



Math Objectives

Students will discover the determinant method for finding the areas of triangles and quadrilaterals.

Vocabulary

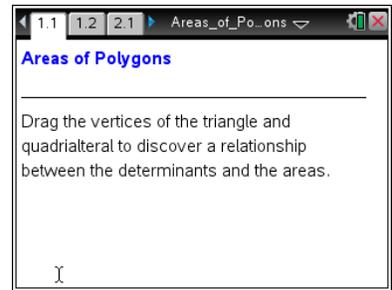
- determinant
- scalar multiple
- convex/concave polygons

About the Lesson

- This lesson involves using determinants of matrices as a tool to find the areas of triangles and quadrilaterals.
- As a result, students will:
 - Identify the correct scalar multiple for the determinant expression to find the area of a triangle.
 - Identify the determinate expression to find the area of a quadrilateral.

TI-Nspire™ Navigator™ System

- Use Live Presenter for student demonstrations.
- Use Quick Poll to share student responses.
- Use Screen Capture to examine patterns that emerge.



TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag points

Tech Tips:

- Make sure the font size on your TI-Nspire handheld is set to Medium.
- Once a function has been graphed, the entry line can be graphed by pressing **ctrl** **G**. The entry line can also be expanded or collapsed by clicking the chevron.

Lesson Materials:

Student Activity

Areas_of_Polygons_Student.pdf

Areas_of_Polygons_Student.doc

TI-Nspire document

Areas_of_Polygons.tns

Optional Materials:

Areas_of_Polygons_Create.pdf

Visit www.mathnspired.com for lesson updates and tech tip videos. (optional)



Discussion Points and Possible Answers

Teacher Tip: Students can either use the premade file *Areas_of_Polygons.tns* or can use *Areas_of_Polygons_Create.doc* to create the file. More detail is given in the follow-up lesson:
Exploration 19: The Determinant of a Matrix from the book by Brendan Kelly: Algebra 2 with TI-Nspire.

Teacher Tip: This activity builds upon students' previous knowledge of a determinant. In question 1, the idea of using determinants for finding area is generalized to polygons with no vertices on the origin, specifically triangles, by adding a third point in the determinant calculation. However, adding additional points to the determinant creates non-square matrices. To fix this, a column of 1s is added as needed.

To provide a common footing for the students, consider using the Scratchpad and having the students find the values of $\det \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$ and

$\det \begin{bmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \\ 1 & 1 & 1 \end{bmatrix}$. They will notice that both determinants return values of -2

despite their different appearances. Remind students that determinants are found through multiplication, which is why the 1s do not affect the value of the determinant.

Tech Tip: If students experience difficulty dragging a point, check to make sure that they have moved the cursor until it becomes a hand () getting ready to grab the point. Also, be sure that the point becomes highlighted. Then press **ctrl** () to grab the point and close the hand (.

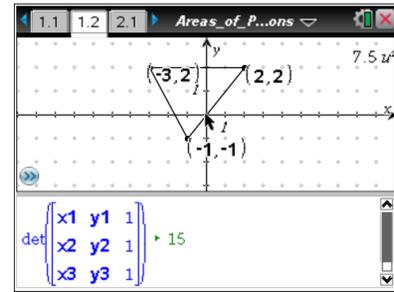
TI-Nspire Navigator Opportunity

Consider using *Live Presenter* to demonstrate how to drag a point on the triangle.



Move to page 1.2.

1. A triangle is given. The coordinates of each vertex are linked to the x- and y-values in the first two columns of the matrix.
 - a. Drag any of the vertices to form a right triangle. What is the base and height of your triangle? Explain how those values justify the area. Show the formula with values used.



Sample answer: If vertices were $(-3, 2)$, $(2, 2)$, and $(2, -2)$, the base would be 5 units and the height would be 4. The formula for the area of the triangle is $5 \cdot 5 \cdot 4 = 10$ square units.

- b. Drag any vertex on the triangle. What values change in the upper and lower parts of the page?

Answer: As a point on the triangle is dragged, the calculated geometric area changes and the answer to the determinant expression changes.

- c. What is the relationship between the determinant and the area of the triangle?

Answer: The value of the determinant expression is a scalar multiple of 2 times the area of the triangle.

- d. Why does the matrix have 1s in the third column?

Answer: Determinants can only be found using square matrices.

- e. Michael says the value of his determinant is -10 , so the triangle has an area of -5 square units. Is he correct?

Answer: Michael is incorrect. He still needs to take the absolute value of his determinant.

- f. What does it mean when the value of the determinate expression is 0? Confirm your answer by dragging the vertices of the triangle until the determinant expression equals zero.

Answer: The triangle has an area of zero, which means that the three points no longer form a triangle but are collinear. Students will confirm this when they drag the vertices.



Teacher Tip: Note that the determinant is only finding a factor of the “signed area” of a region. Students have seen this before when performing expansion by minors when finding determinants of matrices larger than 2×2 . They had to multiply each minor by a signed value.

TI-Nspire Navigator Opportunity

Tell students that you are going to send *Quick Polls (Open Response)* asking their answer to question 1b to check for understanding and to discuss possible misunderstandings. If you are satisfied with the results, continue with the lesson below.

If students had difficulty, take a *Screen Capture* of page 1.2. As a class, discuss the relationship between the geometric area of the triangle and the value of the determinant.

Teacher Tip: As stated earlier, as more points are being added, the matrices are no longer square, which means that columns of 1s need to be added. For a quadrilateral in this activity, students are summing two triangle areas. This method can be expanded to other polynomials.

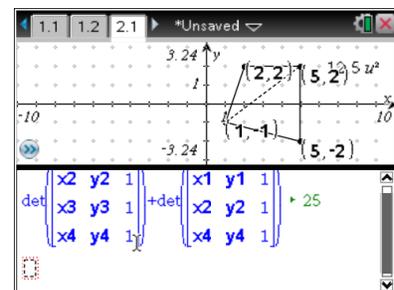
However, to explore a determinant area method specific to quadrilaterals,

have students investigate $\det \begin{bmatrix} x_1 & y_1 & 1 & 1 \\ x_2 & y_2 & 1 & -1 \\ x_3 & y_3 & 1 & 1 \\ x_4 & y_4 & 1 & -1 \end{bmatrix}$. They will notice that the

value for this determinant is four times larger than the actual area. Notice that two columns of 1s were needed to keep the matrix square. However, this results in a matrix whose determinant is zero. This always occurs when two columns in a matrix are identical. To rectify this, the alternating -1 s were added.

Move to page 2.1.

2. A quadrilateral that has been divided into two triangles is given. The coordinates of each vertex are linked to the x - and y -values of the matrix.





- a. Drag any of the vertices to form a rectangle or square. What is the base and height of your figure? Explain how those values justify the area. Show the formula with values used.

Sample Answer: If vertices were $(-4, 2)$, $(2, 2)$, $(-4, -2)$, and $(2, -2)$, the length would have a value of 6 and the width would have a value of 4. The formula for the area of the rectangle is $A = \text{length} \cdot \text{width}$, so $A = 6 \cdot 4$, which equals 24 square units.

- b. What values change when you drag any vertex on the quadrilateral?

Answer: As a point on the quadrilateral is dragged, the calculated, geometric area of the quadrilateral changes as does the determinant expression.

- c. What is the relationship between the determinant and the area of the quadrilateral?

Answer: The value of the determinant expression is a scalar multiple of 2 times the area of the quadrilateral.

- d. Does the method work if the quadrilateral is concave? Try it.

Answer: Yes.

Teacher Tip: The determinant of any matrix with two identical rows is 0. In fact, if a row or column is a constant multiple of any other row or column, the determinant of the matrix is 0. If students have experience with using matrices to solve systems of equations, you can tie this fact about determinants back to dependent systems.

TI-Nspire Navigator Opportunity

Tell students that you are going to use *Screen Capture* to monitor student progress on question 2b and to look for patterns. If students are having trouble, it may be helpful to have them observe the various screen shots of others as they progress in question 2c.

Inform students that you will send a *Quick Poll (Open Response)* asking them to submit their answer to question 2c to check for understanding and to discuss possible misunderstandings.



3. How would this method be used to find the area of a convex pentagon?

Answer: Two line segments are drawn from one vertex forming three triangles. Then determinants are used to find the area of each triangle and summed.

Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand that:

- Given a triangle of vertices (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) , its area can be found by

$$\frac{1}{2} \left| \det \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix} \right|.$$

- Given a quadrilateral of vertices (x_1, y_1) , (x_2, y_2) , (x_3, y_3) , and (x_4, y_4) , its area can be found by

$$\frac{1}{2} \left| \det \begin{bmatrix} x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \\ x_4 & y_4 & 1 \end{bmatrix} \right| + \frac{1}{2} \left| \det \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_4 & y_4 & 1 \end{bmatrix} \right|.$$

Assessment

- Write the determinant expression used to find the area of a triangle with vertices $(3, 5)$, $(-4, 8)$, and $(1, -4)$. Use this expression to calculate the area.

Answer:
$$\frac{1}{2} \left| \det \begin{bmatrix} 3 & 5 & 1 \\ -4 & 8 & 1 \\ 1 & -4 & 1 \end{bmatrix} \right| = 34.5$$

- Write the determinant expression used to find the area of a quadrilateral with vertices $(-4, 2)$, $(6, 1)$, $(2, -2)$, and $(-4, -1)$. Use this expression to calculate the area.

Answer:
$$\frac{1}{2} \left| \det \begin{bmatrix} 6 & 1 & 1 \\ 2 & -2 & 1 \\ -4 & -1 & 1 \end{bmatrix} \right| + \frac{1}{2} \left| \det \begin{bmatrix} -4 & 2 & 1 \\ 6 & 1 & 1 \\ -4 & -1 & 1 \end{bmatrix} \right| = 26$$