NUMB3RS Activity: What is Random?
Episode: "Traffic"

Topic: Random Numbers   **Grade Level:** 9 - 10

Objective: Describe what randomness looks like and use random numbers to find the area of an object using Monte Carlo methods.

Time: 25 - 30 minutes

Materials: TI-83 Plus/TI-84 Plus graphing calculators, TI-Navigator™ system, and the following files: *RandomPoints.act, PhoneNumbers.act, SquareCircle.act, Cluster.8xp* To download these files, go to **http://education.ti.com/exchange** and search for "7760."

Introduction

In "Traffic," Charlie is lecturing a group of students about randomness. He projects two images on a screen showing an array of dots, and asks his students to identify which one represents "raindrops falling on a sidewalk." Most students choose the image showing an almost perfect separation of dots to which Charlie explains that "our brains misperceive evenness as random and wrongly assume that groupings are deliberate." He goes on to say that "because of this, people make all sorts of irrational decisions," such as avoiding working in a high-rise building or choosing not to live in an earthquake prone area.

This idea of randomness extends throughout the episode, as Charlie and the FBI attempt to solve a string of highway attacks that do not follow a predictable pattern. In fact, this lack of a pattern is so unusual that it connects directly with the dot problem portrayed in the beginning of the show. Charlie is led to conclude that the assailant is actually trying to give the impression that his attacks are unrelated.
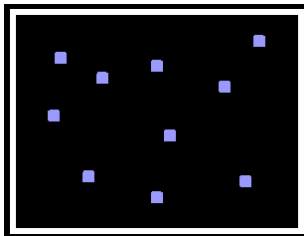
In mathematics, one would expect to see some clustering, or an occasional appearance of a pattern, when investigating truly random phenomena. For example, if you look at a random number table or use the random number generator on a calculator, you will see occasional clusters of numbers, such as a string of three 5s in a row.

In Part I of this activity, students will be asked to plot 10 "random" points on a coordinate grid in Activity Center, much like the raindrops on a sidewalk example from the episode. After discussing the results, you will likely reveal that most students have fallen into the same trap that Charlie's students fell into during the episode. That is, students will instinctively produce examples characterized by "evenness."
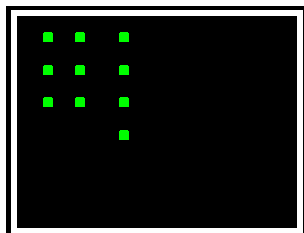
In Part II of this activity, students will generate random data using phone numbers to simulate the locations of raindrops on a sidewalk. Students will investigate the clustering of data and participate in an activity in which their random points can be used to determine the approximate area of a circle.

Part I: Think Randomly

1. a. Tell students that you want them to place what they would consider to be random points in a grid in Activity Center that represent raindrops falling on a sidewalk.

b. Load the ***RandomPoints.act*** activity settings file into Activity Center and click the 'List-Graph' tab. This will display a box that has been loaded in as a background image. Students will be allowed to send 10 points to Activity Center, however, this activity has been configured so that the points will not appear in the Activity Center window until all of them have been marked and sent to the teacher.

c. It is recommended that you also log into NavNet as a teacher and participate in this activity as well. Your goal will be to place points that really do seem to follow some type of pattern.

d. Press ▶ **Start Activity** and tell students that you want them to mark random points within the box.

e. When the students are finished, press ■ **Stop Activity** . Hide all the data contained in **L1** and **L2**. Next, hold down the shift key and highlight the 10 occurrences of a student's name in the list area. This will reveal this student's points in blue in the Activity Center window. Look for results in which the points seem fairly evenly spaced, such as the one shown in the diagram below.
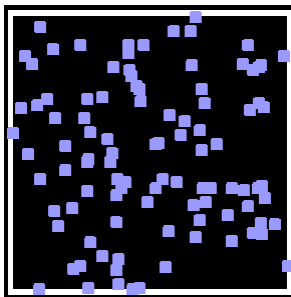

f. Now show your results, which intentionally reveal some type of pattern or clustering, such as the one shown below. Ask students if they think an arrangement of truly random points could look like your diagram. Explain to students that any arrangement of points has an equal probability of occurring. However, it should be made clear that any specific arrangement is extremely rare and highly unlikely to occur.

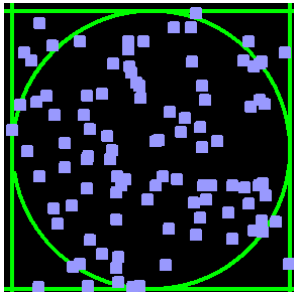## Part II: Looking at Random Data

1. a. In Part I of this activity, students were given a chance to think about randomness. In this section, they will generate a set of random "raindrops on a sidewalk," discuss how it "looks" in relation to the results from Part I, and use the data to estimate the area of a circle with radius 1.

2. **a.** Click "File" and select "Clear Activity Data". Next, load the ***PhoneNumbers.act***
   activity settings file into Activity Center and click the 'Graph' tab. This will display a
   square loaded in as a background image whose corners have coordinates (1, 1),
   (–1, 1), (–1, –1), (1, –1).

   b. Explain to students that you want to generate random ordered pairs with
   coordinates that will range from –1 to 1, whose points, when plotted in Activity
   Center, will always fall within the square. Challenge students to brainstorm some
   possible methods of generating these points. For example, you can use phone
   numbers to create random points that will be graphed in the square. Specifically,
   students can separate the last four digits of some phone numbers into groups of
   two digits, thereby forming two 2-digit numbers. If a number is odd, make it
   negative, and if it is even, make it positive. Finally, a decimal point should be
   placed to the left of the leftmost digit. For example, the phone number
   508-660-7257 has a corresponding ordered pair (0.72, –0.57). This method will
   produce points that will be graphed within the square because each coordinate is a
   number between –1 and 1.

   c. You can also use the serial numbers on a dollar bill or the random number
   generator on a graphing calculator to produce these random points.

   d. Decide in advance exactly how many points you want the class to contribute to this
   activity. It is a good idea to have at least 100 total points, so for a class of 25
   students, have each student submit 4 points. If you are using phone numbers, tell
   students that they can use their own phone number and three others. They must
   not use the phone number of a classmate and they should avoid business phone
   numbers (whose digits are not always random). You may also choose to
   photocopy several pages of a local phone book and have each student work off a
   different page.

   e. Click ▶**Start Activity** and tell students to enter their ordered pairs under lists
   L1 and **L2**. When they are ready, have students **Send** their points to
   Activity Center.

   f. When all the students have finished sending in their points, press
   ■ **Stop Activity** . Below is a sample Activity Center screen in which 100 random
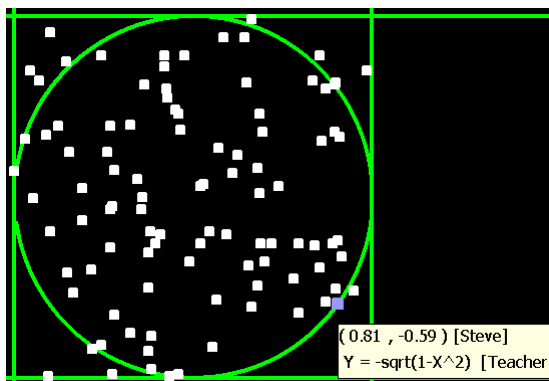   points were submitted.

   g. Discuss the results, placing particular emphasis on the clustering of points seen in
   the diagram.

3. a. Tell students that, while individual points can land anywhere in the square, there is quite a bit of predictability when a large sample of random points is generated. To illustrate this point, load the ***SquareCircle.act*** activity settings file into Activity Center. Click "No" at the prompt to keep the data points that are in Activity Center. You will see the same data points along with a circle inscribed in a square.

   b. Discuss with students that the area of the square is 4 square units and the circle has a radius of 1 unit. Tell students that they will use the random points to estimate the area of the circle, a technique often referred to as the *Monte Carlo Method*.

   c. Ask each student to count the number of points that fell outside the circle (since it is easier to count these rather than the ones that fell inside the circle) and use **Quick Poll** (with *Open Response*) to collect their answers. There will most likely be some discrepancies in the results, particularly since some of the points appear to lie on the circle. You can use the most common result from the students or place your cursor over those points that are in question, revealing its coordinates. If the students know the distance formula, then you can use it to see if the distance from the origin exceeds 1, thereby placing it outside the circle. For example, the diagram below shows a point located at (0.81, –0.59), with a distance equal to

$$\sqrt{\left(0.81-0\right)^2+\left(-0.59-0\right)^2} \approx 1.002\,.$$ Therefore, this point is outside the circle.

   d. Once it has been determined how many points lie outside the circle, students should set up and solve the proportion below, using *x* to represent the area of the circle.

$$\frac{\text{Points inside circle}}{\text{Total points}} = \frac{x}{4} \; \begin{array}{l} \leftarrow \text{area of circle} \\ \leftarrow \text{area of square} \end{array}$$

In the example shown in Step 3a, the value of *x* is 3.36.

   e. Discuss this result with students, making sure that they recognize that this number is close to π, the exact value of the area of a circle with radius 1.
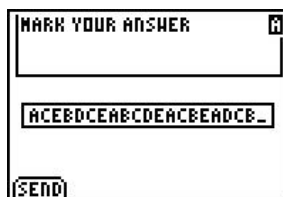
The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extension

<u>For the Student</u>
When answering multiple choice questions on a test, students are often troubled when they encounter a streak of the same answer choice, such as four Bs in a row. Again, there is a perception that there should be "evenness" in a sequence of multiple choice answers, much like there should be "evenness" in raindrops falling on a sidewalk.

*   Tell students to imagine that they are taking a standardized test with multiple choice answers A through E. They are running out of time and decide to randomly guess on 20 consecutive questions (Don't advise doing this in real life!).

*   Click **Quick Poll** (with Open Response) and press ▶ **Start Poll** . Tell your students to press 2nd [A-LOCK] to put their calculators in Alpha-lock mode. Instruct them to enter what they think is a random sequence of the letters A, B, C, D, and E, and send their results to the teacher when complete.

*   After the responses have come in, press ■ **Stop Poll** and click the 'Poll Summary' tab. You will see a row of letters for each student response.

*   Look for results in which it appears an effort was made to avoid clusters of letters. This is a good time to discuss with students what a random collection of letters might really look like. Try to convince students that it is common to see an occasional cluster of the same letter. As with the random point problem, explain that every sequence of letters has an equal chance of occurring, including a sequence (although extremely rare) such as AAAABBBBCCCCDDDDEEEE.

*   The program CLUSTER can be used to look at randomly generated answer choices. This program can be sent to students using **Send to Class**.

- When the program is executed, students will be prompted to enter the number of questions for the test. The program will then randomly generate these answers (where a 1 represents answer choice A, 2 represents answer choice B, and so on), check for clusters of the same answer, and present the results. For example, the screens below show that, for a 30 question test, there were four instances in which the same answer occurred twice in a row and two instances in which the same answer occurred three times in a row.

```
prgmCLUSTER
Enter the number
of questions:
?30
```
```
?30
              4
occurrences of
two in a row.
              2
occurrences of
three in a row.
```
```
              0
occurrences of
four in a row.
              0
occurrences of
five in a row.
          Done
■
```

- The randomly generated answers can be found in list L_2.

Related Topics

- The random integers generated by your graphing calculator are in fact called *pseudo-random numbers* because they are generated from a computer algorithm. For more information on random numbers and pseudo-random numbers, go to **www.random.org** or **www.randomnumbers.org**.

- A random number machine called *ERNIE* (Electronic Random Number Indicator Equipment) is used by the UK to generate true random numbers for their tax free lottery tickets called Premium Bonds. This machine uses thermal noise as its source of randomness. Each output of random numbers is sent to the Government Actuary's Department for statistical randomness checks. To learn more about *ERNIE*, go to **http://www.nsandi.com/pdf/ernie_media_%20guide.pdf**.

- If you would like to learn more about the TI-Navigator™ system, visit **http://education.ti.com/navigator**.