8-1 Exploring Exponential Models

Lesson Preview

What You'll Learn

OBJECTIVE 1 To model exponential growth

OBJECTIVE 2 To model exponential decay

. . . And Why

To model a car's depreciation, as in Example 6

✔ Check Skills You'll Need

(For help, go to Lesson 1-2.)

Evaluate each expression for the given value of x.

1. 2^x for $x = 3$ **8**
2. 4^{x+1} for $x = 1$ **16**
3. 2^{3x+4} for $x = -1$ **2**
4. $3^x 3^{x-2}$ for $x = 2$ **9**
5. $\left(\frac{1}{2}\right)^x$ for $x = 0$ **1**
6. 2^x for $x = -2$ **$\frac{1}{4}$**

New Vocabulary

- exponential function
- growth factor
- decay factor
- asymptote

Interactive lesson includes instant self-check, tutorials, and activities.

OBJECTIVE 1 Exponential Growth

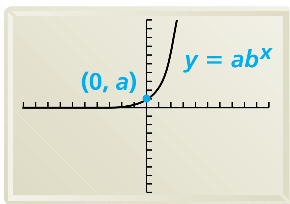
Need Help?

If the value of a function depends on the value of x, then x is the independent variable.

For some data, the best model is a function that uses the independent variable as an exponent. An **exponential function** is a function with the general form $y = ab^x$, where x is a real number, $a \neq 0$, $b > 0$, and $b \neq 1$.

You can use an exponential function to model growth when $b > 1$. When $b > 1$, b is the **growth factor.**

Exponential Growth



Growth factor $b > 1$

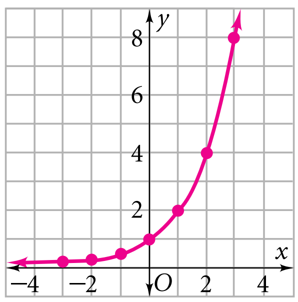
1 EXAMPLE Graphing Exponential Growth

Graph $y = 2^x$.

Step 1 Make a table of values.

x	2^x	y
–3	2^{-3}	$\frac{1}{8} = 0.125$
–2	2^{-2}	$\frac{1}{4} = 0.25$
–1	2^{-1}	$\frac{1}{2} = 0.5$
0	2^0	1
1	2^1	2
2	2^2	4
3	2^3	8

Step 2 Graph the coordinates. Connect the points with a smooth curve.

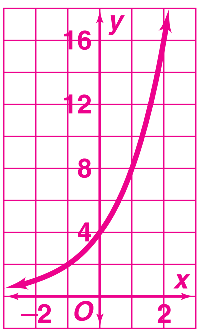


✔ Check Understanding 1 Graph each function. **a–b. See left.**

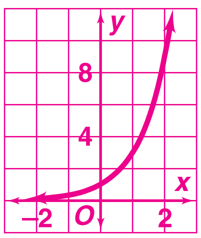
a. $y = 4(2)^x$

b. $y = 3^x$

1a.



b.



You can use an exponential function to model population growth. If you know the rate of increase r, you can find the growth factor by using the equation $b = 1 + r$.

2 EXAMPLE Real-World Connection

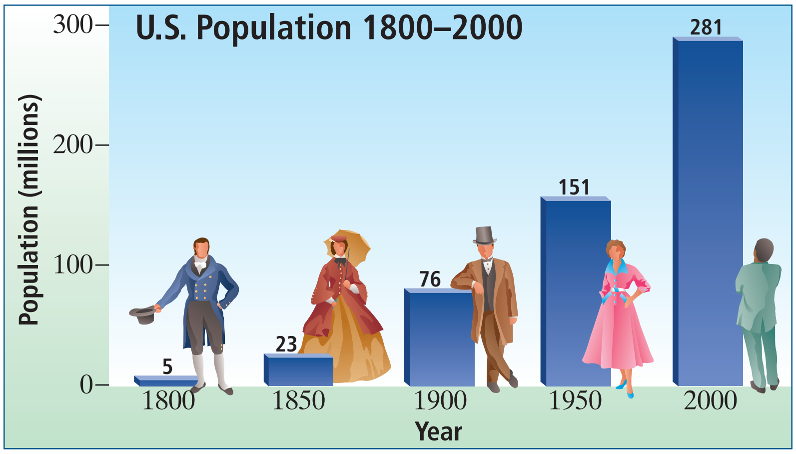
Real-World Connection

The United States counts its population every 10 years.

Population Refer to the graph. In 2000, the annual rate of increase in the U.S. population was about 1.24%.

a. Find the growth factor for the U.S. population.

b. Suppose the rate of increase continues to be 1.24%. Write a function to model U.S. population growth.



SOURCE: U.S. Census Bureau. Go to **www.PHSchool.com** for a data update. Web Code: agg-2041

a. Find the growth factor.

$b = 1 + r$

$= 1 + 0.0124$ **Substitute 1.24%, or 0.0124, for *r*.**

$= 1.0124$ **Simplify.**

b. Write a function.

Relate The population increases exponentially, so use the general form of an exponential function, $y = ab^x$.

Define Let x = number of years after 2000.

Let y = the population in millions.

Write $y = a(1.0124)^x$

$281 = a(1.0124)^0$ **To find *a*, substitute the 2000 values: $y = 281$, $x = 0$.**

$281 = a \cdot 1$ **Any number to the zero power equals 1.**

$281 = a$ **Simplify.**

$y = 281(1.0124)^x$ **Substitute *a* and *b* into $y = ab^x$.**

The function $y = 281(1.0124)^x$ models U.S. population growth.

Check Understanding

2 **a.** Predict U.S. population in 2015 to the nearest million. **about 338 million**

b. **Critical Thinking** Explain why the model and your prediction may not be valid for 2015. **The growth factor may change.**

c. Suppose the rate of population increase changes to 1.4%. Write a function to model population growth and use it to predict the 2015 population to the nearest million. **$y = 281(1.014)^x$; about 346 million**

You can write an exponential function from two points on the function's graph.

3 EXAMPLE Writing an Exponential Function

Write an exponential function $y = ab^x$ for a graph that includes (2, 2) and (3, 4).

$y = ab^x$ Use the general form.

$2 = a \cdot b^2$ Substitute for x and y using (2, 2).

$\frac{2}{b^2} = a$ Solve for a.

$y = ab^x$ Use the general form.

$4 = \frac{2}{b^2}b^3$ Substitute for x and y using (3, 4) and for a using $\frac{2}{b^2}$.

$4 = 2b^{3-2}$ Division Property of Exponents

$4 = 2b$ Simplify.

$b = 2$ Solve for b.

$a = \frac{2}{b^2}$ Use your equation for a.

$a = \frac{2}{2^2}$ Substitute 2 for b.

$a = \frac{1}{2}$ Simplify.

$y = \frac{1}{2} \cdot 2^x$ Substitute $\frac{1}{2}$ for a and 2 for b in $y = ab^x$.

The exponential function for a graph that includes (2, 2) and (3, 4) is $y = \frac{1}{2} \cdot 2^x$.

Check Understanding 3 Write an exponential function $y = ab^x$ for a graph that includes (2, 4) and (3, 16). $y = 0.25(4)^x$

OBJECTIVE 2 Exponential Decay

Real-World Connection

On a day in March, the NCAA announces the selection of 64 teams for its annual tournament.

4. No; the change from one point to the next is not constant.

Investigation: Tournament Play

The National Collegiate Athletic Association (NCAA) holds an annual basketball tournament. The top 64 teams in Division I are invited to play each spring. When a team loses, it is out of the tournament.

1. How many teams are left in the tournament after the first round of basketball games? 32 teams

2. a. Copy, complete, and extend the table until only one team is left. a–b. See margin.

   b. Graph the points from your table on graph paper.

After Round x	Number of Teams Left in Tournament (y)
0	64
1	■
2	■

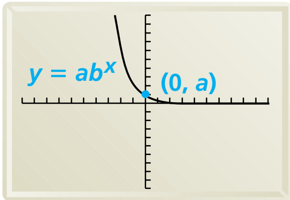
3. How many rounds are played in the tournament? 6 rounds

4. Does the graph represent a linear function? Explain.

5. How does the number of teams left in each round compare to the number of teams in the previous round? There are half as many teams as in the previous round.

An exponential function can be used to model decay when $0 < b < 1$. When $b < 1$, b is a **decay factor.**

Exponential Decay



Decay factor $b < 1$

4 EXAMPLE Analyzing a Function

Without graphing, determine whether the function $y = 14(0.95)^x$ represents exponential growth or exponential decay.

In $y = 14(0.95)^x$, $b = 0.95$. Since $b < 1$, the function represents exponential decay.

Check Understanding 4 Without graphing, determine whether each function represents exponential growth or exponential decay.

a. $y = 100(0.12)^x$ exponential decay **b.** $y = 0.2(5)^x$ exponential growth **c.** $y = 16\left(\frac{1}{2}\right)^x$ exponential decay

Reading Math

In the word asymptote, *asym* means "not together."

An **asymptote** is a line that a graph approaches as x or y increases in absolute value.

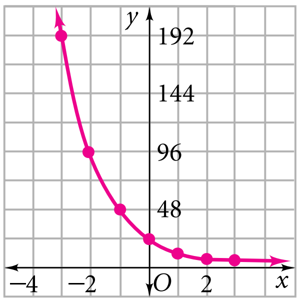
5 EXAMPLE Graphing Exponential Decay

Graph $y = 24\left(\frac{1}{2}\right)^x$. Identify the horizontal asymptote.

Step 1 Make a table of values.

x	–3	–2	–1	0	1	2	3
y	192	96	48	24	12	6	3

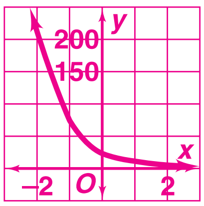
Step 2 Graph the coordinates. Connect the points with a smooth curve.



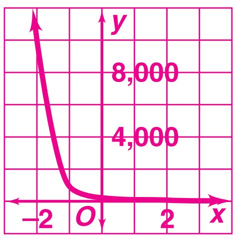
As x increases, y approaches 0.

The asymptote is the x-axis, $y = 0$.

5a.



b.



Check Understanding 5 Graph each decay function. Identify the horizontal asymptote. For graphs, see left.

a. $y = 24\left(\frac{1}{3}\right)^x$ $y = 0$ **b.** $y = 100(0.1)^x$ $y = 0$

Reading Math

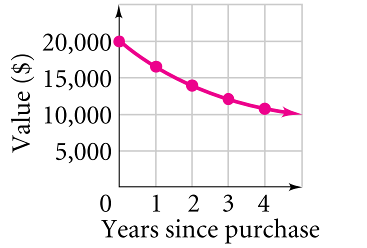
Depreciate comes from a prefix meaning "lower" and a root meaning "price."

Depreciation is the decline in an item's value resulting from age or wear. When an item loses about the same percent of its value each year, you can use an exponential function to model the depreciation.

6 EXAMPLE **Real-World Connection**

Depreciation The exponential decay graph shows the expected depreciation for a car over four years. Estimate the value of the car after six years.

Expected Decrease in Value



The decay factor b equals $1 + r$, where r is the annual rate of decrease. The initial value of the car is \$20,000. After one year the value of the car is about \$17,000.

$r = \dfrac{\text{final value} - \text{initial value}}{\text{initial value}}$	**Write an equation for *r*.**
$= \dfrac{17{,}000 - 20{,}000}{20{,}000}$	**Substitute.**
$= -0.15$	**Simplify.**
$b = 1 + r$	**Use *r* to find *b*.**
$= 1 + (-0.15) = 0.85$	**Simplify.**

Write a function, and then evaluate it for $x = 6$.

Relate The value of the car decreases exponentially; $b = 0.85$.

Define Let x = number of years. Let y = value of the car.

Write $y = ab^x$

$20{,}000 = a(0.85)^0$	**Substitute using (0, 20,000).**
$20{,}000 = a$	**Solve for *a*.**
$y = 20{,}000(0.85)^x$	**Substitute *a* and *b* into $y = ab^x$.**
$y = 20{,}000(0.85)^6$	**Evaluate for $x = 6$.**
≈ 7542.99	**Simplify.**

The car's value after six years will be about \$7540.

Check Understanding **6** Estimate the value of the car after 10 years. **about \$3900**

EXERCISES

For more practice, see *Extra Practice.*

Practice and Problem Solving

A Practice by Example

Example 1 (page 422)

Graph each function. **1–8. See margin.**

1. $y = 6^x$ **2.** $y = 3(10)^x$ **3.** $y = 1000(2)^x$ **4.** $y = 9(3)^x$

5. $f(x) = 2(3)^x$ **6.** $s(t) = 1.5^t$ **7.** $y = 8(5)^x$ **8.** $y = 2^{2x}$

Example 2 (page 423)

9. **Population** The world population in 2000 was approximately 6.08 billion. The annual rate of increase was about 1.26%.

a. Find the growth factor for the world population. **1.0126**

b. Suppose the rate of increase continues to be 1.26%. Write a function to model world population growth. **$y = 6.08(1.0126)^x$, where $x = 0$ corresponds to 2000**