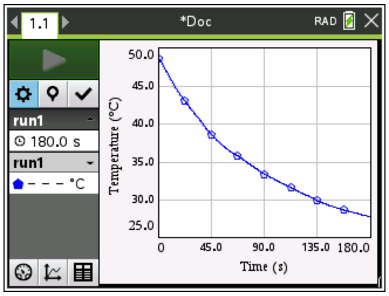
Cooling Down

About the Lesson

A hot drink gradually cools off but how can the cooling process be described? In this activity, students will collect data as a heated temperature sensor cools for three minutes, graph the cooling curve, and determine a model to fit the temperature versus time data.



Students will:

- Collect temperature versus time data of a cooling object and graph a scatter plot.
- Model the temperature versus time data with an exponential equation in the form $f(x) = a(b)^x + c$.

Vocabulary

- Horizontal Asymptote
- Vertical Translation
- Exponential function

Teacher Preparation and Notes

- This activity provides an opportunity for math-science connections.
- Students should be familiar with exponential functions and horizontal asymptotes.
- This activity is best performed with at least two students: one to hold the temperature sensor and one to run the handheld.

Tech Tips:

- This activity includes screen captures taken from the TI-Nspire™ CX II. It is also appropriate for use with the rest of the TI-Nspire CX family. Slight variations to these directions may be required if using other handheld models.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

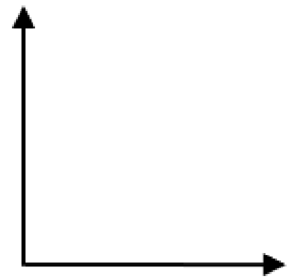
Activity Materials

- TI-Nspire CX II or TI-Nspire CX II CAS
- Vernier® EasyTemp Sensor for each group of students
- Cup of hot water (approximately 45 to 55 degrees Celsius) for each group of students
- Recommended: TI-Nspire™ CX Premium Teacher Software or TI-Nspire™ CX CAS Premium Teacher Software

Introduction

When you heat a liquid and then remove the liquid from the heat, the liquid cools at a certain rate. In this activity, a temperature sensor will be heated in a cup of hot water for approximately 30 seconds and then removed from the water. Temperature versus time data will be collected for three minutes after the sensor is removed from the hot water. A mathematical model will be determined to describe the temperature of the sensor as a function of time.

Before collecting data, predict how the graph of temperature as a function of time would look after the heated sensor is removed from the hot water. Sketch your prediction to the right. Be sure to label the axes.

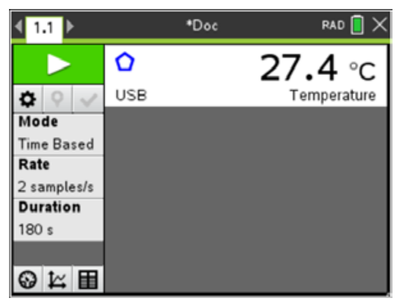


Write a sentence to explain why you think the graph will look like your prediction.

Setup

1. This activity is best performed with at least two students: one to hold the temperature sensor and one to run the handheld.

2. Open a new document on the TI-Nspire™ CX handheld. Connect the Vernier EasyTemp sensor to the handheld. The Vernier DataQuest™ App will start with the Main Screen displayed.



   Note: The default unit of measurement in the DataQuest App for the EasyTemp sensor is degrees Celsius. The ambient room temperature is displayed.

   Note: The current Mode is Time Based, and the default data collection duration is 180 seconds.

3. Before collecting data, record the ambient temperature of the room.

   Ambient temperature: _______________

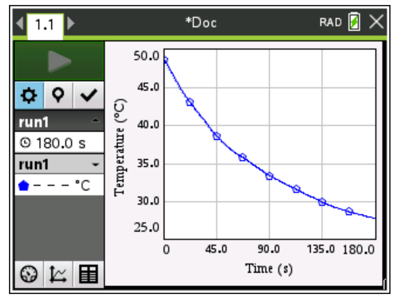
   <u>**Sample Answer:**</u> Answers will vary. For the sample data, the ambient temperature is 27.4 degrees Celsius.

4. Press **Tab** until the **Start Collection** button ▶ is highlighted. Do not click on the button or press enter.

Data Collection

1. Obtain a cup of hot water and place the EasyTemp sensor in the water. After it has heated for approximately 30 seconds, remove the sensor from the water and immediately click the **Start Collection** button or press enter.

   Note: The EasyTemp sensor should be held still.



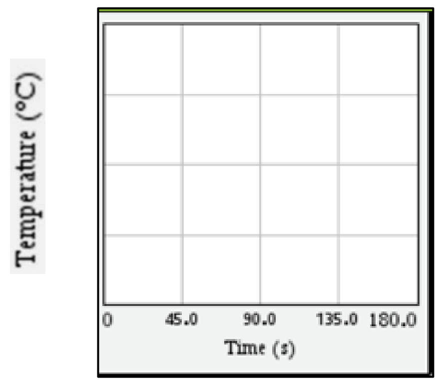
2. After the data collection is complete, the temperature versus time graph is displayed. A graph of sample data is shown at the right.

   Note: If it is necessary to repeat the data collection, press the **Start Collection** button again.)

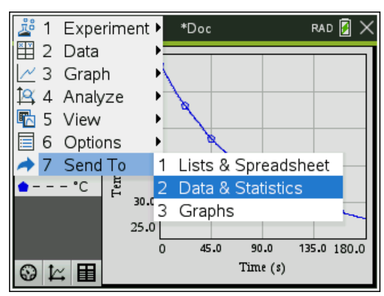
3. Sketch a graph of your data on the grid at the right.

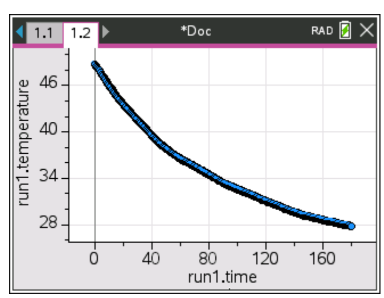
   Note: Label the y-axis to fit your data.

   How does the graph of your data compare to your prediction?

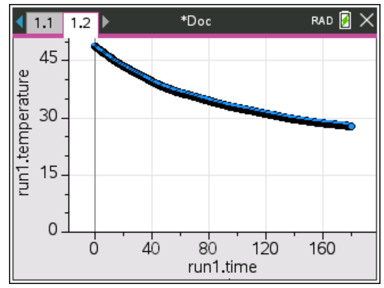


4. Send the data to a Data & Statistics page by pressing **Menu > Send To > Data & Statistics**.





5. Adjust the window to display a minimum temperature of 0 degrees Celsius.
   - Press **Menu > Window/Zoom > Window Settings**.
   - Tab to YMin, and enter 0.
   - Press enter or click **OK**.



Data Analysis

1. You will fit a model to your data.
   a. What type of function might be a good fit for your temperature versus time data?

   Sample Answer: Answers will vary.

   b. As the time increases, the graph of your data should level off and approach (but not necessarily reach) a certain temperature value. What temperature should the sensor be approaching?

   Sample Answer: The temperature should approach the ambient room temperature.

   c. A horizontal asymptote is a horizontal line that a graph approaches. Record the equation of the horizontal asymptote for your graph.

   Sample Answer: Answers will vary, but the equation should be of the form f(x) = k with a value of ***k*** that is the ambient room temperature. For the sample data, f(x) = 27.4.

   d. Graph your horizontal asymptote.

   Tech Tip: To graph the line, select **Menu > Analyze > Plot Function**. Enter the function, and then press enter. To remove the graph of the line, select **Menu > Analyze > Remove Plotted Function.**

2. A model for this cooling curve is an exponential function of the form $f(x) = a(b)^x + c$.

   a. How does the location of the horizontal asymptote relate to the value of ***c***?

   Sample Answer: Answers will vary but the value of ***k*** in the equation of the horizontal asymptote should be close to or equal to the value of ***c***.

   b. Based on the graph of your data, what is the value of ***c***? ___________ (Be sure to include units.)

   Sample Answer: Answers will vary. For the sample data, ***c*** is 27.4 degrees Celsius, the temperature of the room.

   c. What does the value of ***c*** represent in the exponential function?

   Answer: The vertical translation of the function $f(x) = a(b)^x$.

3. Select **Menu > Analyze > Graph Trace**. Trace to the y-intercept of your time versus temperature graph. (**Note:** To exit Graph Trace, press [esc].)

   a. Use the x- and y-coordinates of the y-intercept and the value of *c* to determine the value of *a* in the function $f(x) = a(b)^x + c$.

   Sample Answer: The y-intercept for the sample data is (0, 49). Using c = 27.4, the equation is $49 = a(b)^0 + 27.4$. a = 21.6.

   b. What is the value of ***a***? _____________ (Be sure to include units.)

   Sample Answer: Answers will vary. For the sample data, a = 21.6 degrees Celsius.

   c. What does the value of ***a*** represent?

   Answer: The difference between the temperature of the sensor when it was removed from the hot water (at time t = 0) and the value of *c* (the ambient temperature).

4.

a. Based on the graph of your data, should the value of ***b*** in the function $f(x) = a(b)^x + c$ be between 0 and 1 or greater than 1? Justify your answer.

Sample Answer: Since the temperature values are decreasing as time goes by, the value of ***b*** should be between 0 and 1.

Note: This would be a good time to discuss exponential decay.

b. Estimate a value for ***b***. ______________________

Sample Answer: Answers will vary, but the value should be between 0 and 1.

c. What does this value of ***b*** represent?

Sample Answer: The parameter ***b*** represents the percentage of the temperature the probe retains each second.

5. On the Data & Statistics page, select **Menu > Analyze > Plot Function**. Using the values you determined for ***a*** and ***c*** and your estimate for the value of ***b***, enter the equation of your function in the form $f(x) = a(b)^x + c$. Press **enter**.

6. If the graph of your function does not fit the data well, double-click on the equation, adjust the value of ***b***, and regraph. (If necessary, also adjust the values of ***a*** and ***c***.)

7. What is the equation of the function that best fits the data? ____________________________

   Sample Answer: Answers will vary.

8. If you collected data for more than three minutes, would the graph of the data eventually cross your horizontal asymptote? Justify your answer.

   Answer: No. The graph of the data will approach but not cross the horizontal asymptote. The location of the horizontal asymptote is the room temperature, and the temperature of the sensor could not go below room temperature.