How Random!

ID: 9291

Topic: Probability Simulations & Conjectures

- *Using a spinner, coin or dice, conduct a probability experiment to calculate the relative frequency and the experimental probability of an event.*
- *Use technology to simulate a probability experiment and calculate the experimental probability of an event.*
- *Create a histogram from a probability experiment to determine the distribution of data.*

Activity Overview

In this activity, students use simulations and graphs to explore the common sense notion that repeatedly flipping a coin results in "heads up" about half of the time. First, they simulate an experiment by representing single coin flips with random numbers. Next, they use a given formula to simulate multiple coin flips at once, allowing them to perform a large number of trials easily. Lastly, they create a histogram of their data, observing that the data clusters around the mean.

Teacher Preparation

This activity is designed for use in an Algebra 2 or Statistics classroom. It is best used to introduce the concepts of data clustering about the mean, setting up a framework for future explorations of normal distributions.

- *Prior to beginning the activity, students should know how to interpret scatter plots and histograms and be familiar with binomial probability and the concept of mean as expected value.*

- ***To download the FLIP500 program file, go to education.ti.com/exchange and enter "9291" in the quick search box.***

Classroom Management

- *This activity is intended to have students explore **individually and in pairs** (with the teacher providing assistance when needed). However, an alternate approach would be to use the activity in a whole-class format. By using the graphing calculator and the questions found in the student worksheet, you can lead an interactive class discussion on simulating coin flips.*

- *For new users of the graphing calculator, it would be beneficial to demonstrate how to create scatter plots and histograms from data lists.*

TI-84 Plus Applications

None

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In this activity we will

- *enter data into lists and graph scatter plots*

- *perform a multiple regression on the plots*

- *make predictions or draw conclusions from the quadratic model*

Suppose you flipped a coin 50 times. About how many times would you expect the coin to land heads up?

Most people answer "about 25." The number of heads might be slightly more or less, but it will always be *close* to 25.

But why? Why is it unlikely that you flip only 2 heads or even 48 heads in 50 flips? If flipping a coin is random, shouldn't any number of heads be equally likely?

In this activity, you will simulate flipping a coin to investigate these questions.

Begin by clearing all the data from the lists with the **ClrAllLists** command (found in the **Catalog**.) You should also turn off any functions or statplots.

Next, store a sequence of numbers from 1 to 50 in **L1** using the command shown. This list will number the flips. (1 is the first flip, 2 is the second, etc.) The **seq** command is found in the **List > Ops** menu.

We will use the calculator's **randInt** command (found in the **Math > Prb** menu) to simulate getting heads or tails. The command shown returns a random integer between 0 and 1 inclusive. In our experiment, 1s will represent heads and 0s will represent tails. Try the command a few times to understand what it does.

Now generate a list of 50 integers between 0 and 1 inclusive and store it in **L2**.

```
randInt(0,1,50)→
L₂
```

View the lists in the **List Editor**. You have simulated 50 coin flips!

Think about a formula you could use to quickly calculate the total number of heads from those 50 flips. (Hint: Look at the **List > Math** menu.)

Enter the formula to find the sum in the first row of **L3**.

L1	L2	L3	1
1	1	------	
2	0		
3	1		
4	0		
5	0		
6	0		
7	0		

L1(1)=1

Now we need to repeat the experiment a few more times. Return to the home screen and re-enter the command to generate a list of 50 integers between 0 and 1 inclusive and store it in **L2**. (If the command is still on the screen, press ` + e to copy it.) This will store a new set of random numbers in **L2**.

L1	L2	L3	3
1	1	22	
2	0		
3	1		
4	0		
5	0		
6	0		
7	0		

L3(2) =

Return to the **ListEditor** and recalculate the total number of heads in the second row of **L3**. Repeat for a total of 10 trials.

```
randInt(0,1,50)→
L₂
{1  1  1  1  0  0  1 …
randInt(0,1,50)→
L₂
```

Now make a histogram of the data. Before making the graph, adjust the window settings. The least number of heads we could expect in 50 flips is 0, so make **Xmin** = 0. Set **Xmax** to be the greatest number of heads we could expect? Let **Ymin** = 0 and **Ymax** = 10.

Go to **StatPlot > Plot1**, turn the plot **On**, change the **PlotType** to histogram, and choose **L3** as the **XList**.

You should find that the data points cluster around the "middle," or around 25. If they do not appear this way, it might be that not enough trials of the experiment were performed for a pattern to appear. We will explore even more trials in the next problem.
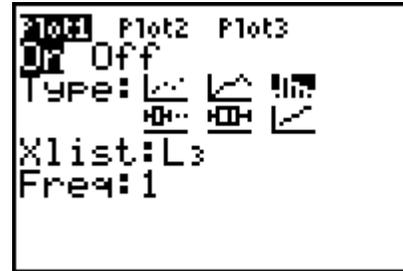
Problem 2 – Even more coin flips

Now suppose that instead of flipping the coin 50 times, you flip it 500 times. And instead of performing 10 trials, you perform 50 trials! How many heads would you expect on each trial?

```
CATALOG        [M]
  X²-Test(
  X²GOF-Test(
  Circle(
  Clear Entries
  ClockOff
  ClockOn
▸ClrAllLists
```

In this problem, each row in the lists will represent one trial or 500 flips. Begin by clearing all the lists.

Store a sequence of numbers from 1 to 50 in **L1**. This list will number the trials. (1 is the first trial of 500 flips, 2 is the second trial of 500 more flips, etc.)

```
seq(X,X,1,50)→L₁
```

Try using the formula **randInt(0,500,50)** to generate random numbers of heads in 500 flips in **L2**.

- Go back and look closely at your data. Of the 50 values, how many are under 200?

- Can you explain why this doesn't make sense?

L1	L2	L3	3
1	196	▉▉▉▉▉	
2	62		
3	214		
4	285		
5	342		
6	41		
7	315		

L3(1)=

The program **FLIP500** correctly represents a random number of heads out of 500 flips and stores the results of 50 trials in **L2**. Run the program. (Note: this program will take several minutes to run, but imagine how long it would take to perform the experiment yourself!)

```
prgmFLIP500
```

- Then go to the **ListEditor** and examine **L2**. Does this data make more sense?

Based on these results, enter a formula in the formula bar of **L3** that will give the number of *tails* for each trial. If you rerun the **FLIP500** program, the numbers in **L3** should update automatically.

L1	L2	L3	1
1	257	74	
2	253	121	
3	248	402	
4	247	434	
5	256	498	
6	253	337	
7	252	295	

L1(1)=1

Now you will make a scatter plot of the number of heads vs. the number of tails. First choose an appropriate window. Enter the minimum and maximum number of heads as **Xmin** and **Xmas** and the minimum and maximum number of tails as **Ymin** and **Ymax**.

Then go to **StatPlot > Plot1**, turn the plot **On**, change the type to **Scatter Plot**, the **XList** to **L2** and the **YList** to **L3**.

- What do you notice about the graph? Do the points fill the viewing window?

Select **Zoom > ZoomStat** to get a closer view of the data.

- Describe this new distribution. What are the minimum and maximum numbers of heads? Are the points evenly spaced? Explain.

Now we'll take a different look at the data. First adjust the window settings as shown.

Go to **StatPlot > Plot1**, change the type to histogram and the **XList** to **L2**, the number of heads.

Press $ to see more detail about the graph.

- Theoretically, how many heads would you expect out of 500 flips?

- What percent of these 50 trials resulted in 240-260 heads?

- What about 230-270 heads?