NUMB3RS Activity: That’s So Random
Episode: “Double Down”

Topic: Randomization **Grade Level:** 8 - 12
Objective: To appreciate the difficulty of constructing random sequences and the mathematical importance of doing so
Time: About 30 minutes
Materials: A deck of 52 playing cards, a fair coin, and a graphing calculator.

Introduction

In “Double Down,” three college students attempt to get an edge in a casino card game by analyzing the mechanical randomization process used to shuffle the cards. As Charlie explains, the auto-shuffler relies on an algorithm to deal the cards in random order. The students are able to crack the algorithm and thus predict the order in which the cards are dealt.

This illustrates two significant observations about randomization in the real world: (1) there are important uses for randomness, and (2) when human minds attempt to produce randomness, the results are not really random. In this activity we explore why randomness is important, what randomness looks like, and why randomness is so hard to produce.

Discuss with Students

Before doing this activity, you may want to review permutations and simple probabilities. For example, there are 3! = 6 possible arrangements of the letters A, B, and C: ABC, ACB, BAC, BCA, CAB, and CBA. If the arrangements are equally likely, the probability of randomly choosing *any* one of these arrangements is therefore $\frac{1}{6}$. While students might think that the arrangement ABC is “less random” than an arrangement such as BCA, it is important to understand that this does not make it less likely to result from a “random” ordering of the letters. This activity will explore the implications of randomness in several contexts, including randomly dealt cards and lottery numbers.

Student Page Answers: **1.** *Option (C) has a more even mix of black and red cards, so it may look more random than the others. However, all three sequences are equally likely to occur.* **2.** *Answers will vary but there should be 26 R’s and 26 B’s.* **3.** *Answers will vary. Students might be surprised by longer “runs” of black or red than they expected to see.* **4.** *Both sequences are 3.141593, which is pi rounded to seven digits* **5.** *All three tickets have an equal chance of winning.* **6.** *Answers will vary.* **7.** *Answers will vary. (If you want to have a little fun with the class, secretly enter the command “1→rand” on two of their calculators before they run the simulation. Because their calculators will be generating pseudo-random numbers starting with the same seed, they will have the exact same sequence of 1’s and 2’s.)*
8. *Answers will vary. (Note: Be sure students are sampling without replacement; otherwise the sequence will not end at 52 cards.)*

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Name: ________________________________ Date: _______________

NUMB3RS Activity: That’s So Random

In “Double Down,” three college students figure out how to get an edge in a casino card game by predicting how the auto-shuffling machine deals the cards. Because the purpose of the auto-shuffler is to deliver the cards in a *random* order, you might think it is strange that anyone could predict that order. In a deck of 52 cards, any one of the 52! possible arrangements of the cards is equally likely to result from a random shuffle.

How could a random shuffle be predicted? The students knew that the machine had to be *programmed* to deliver the cards in a certain way, so it had to use an *algorithm* (a process designed by the programmer). Although the machine could be mathematically designed to *appear* random, the very fact that it used an algorithm made it vulnerable to prediction.

Thinking About Randomness

1. Suppose a standard deck of 52 cards is shuffled, and then dealt from the top, recording which cards are black (B) and which are red (R). Assume that the shuffling has been random. Of the following three arrangements, which is the most likely to occur?

(A) RRRRRRRRRRRRRRRRRRRRRRRRRRBBBBBBBBBBBBBBBBBBBBBBBBBB
(B) RBRBRBRBRBRBRBRBRBRBRBRBRBRBRBRBRBRBRBRBRBRBRBRBRBRB
(C) BBBRRRBRRRRRRRRRBBBBRRRRRBRBBBBBRRRRBBBBBBBBBRBBBRRR

2. Using only your intuition, write a sequence of 52 B’s and R’s that you think might result from cards dealt at random from a shuffled deck.

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3. Shuffle a deck of cards very well, deal the cards from the top of the deck, and record the sequence of the 52 B’s and R’s that results. What surprises do you see in this sequence?

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4. In the slots below, write down the lengths of the seven “runs” of consecutive B's as they occur in order from left to right in shuffle (C) of Question #1. Then do the same for the R's, but count the runs from right to left.

B’s ___ . ___ ___ ___ ___ ___ ___ **R’s** ___ . ___ ___ ___ ___ ___ ___

Do you recognize the number?

If working through Question #4 made you feel that shuffle (C) was less random than you thought, don’t worry. Just realize that detecting the pattern makes the shuffle *no less likely to occur than you originally thought*. All three shuffles in Question #1 and the shuffles you wrote in Questions #2 and #3 are equally likely to occur in a random shuffle. Statisticians do not trust their intuition to create randomness because a human mind attempting to create randomness is far more likely to create some sequences than others. In fact, savvy statisticians can often detect bogus “random” data that people have attempted to invent on their own!

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5. Suppose a lottery ticket consists of six numbers from 1 to 44. The six winning numbers are chosen randomly. Which of the following tickets has the best chance of winning?

(A) 11 12 13 14 15 16
(B) 09 13 22 25 31 43
(C) 17 21 33 34 38 41

Did you recognize that each of the lottery tickets in Question #5 has the same chance of winning? Most people would avoid buying a ticket like (A) because it is not "random" enough – they realize that the odds against the ticket winning are very great. In fact, these people are right. What they might not realize is that the odds are *equally* great against either (B) or (C) winning – the odds against *any* ticket winning are 7,059,051 to 1. When it comes to randomness, intuition can be a dangerous thing to trust.

How You Can Get Random

Let us see how you might construct a random sequence of 26 B's and 26 R's as you were asked to do in Question #3.

6. Separate a deck of cards into a pile of black cards and a pile of red cards. Then flip a fair coin repeatedly to form your sequence of cards; if the coin lands on heads, choose a black card, and if the coin lands on tails, choose a red card. When one of the piles runs out of cards, finish the sequence with the remaining cards in the other pile. You have just generated a random sequence of black and red cards!

7. Perform the same experiment as in question #6, but this time enter the command **randInt(1,2)** on your calculator and press [ENTER] repeatedly. (To find **randInt(** press [MATH] [◄] [5]). If the output is 1, choose a black card, and if the output is 2, choose a red card. When one of the piles runs out of cards, finish the sequence with the remaining cards in the other pile. This sequence is also a random sequence.

8. If you have a TI-83/84 Plus calculator with the **Prob Sim** App, you can watch the calculator draw cards for you. (The **Prob Sim** App can be downloaded for free at **http://education.ti.com/educationportal/sites/US/productDetail/us_prob_sim_83_84.html**.) Choose the **Draw Cards** option from the menu, select **SET**, and answer **No** to **Replace**. Press the **OK** button. You can then press [ENTER] repeatedly to draw cards in a random order. (Remember, clubs and spades are black cards, and hearts and diamonds are red cards.)

The randomness in Question #6 relies on heads and tails being equally probable every time you flip the coin, an assumption that might not be valid. The randomness in Questions #7 and #8 relies on your calculator's ability to generate random numbers, which it can do only because it has a *pseudo-random number generator* programmed into its software. (For more about pseudo-random number generators, see the Extensions page of this activity.)

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The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

More Probability Simulations

- A fair 6-sided die (or number cube) has an equal probability of showing any of the numbers 1 through 6. Use your calculator to generate a random sequence of 50 rolls of a fair die. How many times did you roll the same number three times in a row?

- According to **www.AABB.org**, 45% of blood donors have type O blood, 40% have type A blood, 11% have type B blood, and 4% have type AB blood. Use your calculator to generate a random sequence of 20 donors classified by blood type. [Hint: Generate numbers between 00 and 99. Let 00 through 44 represent type O, since those are 45% of the possible numbers. Assign the remaining numbers to blood types according to their percentages.]

  Suppose a mobile blood unit processes 40 donors in a typical day. Is it unusual for the unit to finish a day with no units of type AB blood? Use your calculator to perform your simulation ten times, each time for 40 donors, and see how many include no AB donors.

Random and Pseudo-Random Numbers

The ability of computers to simulate random processes is useful in many modern applications, ranging from the obvious (e.g., computer games and cryptography) to the less obvious (e.g., weather forecasting and fighting crime). In some applications, particularly cryptography (code design), pseudo-random numbers are dangerously predictable, so "true" random number generators are preferred. These generators derive numerical data from naturally-occurring sources of entropy, two of the best-known examples of which are atmospheric noise and radioactive decay. Other sources that have been used include lava lamps.

- One of the most famous books in applied mathematics is *A Million Random Digits with 100,000 Normal Deviates*, published in 1955 by the RAND Corporation. As the title suggests, it is a book consisting almost entirely of numbers. An intriguing history of the book, along with a sampling of the statistical tests to which it was subjected to prove its true randomness, can be found at **www.rand.org/publications/classics/randomdigits**.

- An article by Ivars Peterson about how SGI used lava lamps to generate random numbers can be found at **www.maa.org/mathland/mathtrek_5_7_01.html**.

- The Web site **www.random.org** has a nice explanation of pseudo-random numbers and true random numbers. It is also a source for true random numbers in various useful forms (they use atmospheric noise to generate random numbers).

- You can find out more about pseudo-random number generators in the activity **Creating "Random" Numbers** that accompanied the *NUMB3RS* episode "Bettor or Worse."

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