EXPLORATIONS

Activity 8

Right Triangles are Cool Because of Pythag's Rule

Objective

- To find the lengths of sides of right triangles

Materials

- TI-73
- Student Activity page (p. 90)

In this activity you will

- Discover the Pythagorean Theorem.
- Determine if a triangle is acute, right, or obtuse.

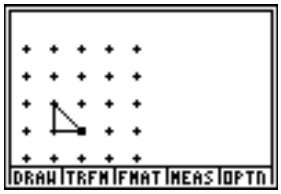
Introduction

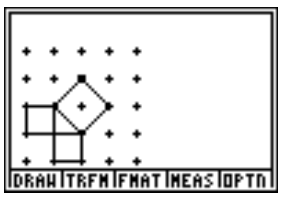
Many interesting discoveries have been made about right triangles. One of the most useful is named after the Greek mathematician Pythagoras and is called the Pythagorean Theorem. The Pythagorean Theorem allows you to find the length of the third side of a right triangle if you are given the lengths of two of the sides. The hypotenuse of a right triangle is the side opposite the right angle. The legs of the right triangle are the other two sides that form the right angle.

Investigation

This investigation will help you learn how to use the Pythagorean Theorem.

1. On a 5×5 board, draw a right triangle with leg lengths of 1 unit. Then draw a square on each side of the triangle.





2. For each row in the table below, draw a right triangle with the given leg lengths on the geoboard. Draw a square on each side of the triangle as in Step 1. Complete the table. Use a 10×10 board.

Length of leg 1	Length of leg 2	Area of square on leg 1	Area of square on leg 2	Area of square on hypotenuse
1	1	1	1	2
1	2			
2	2			
1	3			
2	3			
3	3			

3. Look for a pattern in the relationship of the areas in the table above. Use the pattern you discovered to draw a conclusion about the relationship among the areas. Explain your discovery below.

_____

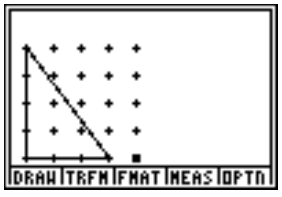
_____

_____

The Pythagorean Theorem states that the sum of the squares of the length of the legs is equal to the square of the length of the hypotenuse. If the length of Leg 1 is A, the length of Leg 2 is B, and the length of the hypotenuse is C, then symbolically this theorem can be written as:

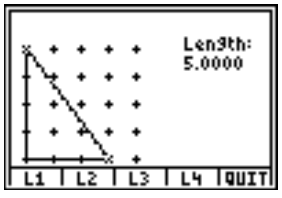
$A^2 + B^2 = C^2$

4. Draw a right triangle with leg lengths of 3 and 4 units and use the **MEAS** tool to test the theorem.



   a. Select **MEAS**, **1:Length**.

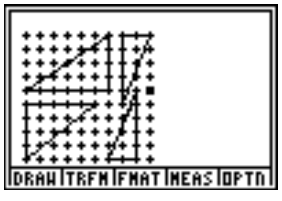
   b. Press [ENTER] on both endpoints of the hypotenuse and the length of the hypotenuse is displayed.



Applying the theorem:

$3^2 + 4^2 = 5^2$
$9 + 16 = 25$

5. On a 10×10 board, draw four different right triangles. Use leg lengths that are different from those already used, such as those in the screen at the right.

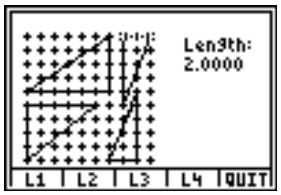


6. Test the theorem using the measure tool and List Editor. You will have the option of storing values in **L1**, **L2**, **L3**, or **L4**. Store the legs in **L1** and **L2** and the hypotenuse in **L3**.

   a. To use the measure tool, press ENTER on each endpoint of the segment you want to measure.

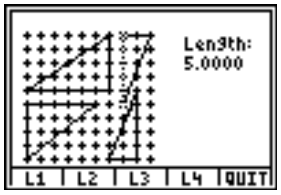
   b. Select **L1** to store the value.

   Leg 1 = 2: Select **L1** to store in **L1**

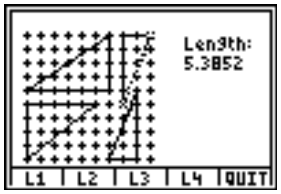
   c. Proceed to the second segment and repeat steps **a** and **b**, but store it in **L2**.

   Leg 2 = 5: Select **L2** to store in **L2**

   d. Repeats steps a and b for the hypotenuse. It does not matter which leg you store in **L1** or **L2**, but make sure that you store the hypotenuse in **L3**.

   Hypotenuse = 5.3852: Select **L3** to store in **L3**

7. Repeat the process in step 6 for the other three triangles, storing legs in **L1** and **L2** and the hypotenuse in **L3**.

8. After completing the measurements for all four triangles, quit the Geoboard application by selecting:

   a. **OPTN 1:Main Menu**

   b. **Exit this board? 2:Yes**

   c. **Save this board? 2:Yes**

   d. **SaveSqr board**
   1:S1
   ENTER

   e. **S1Geobd Created: QUIT**

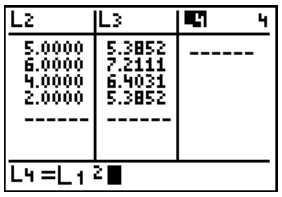
   f. Move over to **QUIT** and select **2:Yes**.

9. Press LIST and you will see **L1**, **L2**, and **L3** with four elements in each list. **L1** is Leg 1, **L2** is Leg 2, and **L3** is the hypotenuse.

L1	L2	L3 1
2.0000	5.0000	5.3852
4.0000	6.0000	7.2111
5.0000	4.0000	6.4031
5.0000	2.0000	5.3852
------	------	------

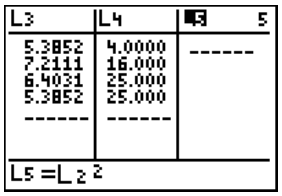
L1(1)=2

10. Move to the top of **L4** and enter **L1**^{2} by pressing 2nd [STAT] **1:L1** x^2. Press ENTER to complete this list.



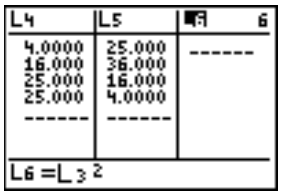
Make sure that you are at the very top of **L4** (that is, **L4** is highlighted as shown at right).

11. Move to the top of **L5** and enter **L2**^{2}. Press ENTER to complete this list.

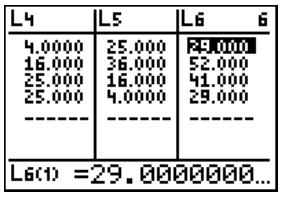


Repeated process of Step 9.

12. Move to the top of **L6** and enter **L3**^{2}. Press ENTER to complete this list.



L4 is Leg 1 squared, **L5** is Leg 2 squared, and **L6** is hypotenuse squared. What relationship do you see among **L4**, **L5**, and **L6**?



13. Since you saved to the variable S1, you can recall the Geoboard file by opening the application and selecting **4:10x10**, **OPTN**, **3:File Open**, **1:S1**.

This file will remain as it is until it is overwritten with a new file.

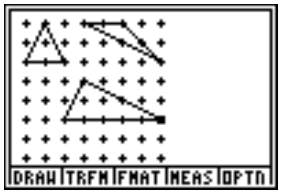
If you know the side lengths of a triangle, you can identify the triangle as acute, right, or obtuse.

If $A^2 + B^2 > C^2$, the triangle is acute.
If $A^2 + B^2 = C^2$, the triangle is right.
If $A^2 + B^2 < C^2$, the triangle is obtuse.

14. Test this concept by drawing the three types of triangles.

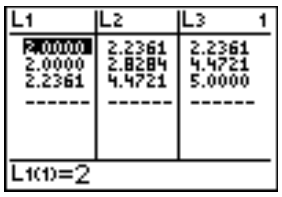
15. Before you reopen the Geoboard application, you will need to clear **L1**, **L2**, and **L3**. One way to clear all of the TI-73 lists is to press 2nd [MEM] **6:ClrAllLists** ENTER.

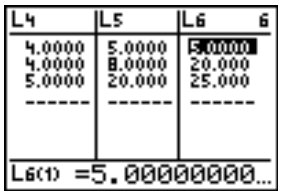
16. Reopen the Geoboard application and draw the three triangles at the right. Use the measure tool and store the sides in Lists. Square the lists as you did earlier to verify these concepts.



Left to right:
Acute, Right, Obtuse

17. Store the length of the longest side (or what appears to be the longest side) in **L3**. A good way to organize this may be to measure in this order: Acute, Right, Obtuse. Start from the shortest side in each triangle and store it in **L1**, store the middle side in **L2**, and store the longest side in **L3**.





The top row is acute, the middle row is right, and the bottom row is obtuse.

L4 = **L1**^{2}; **L5** = **L2**^{2}; **L6** = **L3**^{2}
L4 + **L5** > **L6** (Acute)
L4 + **L5** = **L6** (Right)
L4 + **L5** < **L6** (Obtuse)

Student Activity

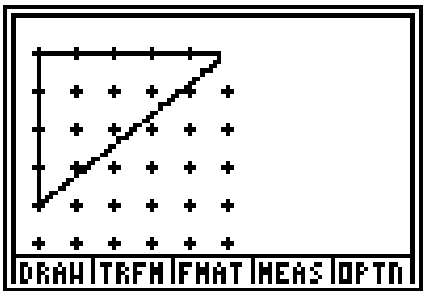
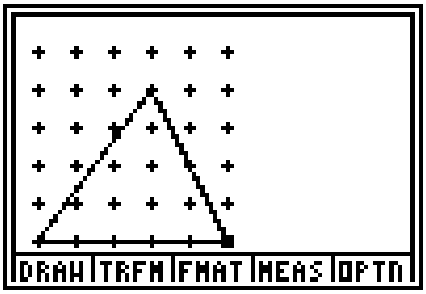
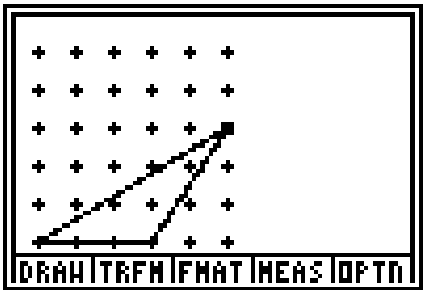
Name _____

Date _____

Activity 8: Right Triangles are Cool Because of Pythag's Rule

To complete this activity:

1. Draw the given triangles on your geoboard.
2. Measure the sides and record the lengths in order from shortest to longest: shortest (a), middle (b), longest (c).
3. Classify each triangle as acute, right or obtuse (d).
4. Write an equation or inequality in terms of a, b, and c to support your answer (e).

1. a. Shortest side: _____ **b.** Middle side: _____ **c.** Longest side: _____ **d.** Acute, right, or obtuse: _____ **e.** Equation or inequality: _____	
2. a. Shortest side: _____ **b.** Middle side: _____ **c.** Longest side: _____ **d.** Acute, right, or obtuse: _____ **e.** Equation or inequality: _____	
3. a. Shortest side: _____ **b.** Middle side: _____ **c.** Longest side: _____ **d.** Acute, right, or obtuse: _____ **e.** Equation or inequality: _____	

⑧ The area is 3 square units 	⑧ Opposite sides are congruent 
⑧ The shape is a hexagon 	⑧ Opposite sides are parallel 
❽ The shape has 2 right angles 	❽ The perimeter is ≈ 6.8284 

Teacher Notes

Activity 8

Right Triangles are Cool Because of Pythag's Rule

Objective

- To find the lengths of sides of right triangles

NCTM Standards

- Model and solve contextualized problems using various representations, such as graphs, tables, and equations
- Use geometric models to represent and explain numerical and algebraic relationships

Investigation

There are several methods for finding the lengths of the sides of triangles in this lesson. It may be helpful for students to complete *Activity 7: Follow the Fence, Then the Hypotenuse* first. That activity goes into more detail on using the method of building squares off of sides and finding areas using the surrounding rectangle method.

When measurements are stored to lists, they will be assigned the first available space. For example, in Step 8, if you do not clear the lists and proceed to measure other objects and store them in **L1**, the measurements are placed in **L4**, **L5**, and so on.

It may be necessary to clear the lists. There are several ways of clearing lists. Refer to the *TI-73 Guidebook* for more instructions on clearing lists. The process used in Step 17 clears all of the lists in the TI-73.

Answers to Student Activity page

1. **a.** 4
   b. 5
   c. 6.4031
   d. Right
   e. 16 + 25 = 41
2. **a.** 4.4721
   b. 5
   c. 5
   d. Acute
   e. 19.9997 + 25 > 25
3. **a.** 3
   b. 3.6056
   c. 5.8310
   d. Obtuse
   e. 9 + 13.0004 < 34.0006

Group Problem Solving: The lengths of sides of right triangles

The Group Problem Solving cards are challenge problems that can be used alone or with the individual sections of this book. The problems are designed to be used in groups of four (five or six in a group are possibilities using the additional cards) with each person having one of the first four clues. Students can read the information on their cards to others in the group but all should keep their own cards and not let one person take all the cards and do the work.

The numbers at the top of the cards indicate the lesson with which the card set is associated. The fifth and sixth clues (the optional clues) have the lesson number shown in a black circle.

The group problems can be solved using the first four clues. The fifth and sixth clues can be used as checks for the group's solution or they can be used as additional clues if a group gets stuck. Some problems have more than one solution. Any shape that fits all the clues should be accepted as correct.

One solution for this problem solving exercise: