

Limit of $\sin(x)/x$

ID: 11068

Time required
15 minutes

Activity Overview

In this activity, students will graph $f(x) = \frac{\sin x}{x}$ in order to visually determine the limit as $x \rightarrow 0$.

They will confirm the answer numerically by tracing left and right limit points to capture values in a spreadsheet.

Topic: Limits

- Trigonometric functions
- Left-hand and right-hand limits
- Graphical limits

Teacher Preparation and Notes

- This activity is meant for students to complete the activity step by step without any previous set-up. Students should start with a blank TI-Nspire document.
- A connection to the formal limit notation should be considered to connect formal mathematical notation to visual representation.
- **To download the TI-Nspire document (.tns file) and the student worksheet, go to education.ti.com/exchange and enter "11068" in the quick search box.**

Associated Materials

- *LimitofSin_Student.doc*

Suggested Related Activities

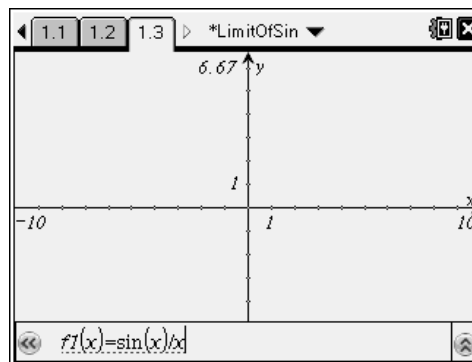
To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *Limits (TI-Nspire CAS technology)* — 8997
- *Sequences in Calculus (TI-Nspire technology)* — 8665
- *Black Bear Cubs From Birth to One Year Old (TI-Nspire CAS technology)* — 8317
- *Exponential Differentiation (TI-Nspire technology)* — 8980

Part 1 – Graphical Limit

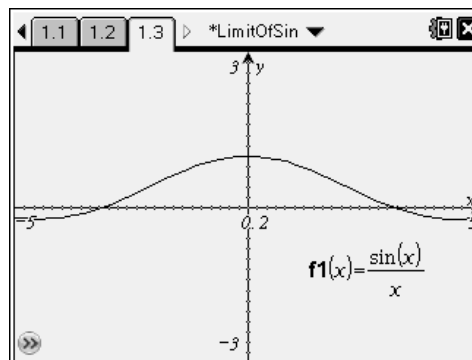
Before beginning the activity, have a discussion with students about the limit of $\frac{\sin(x)}{x}$ before seeing the graph.

Students are to graph the function by typing **sin(x)/x** in the function entry line at the bottom of the screen. Press **enter** to see the graph. Students can press **ctrl** + **G** to hide the entry line.



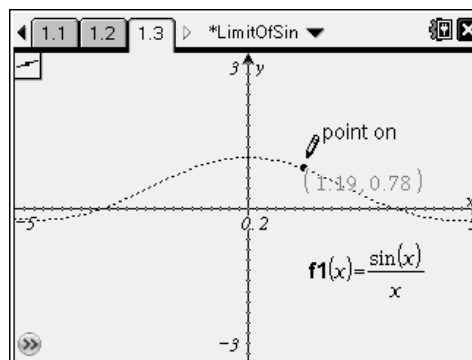
The window settings shown are XMin = -5, XMax = 5, YMin = -3, and YMax = 3.

Discuss with the students what the limit as x approaches zero, appears to be. Explain to students that the graph of the function appears to be continuous even though it is actually undefined at $x = 0$.



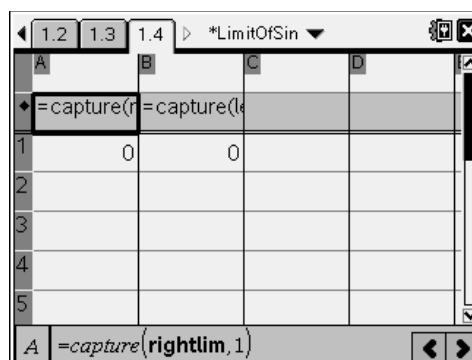
Now students are to use the **Point On** tool to place a point on the function. This tool will also show the point's coordinates on the graph.

Students will move the point to see the values of the function as the x -values approach zero. In order to emphasize a limit must be equal from both sides, move the point from the right toward zero and from the left toward zero.



Part 2 – Numerical Limit

After adding a *Lists & Spreadsheet* page (**ctrl** + **I**), students will set it up to automatically capture data. In the gray formula cell of column A they need to type **=capture(rightlim,1)** and in column B, type **=capture(leftlim,1)**. The 1 denotes automatic whereas 0 denotes manual.

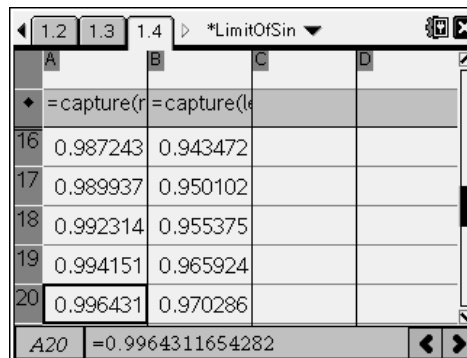
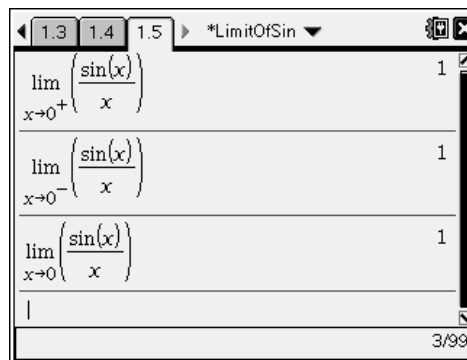


Students are to place a second point on the graph repeating the step in Part 1. They need to have one point to the *left* of zero and one point to the *right* of the zero on the *x*-axis.

To connect the points to the spreadsheet, students are to store the *y*-value of the right point as **rightlim** and store the *y*-value of the left point as **leftlim**. The *y*-value will become bold once it is stored.

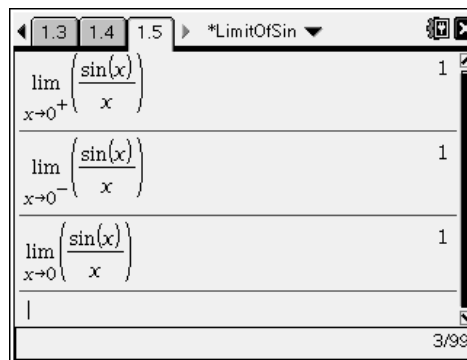
One at a time, students are to move the points toward but not cross over the *y*-axis. The values of the function will be captured on the spreadsheet page.

Students need to move back to the spreadsheet page and scroll down through the columns to that the points are approaching the same value, the limit of $\frac{\sin(x)}{x}$.



Part 3 – Algebraic Limit

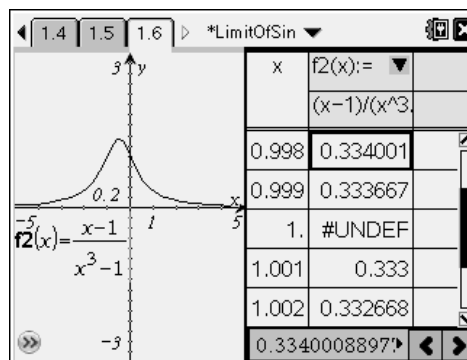
On a *Calculator* page, students are to use the limit template to find the left-hand, right-hand and regular limits.



Practice Problems

Students are given extra practice problems to complete using what they have learned in this activity. They can do the problems for homework or if there is extra time in class.

After graphing the function, students can press **(ctrl) + T** to add a table of values. Press **MENU > Table > Edit Table Settings** to change the Table Start and Table Step.



1. $\lim_{x \rightarrow 1} \frac{x-1}{x^3-1} = \frac{1}{3}$

2. $\lim_{x \rightarrow 0} \frac{1-\cos(x)}{x^2} = \frac{1}{2}$

3. $\lim_{x \rightarrow 0} (1+x)^{\frac{1}{x}} \approx 2.72$