Calculator Blackjack

Introduction

The game of Blackjack allows players to draw cards until they accumulate a total of 21 points. If they exceed 21 points they're bust! In Calculator Blackjack the random number generator is used to produce numbers between 0 and 1. Random numbers are added together and continue to be drawn (generated) until the total exceeds 1.
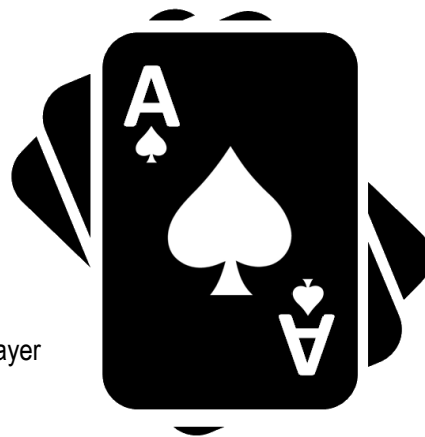
Question: 1.

Estimate, on average, how many numbers need to be generated for a player to be bust?

Answers will vary but should be between 2 and 4.

Teacher Notes:

The theoretical or analytic approach to solving this problem is beyond the scope of high school mathematics courses. A few moments discussing the complexity of the problem can generate an appreciation of how statistics can be a valid approach to exploring the problem and gaining a level of confidence about the solution.

As students record their seed value it is possible to check answers by using the same seed value, assuming students have not generated additional samples than those specified in the investigation.

Playing the Game

Open the TI-Nspire file: Blackjack

Navigate to page 1.2 and seed the random number generator using a four digit number of your choosing.

Probability > **Random** > **Seed**

Enter your own, unique four (or more) digit number for the random seed. This ensures your results will be unique.

It is time to start playing the game. Using the probability menu once again generate a random number.

Probability > **Random** > **Number**

Press **Enter** to generate the random number. This command generates random numbers between 0 and 1. It is therefore not possible to go bust on the first turn.

In the example shown opposite there is a very high probability the game will continue beyond the second random number since the first random number was so small.

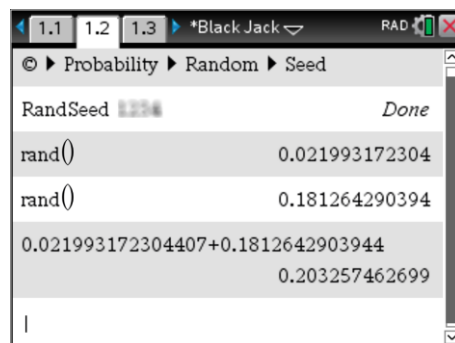Press **Enter** again to generate another random number.

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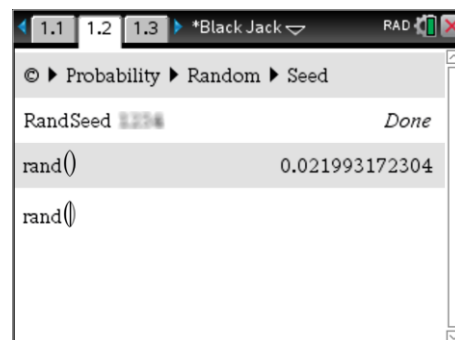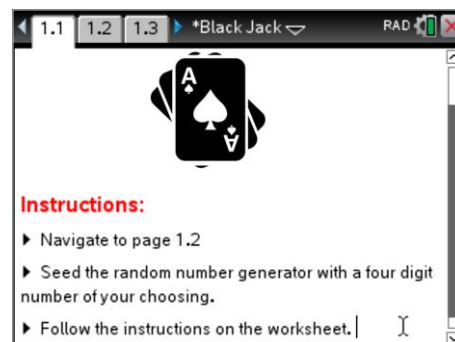
In this hand the first number drawn (generated) was:

0.021993…

The second number was:

0.18126 …

The two numbers can be added together to provide a progressive total, however a faster way to play is to use the 'ANS' command above the (-) key or by typing ANS.

Enter the instructions:

ANS + rand()

This collects the most recent amount and adds a new random number. The game being played here is now up to four random numbers and the progressive total score is still only 0.3262…

The only disadvantage here is that the actual random number that was generated is not displayed, however the focus of this investigation is on how many numbers, on average, will be generated that will cause a player to go 'bust'.
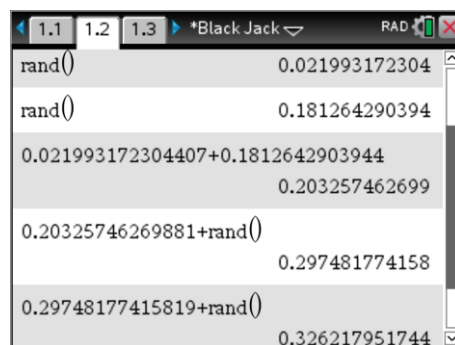
Question: 2.

Play 5 games of Calculator Blackjack and record the results below.

Random Seed number used:
Answers will vary provided students have seeded their random number generator differently.

Teacher Notes:

The purpose of this progressive approach is to give students an idea of the mean of this distribution and the variability when only 25 games are simulated.

Game	#1	#2	#3	#4	#5
Final Total (Bust)					
Quantity of Numbers					

Question: 3.

Based on your small 'sample' of five games, how many random numbers on average will make a player 'bust'?

Answers will vary but students should have narrowed their estimate to be close to 2.5 and certainly be between 2 and 3.

There is a much quicker way to play a large number of games. The Blackjack file includes a program titled: "Blackjack" which can be found in the **VAR** menu.

Select the program and press **Enter** to run it.

Run a simulation of 25 games.

The results of the 25 games displayed simply as the "Sample mean". To see individual game result, select **games** from the **VAR** menu.
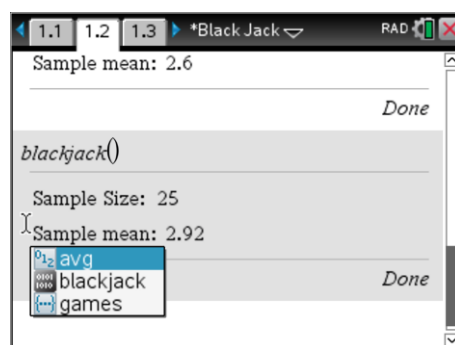
Question: 4.

Run 5 simulations of **25** games using the Blackjack program and record the results below.

Answers will vary. Most sample means will be between 2.4 and 3.

Sample	#1	#2	#3	#4	#5
Sample Mean					

Question: 5.

Imagine repeating the 25 game simulations many times over:

a) What would be the likelihood of obtaining a sample average very close to 2?

Answer: Very unlikely – The purpose of this question is to help students visually construct the shape of the sampling distribution.

b) What would be the likelihood of obtaining a sample average very close to 4?

Answer: Very unlikely – The purpose of this question is to help students visually construct the shape of the sampling distribution.

c) Describe or graph how the sample means might be distributed.

Answers will vary, however students should describe the distribution as approximately symmetrical, centred around the mean … perhaps Normal.

Navigate to page 2.1 and run the blackjack1() program.

When prompted generate 100 samples each consisting of 25 games. Watch closely as the mean of each sample is displayed on the screen.

Navigate to page 2.2 to see a graph of the sampling distribution.
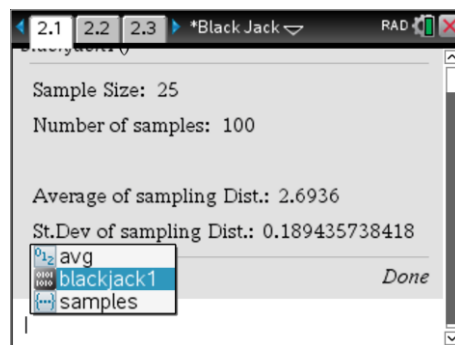
Question: 6.

Describe the sampling distribution.

Answer: Students should recognise that the distribution is symmetrical, perhaps approximately normal, particularly in light of subsequent questions!

Change the dot-plot on page 2.2 to a histogram.

Use the **Analyse** menu to generate a **Normal PDF** for the data.
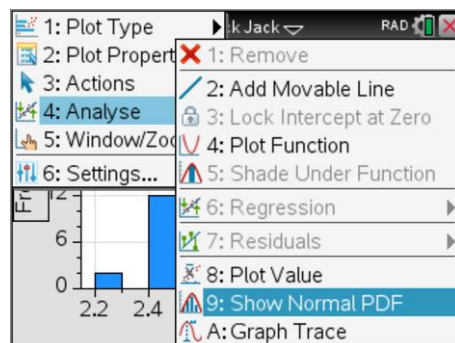
Question: 7.

Record the mean and standard deviation of Normal approximation for the sampling distribution.

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Answers will vary, but the mean should be very close to 2.7 and the standard deviation close to 0.2. Students may be encouraged to repeat this process several times or compare answers with other students to see that there is very little variation for the mean of the sampling distribution.

Question: 8.

Return to page 2.1, run the blackjack1() program again generating 100 samples each consisting of 100 games. (This will take a little time.)

a) Compare and comment on the mean of this sampling distribution.

The mean of this sampling distribution should be approximately the same as the previous sampling distribution involving a smaller sample size.

b) Compare and comment on the standard deviation of this sampling distribution.

c) The standard deviation of this sampling distribution approximately half of the previous value since the sample size has been increased by a factor of 4.

Teacher Notes:

At the conclusion of the investigation this is a great question to discuss and subsequently introduce the formula for the standard deviation of a sampling distribution.

d) The mean of this sampling distribution is close to a common transcendental number. Describe how sampling could be used to create greater certainty about this value?

Answer: The theoretical mean for this distribution is e (Euler's number). The statistical approach used here can only provide a degree of confidence relating to this outcome. Students need to recognise that taking larger samples provides a greater certainty about the value of the mean. Taking more samples does NOT create more certainty as the Central Limit Theorem tells us that the sample means will be approximately normally distributed. Generating more samples simply populates the sampling distribution illustrating its Normality.

Teacher Notes:

This activity provides a good opportunity to introduce hypothesis testing. It has not been included here as this activity is designed as an introduction to sampling distributions, the concept of the central limit theorem (informally) and an *understanding* that larger samples reduce the standard deviation of the sampling distribution, including an informal introduction of this relationship.

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TEXAS INSTRUMENTS