11} + 1 = x$ in the same manner, but this time they will observe the appearance of an extraneous solution during the graphing of the functions in Step 3.

Again, the graphs of the functions at each step in the solution are shown below and on the following page.
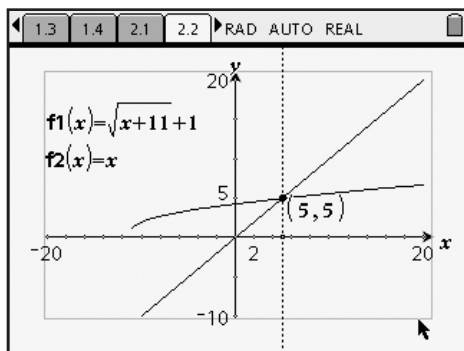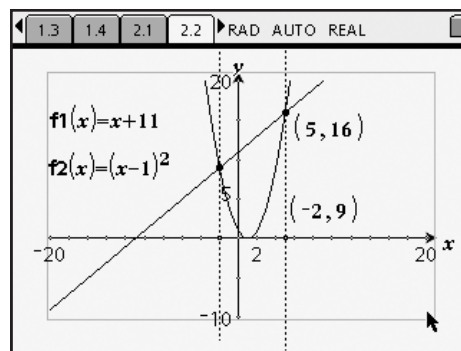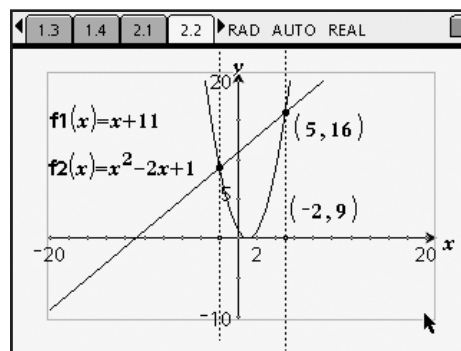
Step 1
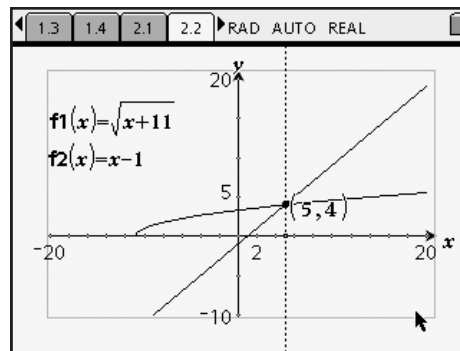
Step 2

When they have graphed the equations in Step 3, students may wish to adjust the window. To accomplish this, one option is to simply grab a tick mark on the positive *y*-axis and drag it down towards the origin. Another way is to select **MENU > Window > Window Settings** and enter the desired values.


Step 3

Students may have to plot the new intersection for the new solution by selecting **MENU > Points & Lines > Intersection Point(s)** and then clicking once on the graph of each function. They may also wish to construct a perpendicular (using the **Perpendicular** tool (⊥) from the Construction menu) to the *x*-axis through this new point. Using the **Attributes** tool (▦) from the Tools menu, students can change the appearance of the line.
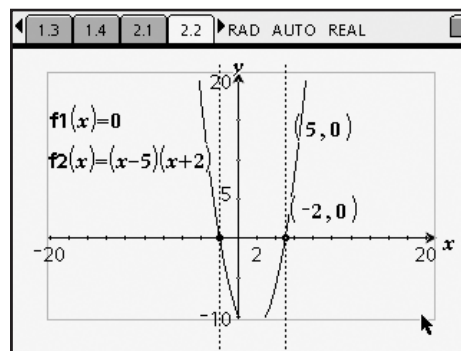
In graphing the equations for Step 4, students should observe that the graph of the function $y = (x - 1)^2$ did not change position as it was altered to the function $y = x^2 - 2x + 1$. Ask students to explain why this happened. (The expressions $(x - 1)^2$ and $x^2 - 2x + 1$ are equivalent—the former is merely the factored form of the latter. Thus, their graphs are identical.)

Step 4
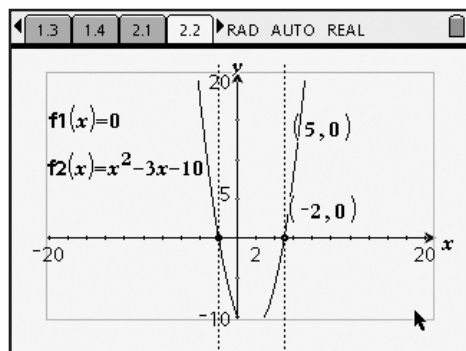
Step 5

Step 6

Students should once again check their solutions—remind them to reset the functions **f1** and **f2** to be as they were in Step 1. This time, students should find that only *one* of their solutions, $x = 5$, makes the equation true. Tell them that $x = -2$ is called an "extra" or *extraneous* solution.

Extension – Solving a rational equation

As an optional extension, Problem 3 provides students with a step-by-step solution to a rational equation. A *Graphs & Geometry* page is provided, for the students to use as before. Have them identify the step in which the extraneous solution appeared. A function table is also provided for students to verify whether the solutions are "true" solutions or extraneous ones. ($x = -3$ is a true solution; $x = 3$ is an extraneous solution—both expressions are undefined for $x = 3$.)

Finding Extraneous Solutions – ID: 8109

(Student)TI-Nspire File: *PreCalcAct1_ExtraneousSolutions_EN.tns*

1.1 1.2 1.3 1.4 ▶ RAD AUTO REAL

FINDING EXTRANEOUS
SOLUTIONS

Precalculus
Solving equations graphically
and algebraically

1.1 1.2 1.3 1.4 ▶ RAD AUTO REAL

Solve graphically: $2x^2 + 3 = 5x$

Step 1: $2x^2 + 3 = 5x$

Step 2: $2x^2 - 5x + 3 = 0$

Step 3: $(2x - 3)(x - 1) = 0$

Step 4: $x = \dfrac{3}{2}$ and $x = -1$

1.1 1.2 1.3 1.4 ▶ RAD AUTO REAL

$f1(x) = 2x^2 + 3$
$f2(x) = 5x$

$(1.5, 7.5)$
$(1, 5)$
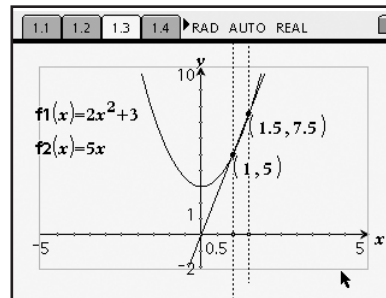
1.1 1.2 1.3 1.4 ▶ RAD AUTO REAL

Check your two solutions algebraically and verify on the table.

x	f1(x):... ▼	f2(x):... ▼	f1
	2*x^2+...	5*x	2*
0.	3.	0.	
.5	3.5	2.5	
1.	5.	5.	

3.   1/99

$f1(1) = f2(1)$
true

1.2 1.3 1.4 2.1 ▶ RAD AUTO REAL

Solve graphically: $\sqrt{x + 11} + 1 = x$

Step 1: $\sqrt{x + 11} + 1 = x$
Step 2: $\sqrt{x + 11} = x - 1$
Step 3: $x + 11 = (x - 1)^2$
Step 4: $x + 11 = x^2 - 2x + 1$
Step 5: $0 = x^2 - 3x - 10$
Step 6: $0 = (x - 5)(x + 2)$
Step 7: $x = 5$ and $x = -2$

1.3 1.4 2.1 2.2 ▶ RAD AUTO REAL

$f1(x) = \sqrt{x + 11} + 1$
$f2(x) = x$

$(5, 5)$
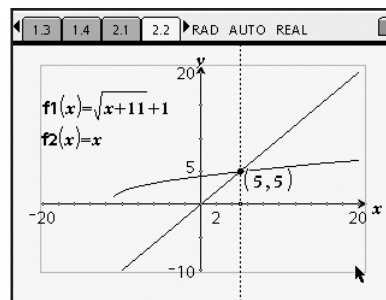
1.4 2.1 2.2 2.3 ▶ RAD AUTO REAL

Check your two solutions algebraically and verify on the table.

x	f1(x):... ▼	f2(x):... ▼
	√(x+11)..	x
0.	4.31662	0
1.	4.4641	1
2.	4.60555	2

0/99   4.3166247903554

2.1 2.2 2.3 3.1 ▶ RAD AUTO REAL

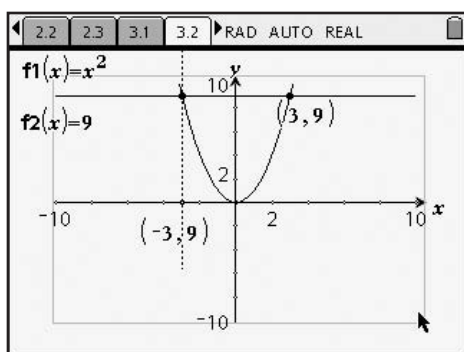Solve graphically: $\dfrac{3x}{x - 3} = \dfrac{2x + 3}{x - 3}$

Step 1: $\dfrac{3x}{x - 3} = \dfrac{2x - 3}{x - 3}$

Step 2: $(3x)(x - 3) = (2x - 3)(x - 3)$

Step 3: $3x^2 - 9x = 2x^2 - 9x + 9$

Step 4: $x^2 = 9$

Step 5: $x = -3$ and $x = 3$

2.2 2.3 3.1 3.2 ▶ RAD AUTO REAL

$f1(x) = \dfrac{3 \cdot x}{x - 3}$
$f2(x) = \dfrac{2x - 3}{x - 3}$

$(-3, 1.5)$
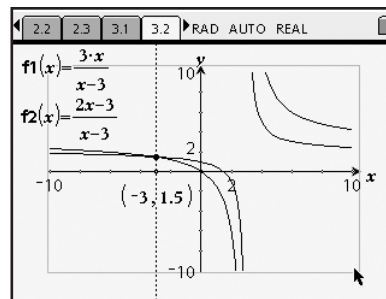
2.3 3.1 3.2 3.3 RAD AUTO REAL

Check your solutions on the function table.

x	f1(x):... ▼	f2(x):... ▼	▼
	3*x/(x-..	(2*x-3)/.	
-1.	.75	1.25	
0.	0.	1.	
1.	-1.5	.5	
2.	-6.	-1.	

0.