Texas Instruments Activity #10
Title: Exploring Taylor's Integrals
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Estimated Time: 40-50 Minutes

NCTM Standards:

Problem Solving Standard – Solve problems that arise in mathematics and other contexts.

Algebra Standard – Understand patterns, relations, and functions. Approximate and interpret rates of change from graphical and numerical data. Understand and compare the properties of classes of functions.

Topics in Calculus:

Integration, Taylor Series, Derivatives

Overview:

In this activity, the students will discover the use of Taylor's polynomials to integrate functions that are not easily integrated by hand. The students discover this by completing a series of calculations that vary from integrating to finding a Taylor polynomial.

Supplies: TI-89 Graphing Calculator

Name: ____ Date: _____________________________

EXPLORING TAYLOR'S INTEGRALS

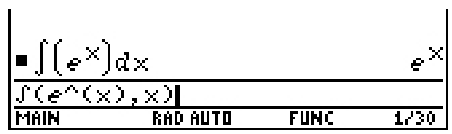
Taylor's Series is an interesting mathematical phenomenon that can help us integrate functions that normally cannot easily be integrated. A fascinating fact is that although this series was named after Brook Taylor (1685-1731), he did not discover the series. Early in Taylor's life, another mathematician, James Gregory actually discovered the theorem. However, Taylor rediscovered this series and published without knowing about its previous discovery. The series became popular when mathematician Colin Maclaurin cited Taylor's work in his book. In this activity, you will discover the magic of Taylor's series and how it applies to Calculus.

Expand the Taylor polynomial for $f(x) = e^x$ by three terms. Press [F3] [9] to select 9:Taylor(, then enter the function, followed by [,][X][,] *order* [)][ENTER]. The order is the number of terms you wish to evaluate.

Write the answer here: **____**

Now for each of the functions listed below, follow the subsequent steps and answer the questions. Write all of your answers in the spaces provided.

STEP ONE: Find the Taylor polynomial for the function. To do this press [F3] [9] to select 9:Taylor(, then enter the function, followed by [,][X][,] *order* [)][ENTER]. The order is the number of terms you wish to evaluate. Use the order asked for below.



STEP TWO: Integrate the function using the TI-89 graphing calculator. Press [2nd][∫] followed by the function, then press [,][X][)][ENTER].

STEP THREE: Now, differentiate the function using the TI-89 graphing calculator. Press [2nd][*d*] followed by the function, then press [,][X][)][ENTER].

STEP FOUR: Find the Taylor polynomial of the integrated function from STEP TWO. Use the order listed minus one. (If the given order was five, then the order for this step would be four.)

STEP FIVE: Integrate the original Taylor polynomial. Press [2nd][∫][F3][9]then enter the function, followed by [,][X][,] *order* [)][,][X][)][ENTER]. Use the same order as in STEP FOUR. The expression should look similar to this:

$$\int (\,taylor(\;expression,x,order\;)\;,x\;)$$

STEP SIX: Similarly, differentiate the original Taylor polynomial. Press [2nd][*d*][F3][9] then enter the function, followed by [,][X][,] *order* [)][,][X][)][ENTER]. Use the order listed plus one.

STEP SEVEN: Now, find the Taylor polynomial of the derivative from STEP THREE.

1

Function: $f(x) = 1/x$	Taylor Polynomial (STEP 1):	Integration of F(x) (STEP 2):	Derivative of F(x) (STEP 3):
Taylor Polynomial from the Integral (STEP 4)		Taylor Polynomial from the Derivative (STEP 7)	
Integral of the Taylor Polynomial (STEP 5)		Derivative of the Taylor Polynomials (STEP 6)	

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Function: $g(x) = e^{2x}$	Taylor Polynomial (STEP 1):	Integration of F(x) (STEP 2):	Derivative of F(x) (STEP 3):
Taylor Polynomial from the Integral (STEP 4)		Taylor Polynomial from the Derivative (STEP 7)	
Integral of the Taylor Polynomial (STEP 5)		Derivative of the Taylor Polynomials (STEP 6)	

3

Function: $h(x) = \tan^{-1}(2x)$	Taylor Polynomial (STEP 1):	Integration of F(x) (STEP 2):	Derivative of F(x) (STEP 3):
Taylor Polynomial from the Integral (STEP 4)		Taylor Polynomial from the Derivative (STEP 7)	
Integral of the Taylor Polynomial (STEP 5)		Derivative of the Taylor Polynomials (STEP 6)	

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Function: $j(x) = \cos(x)$	Taylor Polynomial (STEP 1):	Integration of F(x) (STEP 2):	Derivative of F(x) (STEP 3):
Taylor Polynomial from the Integral (STEP 4)		Taylor Polynomial from the Derivative (STEP 7)	
Integral of the Taylor Polynomial (STEP 5)		Derivative of the Taylor Polynomials (STEP 6)	

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Function: $k(x) = 1/(x^2 + 1)$	Taylor Polynomial (STEP 1):	Integration of F(x) (STEP 2):	Derivative of F(x) (STEP 3):
Taylor Polynomial from the Integral (STEP 4)		Taylor Polynomial from the Derivative (STEP 7)	
Integral of the Taylor Polynomial (STEP 5)		Derivative of the Taylor Polynomials (STEP 6)	

What do you notice about the solutions from STEP FOUR and STEP FIVE? Are they the same or different? How?

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Does this property always hold? ____

What do you notice about the solutions from STEP SEVEN and STEP SIX?   Are they the same or different? How?

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Does this property always hold? ____

In the steps on the first page, why do you need to subtract one from the order when integrating and add one to the order when differentiating?

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How can you use this method for integrating functions?   What advantages would this method give you?

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Using STEP ONE and STEP FIVE find a function that cannot be easily integrated, except by using this new method. Below, write the function, the integral, the orders, and the Taylor series used.