TI-nspire™

Grade level: 9-12
Subject: Geometry
Time required: 20-30 minutes

Minimizing Surface Area of a Cylinder With a Fixed Volume

by – Eric Prowse

Activity overview

Students will discover the relationship between radius and height of a cylinder so that surface area of a cylinder can be minimized while maintaining a fixed volume. This is just an introduction to a project that they will begin after this investigation. Once this is completed they will redesign the packaging of a cylindrical can so that the least amount of material is used to hold the specified volume. Notes regarding the project are not included.

Concepts

G1.8.1 (Michigan High School Content Expectations) Solve multi-step problems involving surface area and volume of cylinders.

Teacher preparation

Students will need to be familiar with the surface area and volume formulas for a cylinder. They should also have some basic understanding on how to solve systems of equations by substitution. Students will need to bring in something packaged in a cylindrical shape (can of soup, vegetables, etc).

Classroom management tips

The first part of this activity is mostly teacher centered. The second part will require students to work independently or in groups.

TI-Nspire Applications

Cylinders.tns

Step-by-step directions (Teacher only, students will have a different handout)

NOTE: Teachers should work on this activity with the students. It is assumed that students will already have a working knowledge of the Nspire, but it is not necessary. Teachers can guide students by doing the activity on the overhead projector. It just may take more time if students are not familiar with the basic operations.

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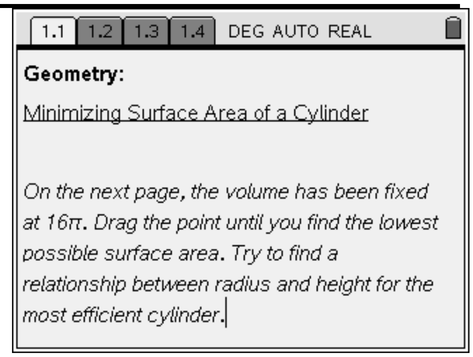
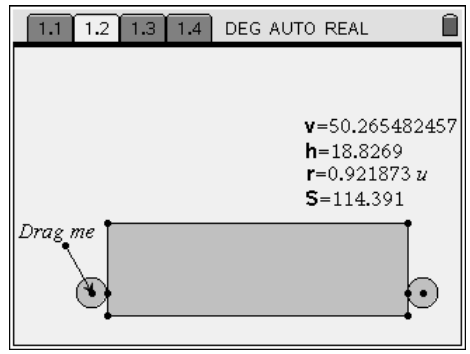
1. Present Scenario

Have students open up the Cylinders.tns file and have thenm read the page. Explain that the goal of the activity is to find the lowest surface area for a fixed volume. It might help to bring in various containers that hold the same volume, but use different amounts of surface area to create the packaging.



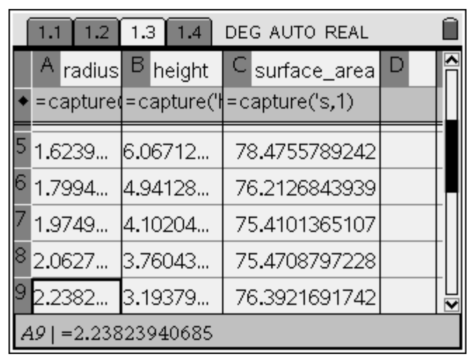
2. Explore Minimum Surface Area for a Fixed Volume

In this window, discuss the variables listed, and how V is actually 16π. Then have students work on problem 1 of the student handout. They are to click and drag the point so that S decreases to a minimum value, and then record their results.



3. Examine Tabular Results

Once the results are recorded, have students turn to page 1.3 on the Nspire. The table automatically records the different values of radius, height, and surface area as the point was dragged in previous step. From this table they will need to predict the relationship between radius and height that will give the minimal surface area for a fixed volume. Have them record their prediction on problem 2 of the student handout.

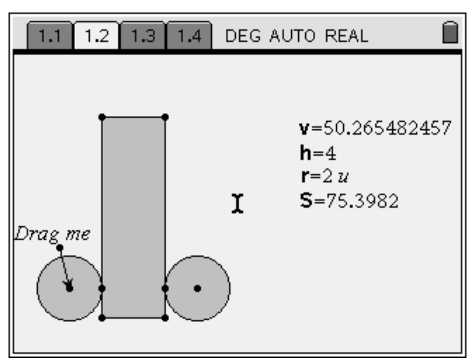


	A radius	B height	C surface_area	D
◆	=capture(	=capture('l	=capture('s,1)	
5	1.6239...	6.06712...	78.4755789242	
6	1.7994...	4.94128...	76.2126843939	
7	1.9749...	4.10204...	75.4101365107	
8	2.0627...	3.76043...	75.4708797228	
9	2.2382...	3.19379...	76.3921691742	

A9 | =2.23823940685

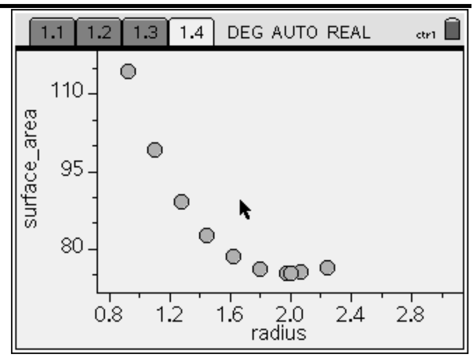
4. Verify Prediction on Diagram

Now students will predict what the values of r and h should be in this problem that will give the lowest surface area. They should do this and write the prediction on problem 3 of the student handout. Students can then turn back to 1.2, and double click on the value for r and edit it to their predicted value (r=2). Have them record the value of r, h, and S in problem 4 of the student handout, and discuss whether their predictions were successful.



5. Verify Prediction on Scatter Plot

This part is just for teacher demonstration, and if all the students have not correctly predicted the relationship, then this should be point where you have the other students explain their findings. Here, students can see how surface area varies with changing radius, and how the minimum surface area occurs at r=2.



6. Model How to Find Radius and Height for Minimum Surface Area for a Fixed Volume Algebraically.

From here you might want to demonstrate how this solution could be found algebraically, and have them fill out problem 5 of the student handout. Knowing that 2r = h for the most "efficient" cylinder, they can use that fact to find the values of r and h for the given volume:

$$2r = h$$
$$V = r^2\pi h = r^2\pi(2r) = 2r^3\pi$$

$$For \quad V = 64\pi,$$
$$64\pi = 2r^3\pi$$

From there demonstrate how r=2, and h=4 for the lowest surface area.

7. Students Redesign Cylinder

Now students may begin their project (this is separate from this activity, and is not included). They are to redesign the cylinder that they brought in so that it uses the least amount of surface area to hold its volume.

Assessment and evaluation

- *Students are assessed by the successful completion of their group design of the cylinder that they brought in. This portion of the activity is part of a whole project, and can either be graded separately or with the project.*

Activity extensions

- *The same activity is done for a rectangular prism before this project is assigned. Eventually we discuss that for a fixed volume that a sphere will result in the lowest surface area of all space figures.*

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Student Handout Name: ______________________________

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Before we begin, open up the document *Cylinders on the TI-Nspire*. Read the first page before continuing.

1.) On page 1.2 drag the point so that surface area decreases to the lowest possible value. Write down the values of r, h, and S that gave you the lowest surface area.

r = _________

h = _________

S = _________

2.) Turn to page 1.3 to view a table of values of r and h as the surface area decreased. What do you predict is the relationship between the radius and height of cylinder so that you minimize surface area for a fixed volume?

3.) For this problem, what do you guess are the exact values of r and h to be so that surface area is minimized?

r = _________

h = _________

4.) Go back to page 1.2 and change the value of r to your predicted value. What is the value of S? What is the value of h? Is your predicted relationship correct from number 2? If not, explain the correct relationship.

r = _________

h = _________

S = _________

ONE MORE PROBLEM ON THE BACK!

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5.) Now that you know the relationship between radius and height for the most "efficient" cylinder, demonstrate algebraically how this could've been solved. You will need to know how to do this before you begin your group project.