Topics in Calculus: Integration

Solving Initial Value Problems

NCTM Principles and Standards

- **Content Standard:** Represent and analyze mathematical situations and structures using algebraic symbols
- **Process Standard**: Use representations to model and interpret physical, social, and mathematical phenomena

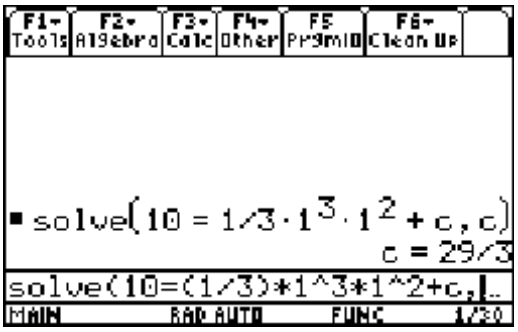
Definition: A function F is an **antiderivative** of a function f over an interval if F'(x)=f(x) at every point of the interval.

If F(x) is an antiderivative of f(x), the family of functions F(x)+C where C is a real number is the general antiderivative of f (x) over the interval. C is called an arbitrary constant. When finding antiderivatives results may be checked by taking the derivative of the general antiderivative to determine if it gives the function, by graphing the general antiderivative for any C to determine if it is the same as the graph of the function, or by using the CAS capabilities of the TI-89.

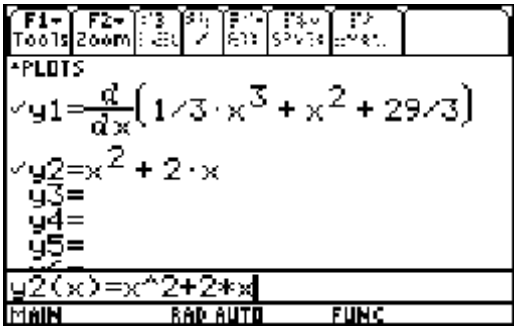
To find the general antiderivative for $f(x)=x^2+2x$ use the power rule in reverse.

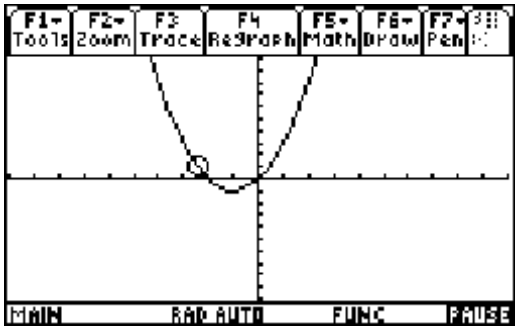
$$F(x)=\frac{1}{3}x^3+x^2+C$$

Finding a function when an initial value (x_0,y_0) and the derivative are known is called an initial value problem. Suppose $y_0=10$ when $x_0=1$. Substituting in F(x) gives $10=\frac{1}{3}(1)^3+(1)^2+C$. To solve for c using the TI-89 press [F2] [1], type the equation followed by [,] [alpha] [)] [ENTER]. Thus the solution for the initial value problem is $F(x)=\frac{1}{3}x^3+x^2+\frac{29}{3}$. The general antiderative is $F(x)=\frac{1}{3}x^3+x^2+C$, **any** real number C is an antiderivative, but there is **only one** value of C for which F(1)=10 namely $\frac{29}{3}$.



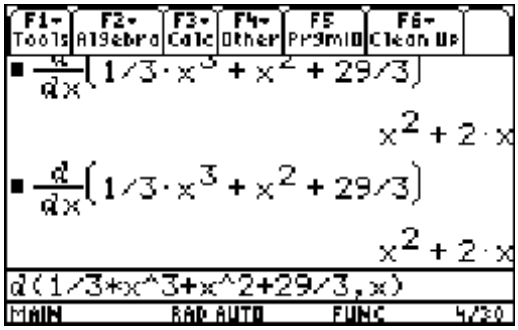
To check the antiderivative graphically press [◆][F1] [2nd][8] and type the function followed by [,] [X] [)] [ENTER]. In y2 type the function f(x). Press [2nd][F1][6] to choose "path" format so that students can see y2 as it graphs.





To check the antiderivative algebraically, press [2nd][QUIT]to return to the home screen. To find the derivative press [F3] [1]or [2nd] [8], type the function, type [,] [X] [)] and then press [ENTER].

.



Find a function y=f(x) for the following differential equations and initial conditions. Support answers using CAS on the TI-89 and/or by graphing on the TI-89.

1. $\frac{dy}{dx} = 2\sqrt{x} \quad y = 4 \, when \, x = 8$
2. $\frac{dy}{dx} = 2^x \quad y = 1.5 \, when \, x = 0$
3. $\frac{d_2 y}{dx^2} = \frac{2}{x^3} \quad \frac{dy}{dx} = 1 \, and \, y = 1 \, when \, x = 1$