

Bell Ringer: The Path of a Projectile – ID: 13456

Time required
15 minutes

Topic: Kinematics

- *Use the kinematic equations to plot the motion of a projectile in two dimensions.*

Activity Overview

In this activity, students will use a parametric plot to make a graph of the path of a projectile. They will examine how changing the initial position and velocity of the projectile affects its path.

Materials

To complete this activity, each student will require the following:

- *TI-Nspire™ technology*
- *pen or pencil*
- *blank sheet of paper*

TI-Nspire Applications

Graphs & Geometry, Notes

Teacher Preparation

Before carrying out this activity, review with students the kinematic equations for the position of an object.

- *The screenshots on pages 2–5 demonstrate expected student results. Refer to the screenshots on page 6 for a preview of the student TI-Nspire document (.tns file). The solution .tns file contains sample responses to the questions posed in the student .tns file.*
- ***To download the student .tns file and solution .tns file, go to education.ti.com/exchange and enter “13456” in the search box.***
- *This activity is related to activity 9601: Potential and Kinetic Energy. If you wish, you may extend this bell-ringer activity with the longer activity. You can download the files for activity 9601 at education.ti.com/exchange.*

Classroom Management

- *This activity is designed to be **teacher-led**, with students following along on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Note that the majority of the ideas and concepts are presented only in **this** document, so you should make sure to cover all the material necessary for students to comprehend the concepts.*
- *If you wish, you may modify this document for use as a student instruction sheet. You may also wish to use an overhead projector and TI-Nspire computer software to demonstrate the use of the TI-Nspire to students.*
- *The extension questions may be answered for homework or in small groups during class time. If students do not have sufficient time to complete the main questions, they may also be completed as homework.*
- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

The following questions will guide student exploration in this activity:

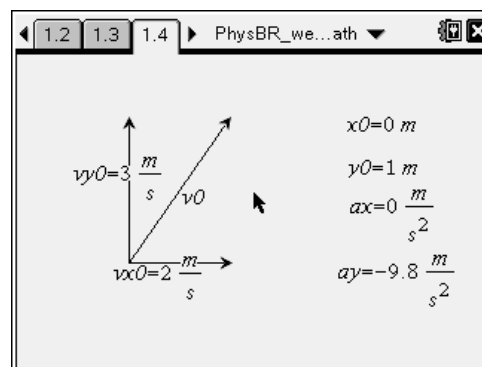
- What is the shape of a projectile's path?
- What factors affect the path of a projectile?

Students will use a parametric plot to make a graph of a projectile's path in x- and y-coordinates. They will vary the initial position and velocity of the projectile and observe the effects on the projectile's path.

Step 1: Students should open the file

PhysBR_week04_projpath.tns and read the first three pages. (Students can press $\text{ctrl} \rightarrow$ and $\text{ctrl} \leftarrow$ to move between pages in the .tns file.)

Step 2: Students should study the information on page 1.4, which shows some of the parameters of a sample projectile. The vectors show the initial velocity (v_0) and initial horizontal and vertical velocities (v_{x0} and v_{y0} , respectively) of the projectile. The variables to the right give additional information: the initial horizontal and vertical positions (x_0 and y_0 , respectively) and the horizontal and vertical accelerations (a_x and a_y , respectively). Students should examine these parameters and then answer questions 1 and 2 on pages 1.5 and 1.6.



Q1. Write the equations for the projectile's horizontal and vertical positions over time.

A. The general equations for horizontal (x) and vertical (y) position are:

$$x(t) = x_0 + v_{x0} \cdot t + \frac{1}{2} a_x \cdot t^2$$

$$y(t) = y_0 + v_{y0} \cdot t + \frac{1}{2} a_y \cdot t^2$$

For the values given on page 1.4, these equations reduce to:

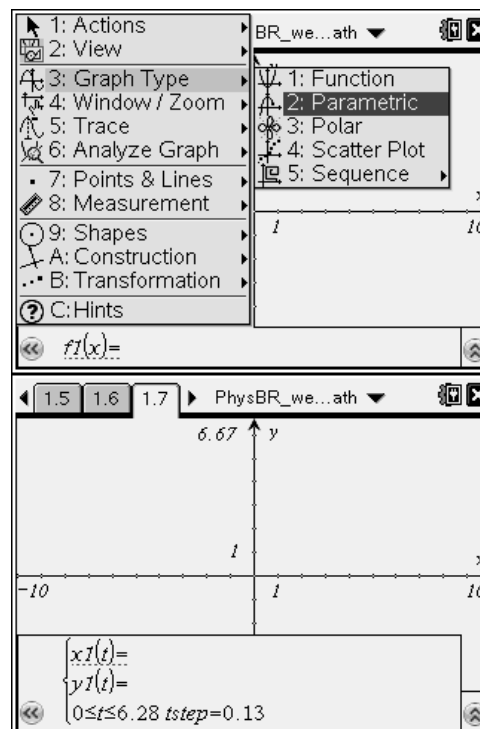
$$x(t) = 2t$$

$$y(t) = 1 + 3t - 4.9t^2$$

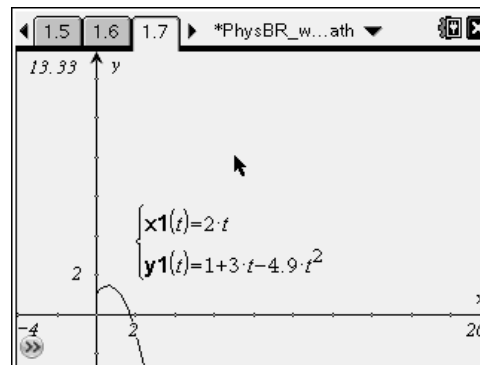
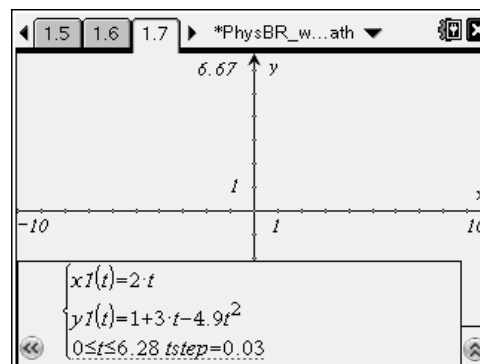
Q2. Predict the shape of the projectile's path as it flies through the air. Explain your answer.




A. Students' answers will vary. If time allows, encourage students to discuss their predictions with each other.

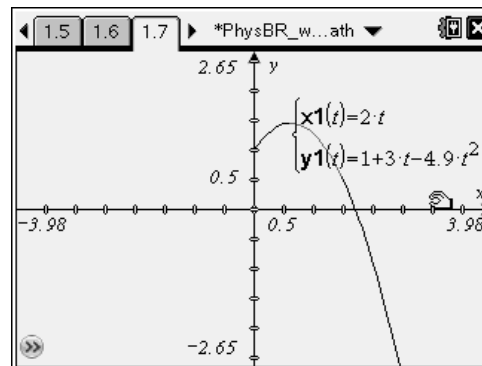
Step 3: Next, students should move to page 1.7. This page contains an empty *Graphs & Geometry* page, which the students will use to plot the path of the projectile. The equations students wrote in question 1 are parametric equations, so students will use a parametric graph to plot them. Students should change the graph type to **Parametric** (Menu > Graph Type > Parametric). When students select the parametric graph, the function entry line at the bottom of the screen should change to show an entry line for $x1(t)$, an entry line for $y1(t)$, and a line that controls the range of t values and the step size for the plot.



Step 4: Students should enter the equations they identified in question 2 into the function entry line. They can use the NavPad to move between lines on the function entry line. They should change the step size ($tstep$) to a smaller value to produce a smoother curve. When they have entered the equation and changed the step size, they should press enter to plot the equations.



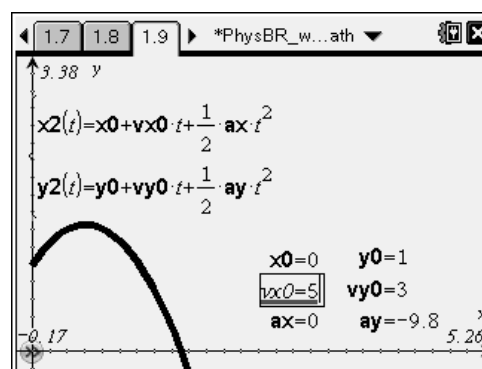
Step 5: By default, the graph shows all four quadrants. However, the projectile's path has physical meaning only within the first quadrant (because, in this case, the x-axis represents the ground, and the y-axis represents the point of launch of the projectile). Students should therefore zoom the graph to better study the physically relevant region of the graph. (Students can zoom the graph in a number of ways. They can zoom in on the first quadrant using **Menu > Window/Zoom > Zoom – Quadrant 1**. Or they can set the window limits manually using **Menu > Window > Window Settings**. A third way to change the window view is to drag the graph area so only the first quadrant is visible and then drag the axes to make the graph larger. To drag the graph area, students should move the cursor to a blank area of the graph in the first quadrant. They should press and hold  until the cursor changes to a closed hand. They can then use the NavPad to drag the graph area. They should press  again to release the graph. To drag the axes, students should move the cursor over one of the axis tick marks, press and hold  to grab the axis, and then drag to extend or shrink the axis range.)



Step 6: Students should study their graphs and then answer question 3 on page 1.8.

- Q3.** Was your prediction correct? If not, explain any errors in your reasoning.
- A.** *Students' answers will vary. Encourage students to identify errors in their reasoning.*

Step 7: Students should move to page 1.9, which shows a blank *Graphs & Geometry* application. On this page, the various variables for the two equations are listed. The graph of the projectile's motion is also shown. Students should modify the variables and study the effects on the projectile's path. Encourage students to experiment with extreme values (e.g., $v_{x0} = 0$), but to keep their values physically relevant (e.g., no negative initial positions). Once students have experimented with manipulating the equations, they should answer questions 4–6.

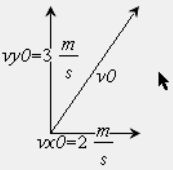
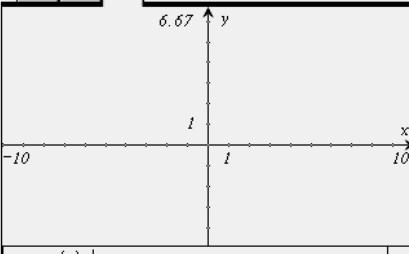
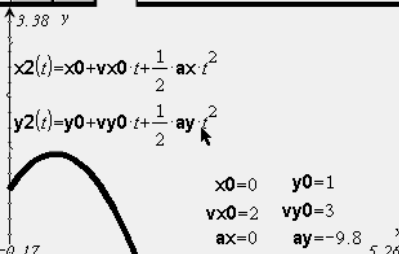


- Q4.** Which of these conditions corresponds to a projectile fired vertically?
- A.** *B; When the projectile is fired vertically (straight up), it has no horizontal velocity, so $v_{x0} = 0$.*
- Q5.** Which of these variables does NOT affect how long the projectile is in the air?
- A.** *C; The duration of the projectile's flight is determined by how long it takes to go up and come back down, which, in turn, depends on the initial vertical velocity (which is a component of initial velocity, v_0) and vertical acceleration. The initial horizontal velocity determines the horizontal distance the projectile covers, but not how long it is in the air.*
- Q6.** For a projectile launched horizontally, what variables determine how far the projectile travels? Explain your answer.
- A.** *A projectile launched horizontally has no initial vertical velocity. If there is no horizontal acceleration, the only variables that affect its flight are its initial vertical position, its horizontal velocity, and its horizontal and vertical acceleration. The higher the initial vertical position (and the larger the vertical acceleration), the longer the projectile will be in the air (because it will take longer to fall to the ground). The horizontal velocity determines the horizontal distance the projectile will cover during that time. If the projectile accelerates horizontally, that acceleration will also affect how far it travels.*

Suggestions for Extension Activities: If time allows, you can have students identify the equations for horizontal and vertical velocity over time, and plot those equations on the parametric graph. You can also have students experiment with adding horizontal acceleration or with changing the gravitational acceleration. Another useful exploration is to use vector resolution to convert an initial velocity vector (i.e., magnitude and direction) into horizontal and vertical vectors, and then use those vectors to determine the projectile's path.

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(Student)TI-Nspire File: *PhysBR_week4_projpath.tns*

<p>1.1 1.2 1.3 PhysBR_we...ath</p> <h2>THE PATH OF A PROJECTILE</h2> <p>Physics</p> <p>Kinematics</p>	<p>1.1 1.2 1.3 PhysBR_we...ath</p> <p>A projectile is an object that is launched into the air and, once it is airborne, experiences only the forces of gravity and air resistance. For a small, dense object, such as a baseball or cannonball, we can neglect the force of air resistance when predicting the motion of the projectile.</p>	<p>1.1 1.2 1.3 PhysBR_we...ath</p> <p>Typically, the motion of a projectile in two dimensions is described by a pair of equations: $x(t)$, which describes the projectile's horizontal position; and $y(t)$, which describes the projectile's vertical position. For example, consider the projectile on the next page. In this diagram, the following variables are shown:</p> <p>x_0: initial horizontal position</p>
<p>1.2 1.3 1.4 PhysBR_we...ath</p>  <p>$x_0 = 0 \text{ m}$ $y_0 = 1 \text{ m}$ $ax = 0 \frac{\text{m}}{\text{s}^2}$ $ay = -9.8 \frac{\text{m}}{\text{s}^2}$</p>	<p>1.3 1.4 1.5 PhysBR_we...ath</p> <p>1. Write the equations for the projectile's horizontal and vertical positions over time.</p>	<p>1.4 1.5 1.6 PhysBR_we...ath</p> <p>2. Predict the shape of the projectile's path as it flies through the air. Explain your answer.</p>
<p>1.5 1.6 1.7 PhysBR_we...ath</p>  <p>$f(x) =$</p>	<p>1.6 1.7 1.8 PhysBR_we...ath</p> <p>3. Was your prediction correct? If not, explain any errors in your reasoning.</p>	<p>1.7 1.8 1.9 PhysBR_we...ath</p> <p>$x_2(t) = x_0 + vx_0 t + \frac{1}{2} ax t^2$ $y_2(t) = y_0 + vy_0 t + \frac{1}{2} ay t^2$</p> <p>$x_0 = 0$ $y_0 = 1$ $vx_0 = 2$ $vy_0 = 3$ $ax = 0$ $ay = -9.8$</p> 
<p>1.8 1.9 1.10 PhysBR_we...ath</p> <p>4. Which of these conditions corresponds to a projectile fired vertically?</p> <p><input type="radio"/> $v_0 = vx_0$ <input type="radio"/> $vx_0 = 0$ <input type="radio"/> $vx_0 = vy_0$ <input type="radio"/> $vy_0 = 0$ <input type="radio"/> $v_0 = 0$</p>	<p>1.9 1.10 1.11 PhysBR_we...ath</p> <p>5. Which of these variables does NOT affect how long the projectile is in the air?</p> <p><input type="radio"/> vy_0 <input type="radio"/> ay <input type="radio"/> vx_0 <input type="radio"/> v_0</p>	<p>1.10 1.11 1.12 PhysBR_we...ath</p> <p>5. For a projectile launched horizontally, what variables determine how far the projectile travels? Explain your answer.</p>