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| **Math Objectives*** Students will determine the domain of rational functions.
* Students will use algebraic concepts to determine the vertical asymptotes of a rational function.
* Students will determine the removable discontinuities of a rational function.
* Students will describe the graph of a rational function given the equation.
* Students will use appropriate tools strategically (CCSS Mathematical Practice).

**Vocabulary*** rational function • domain • removable discontinuity
* factors • zeros

**About the Lesson*** This lesson involves observing how changing the values in a rational function affects the continuity of the graph of the function.
* As a result, students will:
* Manipulate the factors of the numerator and denominator to observe the effects of changes in the factors.
* Explain how the values in a rational function determine the vertical asymptotes.
* Identify the conditions that must be met for a rational function to have a removable discontinuity.

**Teacher Preparation and Notes**.* This activity is done with the use of the TI-84 family as an aid to the problems.

**Activity Materials*** Compatible TI Technologies: TI-84 Plus\*, TI-84 Plus Silver Edition\*, TI-84 Plus C Silver Edition, TI-84 Plus CE

 *\* with the latest operating system (2.55MP) featuring MathPrintTM  functionality.* | **Tech Tips:*** This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
* Watch for additional Tech Tips throughout the activity for the specific technology you are using.
* Access free tutorials at <http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials>

**Lesson Files:***Student Activity*Exploring Vertical Asymptotes\_84CE\_Student.pdfExploring Vertical Asymptotes\_84CE\_Student.doc |

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| Given the equation of a rational function, will you always be able to determine the domain? In this activity, you will explore vertical asymptotes and removable discontinuities using the **Transformation Graphing App** on the handheld. |  |

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| **Problem 1**To turn on the **Transformation Graphing** app, press **apps**, **:Transfrm**, and press any key. Press $y=$ and in $Y\_{1}$, type in the equation $Y\_{1}= \frac{C}{(X-A)(X-B)}$. |
| 1. Use the up/down arrows to change between the values of *A, B, and C*. Use the left/right arrows to change each individual value. Change the value of *A.* Describe how the graph changes.**Sample answers:** One of the vertical asymptotes moves. The other vertical asymptote stays the same. When *A* = *B*, there is only one vertical asymptote. The forms of the curves bounded by the asymptotes dilate but do not reflect.  |
| 2. Change the value of *B*. Describe how the graph changes.**Sample answers:** The other vertical asymptote moves, while the first vertical asymptote stays the same. When *A* = *B*, there is only one vertical asymptote. The forms of the curves bounded by the asymptotes dilate but do not reflect.  |
| 3. What do the values of *A* and *B* represent in the function?**Answer:** *A* and *B* are the zeros of the denominator or the values of x at which the function is undefined.  |
| 4. What are the equations of the vertical asymptotes?**Answer:** *x* = *A* and *x* = *B*. When *A* = *B*, there is only one asymptote, *x* = *A* = *B*.   |
| 5. State the domain of the function in terms of *A*, *B*, and *C*. **Answer:** $\left(-\infty , A\right)∪\left(A, B\right)∪(B, \infty )$ when $A<B$, $\left(-\infty , B\right)∪\left(B, A\right)∪(A, \infty )$ when $B<A$,  or $\left(-\infty , A\right)∪(A, \infty )$ when $A=B$ |
| 6. Change the value of *C*. How does changing *C* affect the domain?**Answer:** Changing *C* only dilates the curves bounded by the asymptotes. It does not move the asymptotes. Therefore, the domain is not affected.  |

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| 7. Describe how you could find the vertical asymptotes for any rational function with a constant numerator. **Answer:** Factor the denominator. Solve the denominator to find the zeros of the  denominator.**Teacher Tip:** Removable discontinuity will be addressed in the next section. Students could discuss the need for the “with a constant in the numerator” qualifier.  |
| **Problem 2** |
| 8. For problem 2, type the following equation into $Y\_{1}$, $Y\_{1}= \frac{(X-A)(x-B)}{(X-C)}$. Using the arrows, set  *A* = 2 and *B* = –1, and then change the value of *C*. For which values of *C* are there no asymptotes? Explain why there are no asymptotes for these values of *C*. |

**Answer:** When *C* = *B* = –1 and *C* = *A* = 2, there is no asymptote. The graph looks like a line with a “hole” at *x* = *C* = *A* or *x* = *C* = *B*.

When *A* = *C*, the factor (*x – A*) in the numerator reduces with the factor (*x* – *C*) in the denominator, leaving the graph *y* = *x* – *B*. When *B* = *C*, the factor (*x* – *B*) in the numerator reduces with the factor (*x* – *C*) in the denominator, leaving the graph *y = x* – *A*.

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|  9. The “hole” in the graph is called a removable discontinuity. Explain why the hole exists and how you might remove it by modifying the function definition.**Sample answer:** The domain of the function is $\left(-\infty , C\right) ∪(C, \infty )$. The value of *C* will always be undefined regardless of the values of *A* and *B*. When the value of *C* does not equal either *A* or *B*, there is an asymptote at *x* = *C*. However, when *C* equals either *A* or *B*, there is only a hole. If that hole was removed, the graph would be continuous. Suppose that there is a hole at x = C. If the function definition is modified so that f(C) is defined to equal the limit of the function as x approaches C, the discontinuity is removed. For example, in question 10, there is a hole at x = -6. Since the limit of the function as x approaches -6 is -9, modifying the function definition to include f(-6) = -9 will remove the discontinuity.

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| **Extension:** This could be a nice place to discuss limit notation with a hole. $\lim\_{x\to -6}f\left(x\right)= -9$. |

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| 10. Answer the following question:The function $f\left(x\right)= \frac{(x+6)(x-3)}{(x+6)}$ has1. an asymptote at x = -6 (b) a removable discontinuity at x = -6

 (c) an asymptote at x = 6 (d) a removable discontinuity at x = 6 (e) continuity  |
|  **Answer:** (b) a removable discontinuity at x = -6 due to the common factors of (x + 6)  in the numerator and the denominator.**Problem 3** |
| 11. For problem 3, type the following equation into $Y\_{1}$, $Y\_{1}= \frac{(X-A)}{(X-B)(X-C)}$. Using the arrows, set *B* = –1 and  *C = 4*, and then change the value of *A*.  a. Describe how the graph changes as the value of *A* changes.**Sample answer:** The graph bounded by the two asymptotes *x* = –1 and *x* = 4 dilates, but the asymptotes do not move. For certain values of *A*, there is only one asymptote. b. What is the domain of the function in terms of *A*, *B*, and *C*?**Answer:** $\left(-\infty , B\right)∪\left(B, C\right)∪(C, \infty )$ when $B<C$ .  **Teacher Tip:** It is important to note that the domain would be $\left(-\infty , C\right)∪\left(C, B\right)∪(B, \infty )$ when $C<B$ or $\left(-\infty , B\right)∪(C, \infty )$ when $B=C$. c. For which values of *A* is there only one asymptote? Describe the graph at these values.**Answer:** When *A* = *B* = –1 or *A* = *C* = 4, there is only one asymptote. The graph looks like a translation of an inverse variation. There is also a hole (a removable discontinuity). d. Explain algebraically why the graph looks as it does at these points. **Answer:** When *A = B*, the factor (*x – A*) in the numerator reduces with the factor (*x – B*) in the denominator, leaving the graph $y= \frac{1}{X-C}$ . When *A* = *C*, the factor (*x* – *A*) in the numerator reduces with the factor (*x* – *C*) in the denominator, leaving the graph $y= \frac{1}{x-b}$ The domain of the function remains the same regardless of the factors that reduce. When *A* equals either *B* or *C*, there is a removable discontinuity instead of an asymptote.  |
| 12. Describe how the domain would change if you changed the values of *B* and *C*. **Answer:** The domain is $\left(-\infty , B\right)∪\left(B, C\right)∪(C, \infty )$ when $B<C$, $\left(-\infty , C\right)∪\left(C, B\right)∪(B, \infty )$  when $C<B$, or $\left(-\infty , B\right)∪(C, \infty )$ when $B=C$ **Teacher Tip:** This is a good time to revisit Question 7. Students could discuss how to find the vertical asymptotes for *any* rational function.  |
| **Move to page 3.2.**13. Answer the following question: The function $f\left(x\right)= \frac{(x-3)}{(x+6)(x-3)}$ has1. one asymptote at x = 3 (b) a removable discontinuity at x = 3

 (c) two asymptotes at x = -6 and x = 3 (d) one asymptote at x = -6 (e) continuity |

**Answer:** (b) and (d) as it has a common factor of (x – 3) in the numerator and the denominator and the factor of (x + 6) only in the denominator.

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| **Problem 4**For problem 4, type the following equation into $Y\_{1}$, $Y\_{1}= \frac{(X+1)^{A}}{(X+1)^{B}}$. Using the arrows, set *B* = –1 and *C = 4*, and then change the value of *A*. |
| 14. Answer the following questions: Holes were discussed in question 9. While manipulating *A* and *B* on your graph, what would *A* and *B* have to be for *f1(x)* to have a hole?  1. If *A* < *B*
2. If *A* = *B*
3. If *A* > *B*

 **Answers:** (b) and (c), both result in the scenario of having 1 or more common factors in the numerator and the denominator, but once reduced, there remains zero or more factors in the numerator and none in the denominator.  What would *A* and *B* need to be to have a vertical asymptote? 1. If *A* < *B*
2. If *A* = *B*
3. If *A* > *B*

 **Answer:** (a) Even though common factors may be reduced, this scenario leaves one or more factors in the denominator, resulting in a vertical asymptote and not a hole. |

# Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to understand:

  How to determine the vertical asymptotes of a rational function.

  What conditions must be true for a rational function to have a removable discontinuity.