Euler Line
Guided Investigation

Teacher Notes & Answers

7 8 9 **10 11** 12

TI-Nspire™ Investigation Student 50 min

Introduction

The three triangle centres: orthocentre, centroid and circumcentre have many amazing properties, however there is one super property that connects them all! In this activity you will combine your learning from the three activities:

- Circumcentre
- Centroid
- Orthocentre

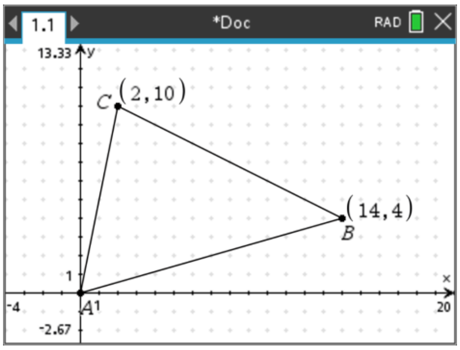
Spoiler alert: You can the QR code to watch a video and review the three centres and see what happens when all three centres are determined.

Geometry

Open a New TI-Nspire Document and insert a **Graphs Application**.

Draw a triangle with vertices:

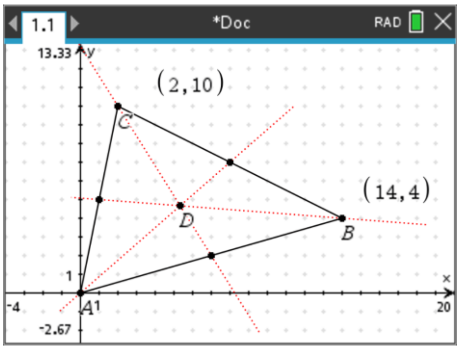
A:(0, 0) B:(14, 4) C:(2, 10)



Question: 1.

Construct and determine equation for the three median lines and hence their point of intersection. (Centroid)

Note: In the diagram shown opposite the median lines have been changed to red dotted lines using the attributes and colour options. The centroid as at point D.



Answers: $y=\frac{7}{8}x$, $y=-\frac{8}{5}x+\frac{66}{5}$, $y=-\frac{1}{13}x+\frac{66}{13}$

Point of intersection: $\left(\frac{16}{3},\frac{14}{3}\right)$

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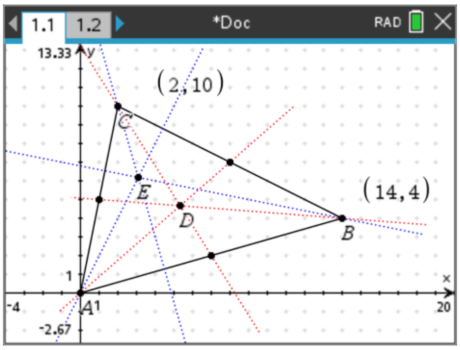
Question: 2.

Construct and determine the equation for the three altitudes and hence determine their point of intersection. (Orthocentre)

Note: In the diagram shown opposite the altitudes have been changed to blue dotted lines using the attributes and colour options. The orthocentre is at point E.



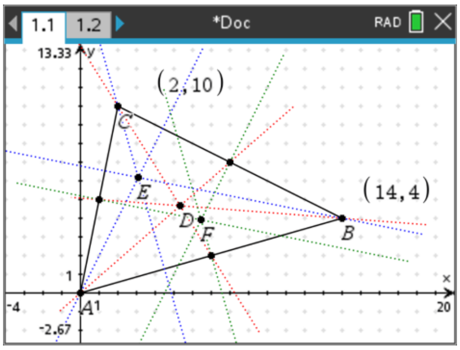
Answer: $y=-\frac{7}{2}x+17$, $y=-\frac{1}{5}x+\frac{34}{5}$, $y=2x$

Point of Intersection: $\left(\frac{34}{11},\frac{68}{11}\right)$

Question: 3.

Construct and determine the equation for the three perpendicular bisectors and hence determine their point of intersection. (Circumcentre)

Note: In the diagram shown opposite the perpendicular bisectors have been changed to green dotted lines using the attributes and colour options. The circumcentre is at point F.



Answer: $y=-\frac{7}{2}x+\frac{53}{2}$, $y=-\frac{1}{5}x+\frac{26}{5}$, $y=2x-9$

Point of intersection: $\left(\frac{71}{11},\frac{43}{11}\right)$

Question: 4.

Determine the equation to the line passing through points D (centroid) and E (orthocentre).

Answer: $y=\left(\frac{\frac{68}{11}-\frac{14}{3}}{\frac{34}{11}-\frac{16}{3}}\right)\left(x-\frac{34}{11}\right)+\frac{68}{11}$ which simplifies to: $y=-\frac{25}{37}x+\frac{306}{37}$

Question: 5.

Determine the equation to the line passing through points E (orthocentre) and F (circumcentre) and comment on the result.

Answer: $y=\left(\frac{\frac{68}{11}-\frac{43}{11}}{\frac{34}{11}-\frac{71}{11}}\right)\left(x-\frac{34}{11}\right)+\frac{68}{11}$ which simplifies to: $y=-\frac{25}{37}x+\frac{306}{37}$

Comment: The two lines are the same, the points D, E & F are collinear.

Question: 6.

The triangle vertices are dynamic. Explore what happens to points D, E and F when the original triangle vertices are changed.

Answer: Points D, E & F remain collinear. [This is the Euler line.]

Question: 7.

Change vertex C on the original triangle to: (5, 9). Describe what happens to points D, E & F.

Answer: The points remain in a straight line [Euler line] and the line passes through C. This is because AC and BC are at right angles, therefore the altitude through A and B intersect at C.

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Question: 8.

Let m_1 = Gradient of line AB; m_2 = Gradient of line BC and m_3 = Gradient of line AC.

Determine the following: $-\dfrac{m_1m_2+m_2m_3+m_1m_3+3}{m_1+m_2+m_3+3m_1m_2m_3}$

Answer: $m_1=\dfrac{2}{7}$, $m_2=-\dfrac{1}{2}$ and $m_3=5$. $m_E=-\dfrac{\frac{2}{7}\times\frac{-1}{2}+\frac{-1}{2}\times5+\frac{2}{7}\times5+3}{\frac{2}{7}-\frac{1}{2}+5-3\times\frac{2\times1\times5}{7\times2}}=-\dfrac{25}{37}$

Note that this is the same as the gradient of the Euler line.

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