Stay Tuned:
Sound Waveform Models

If you throw a rock into a calm pond, the water around the point of entry begins to move up and down, causing ripples to travel outward. If these ripples come across a small floating object such as a leaf, they will cause the leaf to move up and down on the water. Much like waves in water, sound in air is produced by the vibration of an object. These vibrations produce pressure oscillations in the surrounding air which travel outward like the ripples on the pond. When the pressure waves reach the eardrum, they cause it to vibrate. These vibrations are then translated into nerve impulses and interpreted by your brain as sounds.

These pressure waves are what we usually call *sound waves*. Most waves are very complex, but the sound from a tuning fork is a single tone that can be described mathematically using a cosine function $y = A\cos\big(B(x - C)\big)$. In this activity you will analyze the tone from a tuning fork by collecting data with a microphone.
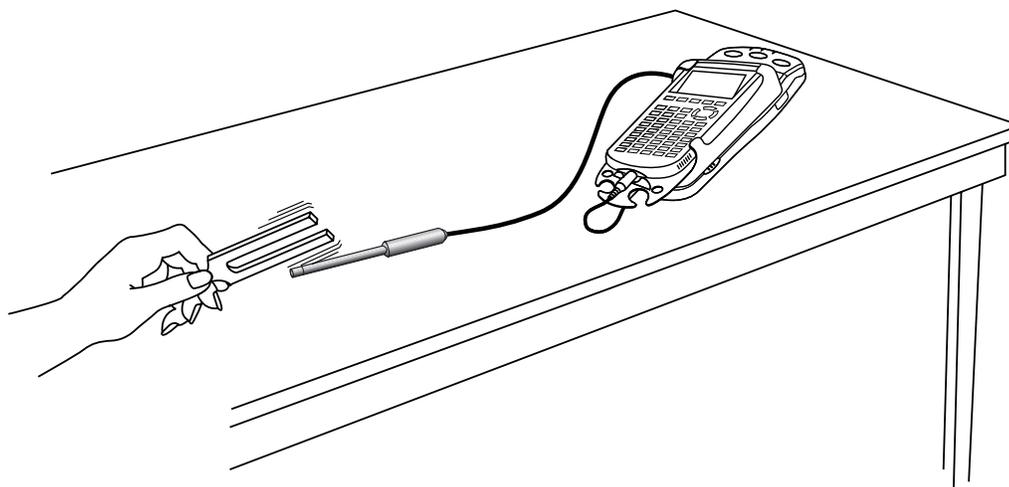
OBJECTIVES

- Record the sound waveform of a tuning fork.
- Analyze the waveform to determine frequency, period and amplitude information.
- Model the waveform using trigonometric functions.

MATERIALS

TI-83 Plus or TI-84 Plus graphing calculator	Vernier Microphone
EasyData application	tuning fork or electronic keyboard
data-collection interface	

PROCEDURE

1. Turn on the calculator. If the calculator is in degree mode, change it to radian mode, and return to the home screen.

2. Connect a Microphone, data-collection interface, and calculator.

3. Set up EasyData for data collection.

   a. Start the EasyData application, if it is not already running.
   b. Select ⸢File⸣ from the Main screen, and then select **New** to reset the application.

4. To center the waveform on zero, zero the microphone channel.

   a. Select ⸢Setup⸣ from the Main screen, and then select **Zero…**
   b. With the room quiet, select ⸢Zero⸣ to zero the channel.

5. If you are using a keyboard, set it to a flute sound. Use middle C as the note. If you are using a tuning fork, strike it against a soft object such as a rubber mallet or the rubber sole of a shoe.

   C*aution: Striking it against a hard object can damage it. If you strike it too hard or too softly, the waveform may be rough.*

   Produce a sound with a tuning fork or keyboard, hold it close to the Microphone and select ⸢Start⸣ to collect data.

6. After data collection ends, a graph will appear. The waveform should resemble a sine function. Check with your instructor if you are not sure if you need to repeat data collection. If you need to repeat data collection, select ⸢Main⸣ to return to the Main screen; then repeat Step 5.

7. Once you are satisfied with the data, select ⸢Main⸣ to return to the Main screen. Exit EasyData by selecting ⸢Quit⸣ from the Main screen and then selecting ⸢OK⸣.

ANALYSIS

1. Redisplay the graph.

   a. Press ⸢2nd⸣ [STAT PLOT].
   b. Press ⸢ENTER⸣ to select Plot1 and press ⸢ENTER⸣ again to select On.
   c. Press ⸢ZOOM⸣.
   d. Press ⸢⌄⸣ until ZoomStat is highlighted; press ⸢ENTER⸣ to display a graph with the *x* and *y* ranges set to fill the screen with data.
   e. Press ⸢TRACE⸣ to determine the coordinates of a point on the graph using the cursor keys.

2. Use the cursor keys to trace across the graph. Record the times for the first and last peaks of the waveform. Record the number of complete cycles that occur between the first measured time and the last. Divide the difference, Δt, by the number of cycles to determine the period of the waveform.

   Record the period, to three significant figures, in the Data Table on the *Data Collection and Analysis* sheet. For example, 0.00230 has three significant figures.

3. Trace across the graph again, and note the maximum and minimum *y* values for an adjacent peak and trough. Calculate the amplitude of the wave by taking half of the absolute value of the difference between the maximum and minimum *y* values.

   Record the amplitude *A*, to two significant figures, in the Data Table on the *Data Collection and Analysis* sheet.

4. Since the cosine function starts at a maximum value when its argument is zero, you can use the location of a maximum to determine the value of *C*, which represents the horizontal shift of the data. Trace across your data to any maximum and read the time (*x*) value. Record this value as *C* in your Data Table.

5. To compare the model to the data, enter the model equation into the calculator.

   a. Press ⬭Y=.
   b. Press ⬭CLEAR to remove any existing equation.
   c. Enter the expression A∗cos(B∗(X–C)) in the Y1 field.
   d. To make the model equation plot with a bold line, press ◀ until the diagonal line to the left of Y1 is highlighted. Press ⬭ENTER until the line is shown as a bold line.
   e. Press ⬭2nd [QUIT] to return to the home screen.

6. The parameter *A* represents the amplitude of the waveform. Enter your value for the parameter *A* in the calculator's memory. To do this, enter the value for the amplitude from the Data Table. Press ⬭STO▸ A ⬭ENTER to store the value in the variable A.

7. The sinusoidal model has a parameter *B* that represents the number of cycles the sinusoidal function makes during the natural period of the sine function. Find *B* by taking 2π (the natural period of the sine function) divided by the period of the waveform (the time for one cycle). Store this value in the calculator after performing the division by pressing ⬭STO▸ B ⬭ENTER to store the value in the variable B.

8. Enter the value for C from the Data Table, and press ⬭STO▸ C ⬭ENTER to store the value in variable C.

9. You have now entered the model equation and values for the three parameters *A*, *B*, and *C*. Press ⬭GRAPH to display a graph of the Microphone data with the model equation.

⇒ Answer Questions 1 and 2 on the *Data Collection and Analysis* sheet.

10. Most tuning forks are marked with the frequency. Check the tuning fork you used and record its frequency in the Data Table. If you used a keyboard, note that middle C is approximately 263 Hz.

⇒ Answer Questions 3–7 on the *Data Collection and Analysis* sheet.

EXTENSION

The calculator can automatically fit a sine function to the waveform data. The format of the calculator's fit is a little different than the one you used: $y = A\sin(Bx + C)$. You can work out the translation for the new usage of the parameter C.

Use the calculator to fit a sine curve to the waveform data. How do the fit parameters compare to those of your model?

    a. Press ⬭STAT⬭ and use the cursor keys to highlight CALC.
    b. Press ⬭ to highlight SinReg; press ⬭ENTER⬭ to copy the command to the home screen.
    c. Press ⬭2nd⬭ [L1] ⬭,⬭ ⬭2nd⬭ [L2] ⬭,⬭ to enter the lists containing the data.
    d. Press ⬭VARS⬭ and use the cursor keys to highlight Y-VARS.
    e. Select Function by pressing ⬭ENTER⬭.
    f. Press ⬭ENTER⬭ to copy Y1 to the expression.
    g. On the home screen, you will now see the entry SinReg L1, L2, Y1. This command will perform a sine regression with L1 as the *x* and L2 as the *y* values. The resulting regression line will be stored in equation variable Y1. Press ⬭ENTER⬭ to perform the regression. Use the parameters a, b, c, and d that appear on the calculator screen to write down the fitted equation. Write the fitted parameters to three significant digits.
    h. Press ⬭GRAPH⬭ to see the graph.

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DATA COLLECTION AND ANALYSIS

Name _____

Date _____

DATA TABLE

period (s)	
amplitude **A**	
time offset **C**	
frequency (measured)	
frequency (marked)	

QUESTIONS

1. How well does your model equation fit the data?

    If your fit is acceptable, write the model equation below, and suggest explanations for any discrepancies. If the fit of the model is not acceptable, deduce which of the parameters is producing the problem. Make changes as necessary to the parameters, and discuss why the changes were necessary. Write out the equation that produced a good fit.

2. The frequency of a sound wave is the number of cycles per second. The period is the number of seconds per cycle. Explain the relationship between frequency and period.

    The unit Hertz, or Hz, is equivalent to cycles per second. Calculate the frequency of the tuning fork in Hz and record it in the Data Table above.

3. The amplitude of a sound wave increases with the loudness of the sound. Explain how you could alter the value of *A* if you repeated this investigation.

4. *Pitch* is associated with the frequency of the tuning fork. A higher pitched tone would have a higher frequency. Explain how your graph would change if you used a tuning fork of higher frequency.

   How would the value of the period change if the frequency were higher? Explain your reasoning clearly.

5. How many different values of *C* are possible in order to match this graph? Explain your reasoning.

   Find another value of *C* that will work and record it below. Check this in the equation, and discuss your reasoning.

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6. How would the parameters *A*, *B*, and *C* change if you were to use the sine function $y = A\sin(B(x - C))$ instead of the cosine function? Predict the values below and explain your reasoning for each.

7. Test your predictions by storing any changed values in the three parameters *A*, *B*, and *C* using the same method you used earlier. Also change the model equation to a sine function using the method you used earlier. Redisplay the graph to compare your data and sine model. How well does your sine model fit the data? Explain any discrepancies.