Breaking Up Is Not Hard To Do

About the Lesson

In this activity, students will split rational functions into sums of partial fractions. As a result, students will:

- Utilize graphing to verify accuracy of results and to support the understanding of functions being represented in multiple ways.

Vocabulary

- rational function
- partial fractions
- Heaviside method

Teacher Preparation and Notes

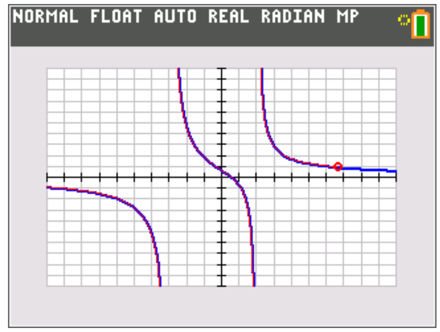
- Problems 1–3 of this activity should be done in class as guided practice or small group work. Several problems are provided on the student worksheet for additional practice.
- Before beginning the activity, make sure that all plots have been turned off and all equations have been cleared from the **Y=** screen.
- As an extension, the teacher could include a discussion of the placement of vertical asymptotes.

Activity Materials

- Compatible TI Technologies:

  TI-84 Plus*
  TI-84 Plus Silver Edition*
  TI-84 Plus C Silver Edition
  TI-84 Plus CE

*** *with the latest operating system (2.55MP) featuring MathPrint™ functionality.*



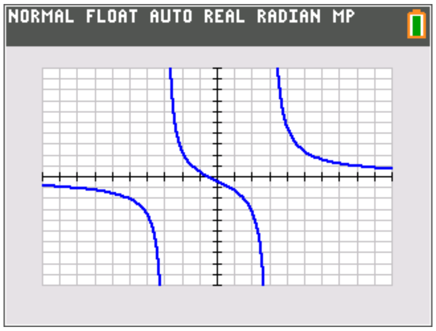
Tech Tips:

- This activity includes screen captures taken from the TI-84 Plus CE. It is also appropriate for use with the rest of the TI-84 Plus family. Slight variations to these directions may be required if using other calculator models.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials
- Any required calculator files can be distributed to students via handheld-to-handheld transfer.

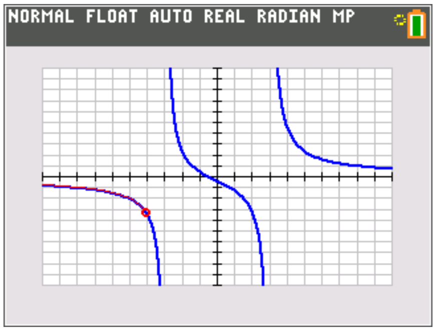
Lesson Files:

- Breaking_Up_Is_Not_Hard_To_Do_Student.pdf
- Breaking_Up_Is_Not_Hard_To_Do _Student.doc

Problem 1 – Introduction



This part of the activity involves an exploration of equivalent ways to express a rational function. Students will generate function graphs from which they will learn that a rational function can be represented as the sum of individual fractions, known as partial fractions.



To make it clear to students that the graphs of **Y1** and **Y2** are identical, show students how to place the tracing circle in front of **Y2=** by moving the cursor to the left of **Y2=** and pressing [enter] to bring up the graphing options screen. Select **Line:** and choose the -0 option.

Students will answer questions regarding their observations of the graphs of the given equations. They are also asked to observe the denominators of the two functions.

1. How do the graphs of the two given equations compare?

Answer: The graphs are the same.

2. What do the graphic results tell us about the two functions?

Answer: The functions appear to be equal.

3. How are the denominators in $\frac{3}{x+3}+\frac{4}{x-3}$, the partial fractions, related to the denominator of the original expression $\frac{7x+3}{x^2-9}$?

Answer: The denominators of $\frac{3}{x+3}+\frac{4}{x-3}$ are factors of the denominator of $\frac{7x+3}{x^2-9}$.

Because the graphic results show that the two functions are equivalent, they are set equal to each other and a framework is established for finding the numerators of the partial fractions of a rational function. Directions are provided to help students through the process.

$$Y_1(x) = Y_2(x)$$
$$\frac{7x+3}{x^2-9} = \frac{A}{x+3} + \frac{B}{x-3}$$

Students proceed to solve for *A* and *B* by substituting in values for *x* that will simplify the work to be done. For example, substituting –3 for *x* will eliminate the *B* term and simplify the process of solving for *A*. Similarly, substituting 3 for *x* will simplify solving for B.

This technique is sometimes called the "Heaviside method."

Discuss with students why it might be helpful to decompose a rational expression into a sum of partial fractions. Students may note that the partial fractions, being less complex, will be easier to work with for certain mathematical applications.

$$(x+3)(x-3)\left(\frac{7x+3}{x^2-9} = \frac{A}{x+3} + \frac{B}{x-3}\right)$$

$$7x-3 = A(x-3) + B(x+3)$$

$$Let\ x = 3$$
$$7(3)-3 = A(3-3) + B(3+3)$$
$$18 = 6B$$
$$3 = B$$

$$Let\ x = -3$$
$$7(-3)-3 = A(-3-3) + B(-3+3)$$
$$24 = -6B$$
$$-4 = B$$

4. What is the LCD (least common denominator) for $\frac{7x+3}{x^2-9} = \frac{A}{x+3} + \frac{B}{x-3}$?

Answer: $x^2 - 9$ or $(x - 3)(x + 3)$

5. What is the result of multiplying both sides of $\frac{7x+3}{x^2-9} = \frac{A}{x+3} + \frac{B}{x-3}$ by the LCD?

Answer: $7x + 3 = A(x - 3) + B(x + 3)$

6. Substitute in a convenient number for *x* and solve for *A*. What value did you obtain for *A*?

Sample Answer: 3

7. Similarly substitute in a convenient number for *x* and solve for *B*. What value did you obtain for *B*?

Sample Answer: 4

8. Now substitute the values you found for both *A* and *B* into the equation shown in Question 4 to show the equivalent rational function and sum of partial fractions.

Sample Answer: $\frac{7x+3}{x^2-9} = \frac{3}{x+3} + \frac{4}{x-3}$

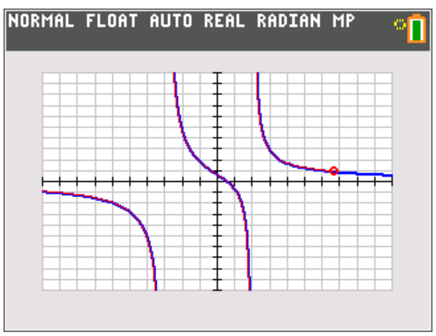
9. How do your results for Question 8 support your answer to the Question 2 regarding what the graphs of the functions Y_1 and Y_2 tell us about the two functions?

Answer: The results verify algebraically that the two functions are equivalent.

Problem 2 – Practice

Students apply what was learned in Problem 1 to find a sum of partial fractions equivalent to a given rational function.

Once the algebraic work is completed, students can verify the equivalence of their solution to the original function via graphing the two functions. Remind students to use the show/hide feature to the left in the function entry bar of the graph page to be certain that the graphs of the two functions are identical.



$$\frac{7x-4}{x^2+x-6} = \frac{A}{(x+3)} + \frac{B}{(x-2)}$$

$$(x+3)(x-2)\left(\frac{7x-4}{x^2+x-6} = \frac{A}{(x+3)} + \frac{B}{(x-2)}\right)$$

$$7x-4 = A(x-2) + B(x+3)$$

$$Let\ x = 2; \quad 7(2) - 4 = A(2-2) + B(2+3)$$
$$10 = 5B \quad B = 2$$

$$Let\ x = -3; \quad 7(-3) - 4 = A(-3-2) + B(-3+3)$$
$$-25 = -5A \quad A = 5$$

10. Express the rational function, $(x) = \frac{7x-4}{x^2+x-6}$, as a sum of partial fractions.

Answer: $\frac{7x-4}{x^2+x-6} = \frac{5}{x+3} + \frac{2}{x-2}$

11. Graph the initial function and your sum of partial fractions using the graphing calculator as outlined in Problem 1. How does this verify your results? Explain your reasoning.

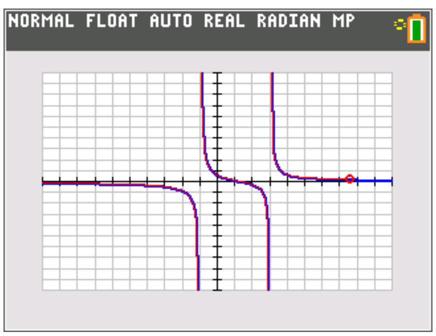
Answer: The graphs are the same, which also verifies equivalent algebraic results.

Problem 3 – The Next Level

Students again apply what has been learned, but the challenge level increases.

In this situation, the denominator has a constant factor in addition to two binomial factors. A hint is given to prompt students to use the algebraic binomial factors as denominators for the partial fractions to be determined.

When students solve for *A* and *B*, they will find that the value for *B* is a fraction, which will result in a need for simplification of the partial fractions obtained.



$$\frac{5x-7}{4x^2-8x-12} = \left(\frac{A}{(x-3)} + \frac{B}{(x+1)}\right)$$

$$4(x-3)(x+1)\left(\frac{5x-7}{4x^2-8x-12} = \frac{A}{(x-3)} + \frac{B}{(x+1)}\right)$$

$$5x-7 = 4A(x+1) + 4B(x-3)$$

$$Let\ x = -1; \qquad 5(-1)-7 = 4A(-1+1) + 4B(-1-3)$$

$$-12 = -16B \qquad B = \frac{3}{4}$$

$$Let\ x = 3; \qquad 5(3)-7 = 4A(3+1) + 4B(3-3)$$

$$8 = 16A \qquad A = \frac{1}{2}$$

12. Express the rational function, $(x) = \frac{5x-7}{4x^2-8x-12}$, as a sum of partial fractions.

Answer: $\frac{5x-7}{4x^2-8x-12} = \frac{1}{2x-6} + \frac{3}{4x+4}$

13. Graph the initial function and your sum of partial fractions using the graphing calculator. How does this verify your results? Explain your reasoning.

Answer: The graphs are the same, which also verifies equivalent algebraic results.

Problem 4 – Additional Practice Problems

Represent each of the following rational functions as a sum of partial fractions. Verify your results graphically.

14. $f(x) = \frac{-7x-11}{x^2+4x+3}$

Answer: $\frac{-7x-11}{x^2+4x+3} = \frac{-2}{x+1} - \frac{5}{x+3}$

15. $f(x) = \frac{2x+42}{x^2+2x-24}$

Answer: $\frac{2x+42}{x^2+2x-24} = \frac{5}{x-4} - \frac{3}{x+6}$

16. $f(x) = \frac{x}{x^2+2x-8}$

Answer: $\frac{x}{x^2+2x-8} = \frac{2}{3x+12} + \frac{1}{3x-6}$