Watch Your P's and Q's

ID: 8517

Time required
30 minutes

Activity Overview

Students will use the Rational Zero Theorem to find all rational zeros of a polynomial.

Topic: Polynomials & Polynomial Equations

- *Divide one polynomial by another to obtain a quotient and a remainder.*
- *Prove and apply the Remainder Theorem and the Factor Theorem.*
- *Approximate the real roots of a polynomial equation by graphing and identifying the number of real roots.*

Teacher Preparation and Notes

- *Students should have already begun to observe graphs of quadratic functions. Students should also be able to use the quadratic formula and synthetic division to solve polynomial equations.*
- *Remind students that the zeros of a polynomial are those values for which the polynomial is equal to zero (f(x) = 0).*
- *This activity is intended to be **teacher-led** with students in **small groups**. You should seat your students in pairs so they can work cooperatively on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds, although the majority of the ideas and concepts are only presented in **this** document; be sure to cover all the material necessary for students' total comprehension.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student .tns file and student worksheet, go to education.ti.com/exchange and enter "8517" in the quick search box.***

Associated Materials

- *WatchYourPsandQs_Student.doc*
- *WatchYourPsandQs.tns*

Suggested Related Activities

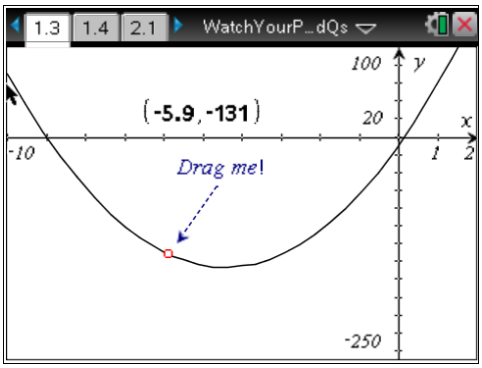
To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *One of the Many Ways (TI-Nspire technology) — 11885*
- *It's A Rat Race For The Zeros (TI-Nspire technology) — 11687*
- *Asymptotes and Zeros (TI-Nspire technology) — 9286*

This investigation guides the students through using the Rational Zero Theorem by way of the *Lists & Spreadsheet* application to find the rational zeros of a polynomial.

Problem 1 – Zeros of a parabola

On page 1.3, have students move the point on the graph to find all zeros of the polynomial, recording their results. When the point has reached a zero, the word "zero" appears. Remind students that ***zeros*** of a polynomial are the ***x-values*** when the *y*-values are zero. The location (*x*, 0) is referred to as the *x-intercept*.



- zeros at $x = -9$ and 0.1

Students should notice that one of these zeros is not exact. This is a limitation of finding results graphically. To find the exact value of this zero (if it is rational), the Rational Zero Theorem must be applied. Explain to students how the theorem is used to find the potential zeros.

TI-Nspire Navigator Opportunity: *Screen Capture*

See Note 1 at the end of this lesson.

The ***Rational Zero Theorem*** states that all <u>*potential*</u> rational zeros of a polynomial are of the form $\frac{P}{Q}$, where *P* represents all positive and negative factors of the *last* term of the polynomial and *Q* represents all positive and negative factors of the *first* term of the polynomial.

For this polynomial, $7x^2 + 62x - 9$, the possible rational zeros are:

$$\frac{P}{Q} = \frac{\pm 1, \pm 3, \pm 9}{\pm 1, \pm 7} = \pm 1, \pm 3, \pm 9, \pm \frac{1}{7}, \pm \frac{3}{7}, \pm \frac{9}{7}$$

TI-Nspire Navigator Opportunity: *Quick Poll*

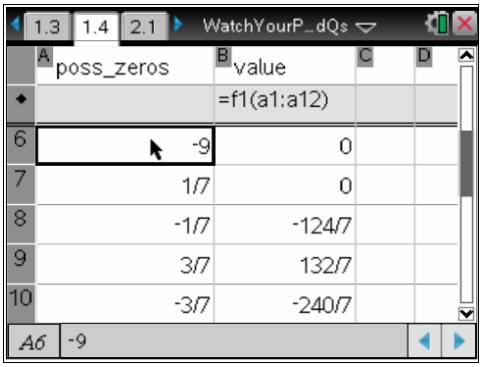
See Note 2 at the end of this lesson.

On page 1.4, possible zeros have been entered into the first column of the spreadsheet.

To evaluate the polynomial from the previous page for each value, have students enter the formula, **f1**(a1:a12), into the formula cell (gray) for Column B.

Remind students again that the zeros are those values for which the polynomial equals zero.

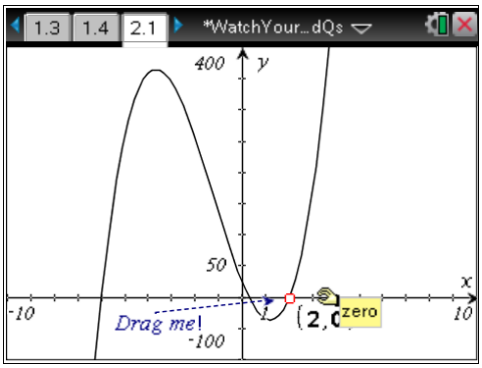
- zeros at $x = -9$ and 1/7



Problem 2 – Zeros of a cubic function

On page 2.1, the students will once again find the zeros by dragging the point on the graph.

- zeros at $x = -6$, 0.3, and 2



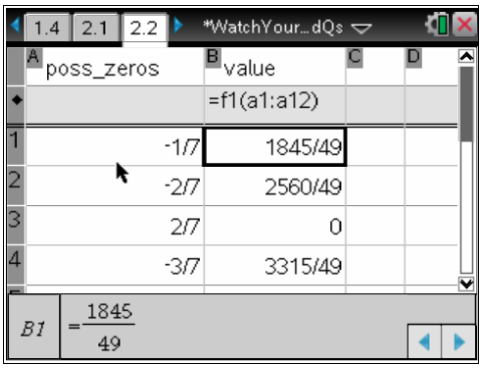
Students may use the spreadsheet on page 2.2 to find the exact rational zeros by first identifying P and Q and then determining all 32 values of $\frac{P}{Q}$.

The possible zeros should be entered into Column A. These should be as follows:

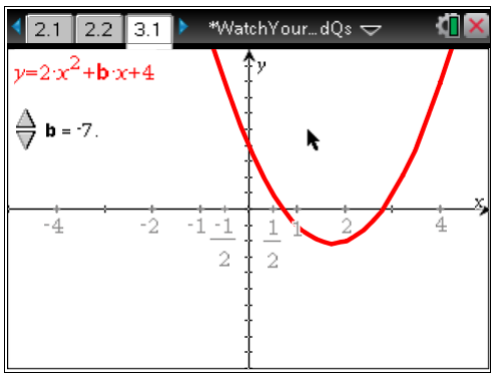
{±1, ±2, ±3, ±4, ±6, ±8, ±12, ±24, ±1/7, ±2/7, ±3/7, ±4/7, ±6/7, ±8/7, ±12/7, ±24/7}

The formula to be entered into Column B is **f1**(a1:a32).

- zeros at $x = -6$, 2/7, and 2



Problem 3 – A graphical look at the possible zeros of a parabola



- Click the slider to change the value of b (which changes the parabola). How is it possible that the graph sometimes has zeros that don't cross at any of the possible rational zeros that are labeled on the graph?

Answer: Sometimes the graph crosses at irrational zeros. The students may not make the connection that the rational zero theorem is only good for finding possible rational zeros.

Exercises

The worksheet exercises are designed to test the understanding of the Rational Zero Theorem. Have students complete the remaining exercises individually or in small groups. A graph and spreadsheet for student use are provided on pages 4.1 and 4.2.

Student Worksheet Solutions

1. 2
2. Use synthetic division with the known zero of 2. This would reduce the polynomial to a quadratic which can be solved using the quadratic formula.

   The remaining zeros are then $\frac{-5 \pm \sqrt{265}}{20}$.
3. Yes; yes. For example, the polynomial $f(x) = x^2 - 5$ has only irrational roots, and the polynomial $f(x) = x^2 + 5$ never intersects the x-axis and has no [real] roots.
4. 18.8 seconds

TI-Nspire Navigator Opportunities

Note 1

Problem 1, *Screen Capture*

This would be a good place to do a screen capture to verify students are able to grab and drag the point and find all the zeros.

Note 2

Problem 2, *Quick Poll*

You may choose to use Quick Poll throughout the activity to assess student understanding. The worksheet questions can be used as a guide for possible questions to ask. In Problem 1, you may ask students to list the zeros of the graph on page 1.3, or to list several of the rational zeros possible using the Rational Zero Theorem.