NUMB3RS Activity: The Chase
Episode: "Spree, Part I"

Topic: Introduction to Pursuit Curves **Grade Level:** 9 - 10
Objective: Students will be able to use linear functions, rates of change, Pythagorean Theorem, measurement, and graphing to solve the problems involving distance, rate, and time.
Time: 15 - 20 minutes
Materials: TI-83 Plus/TI-84 Plus graphing calculator, a ruler (marked with centimeters)

Introduction

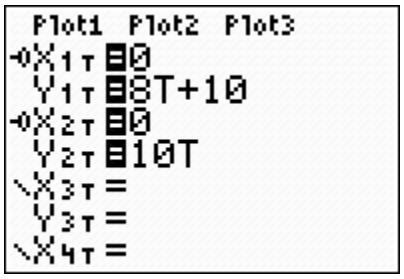
In this activity, students will be introduced to pursuit curves. A pursuit curve is the path a person might follow if they were chasing someone else. In "Spree, Part I," Agent Edgerton is chasing Buck and Crystal, a couple who is on a Bonnie and Clyde type crime spree as they travel across the United States. Charlie is tracking all of the places that Buck and Crystal have visited and is using the information to predict the next place the couple will visit. He explains that Buck and Crystal's path is changing because they are being chased by Agent Edgerton. Although this activity does not explain how Buck and Crystal's path is being changed, it does explain how Agent Edgerton's path is changing while he is pursuing the couple.

Students will also create graphs of parametric equations, but this vocabulary is not mentioned in the activity. There are many variables that influence the paths of the two groups; in this activity, the speed and direction of the two groups will be explored.

Discuss with Students

Make sure students know that Buck and Crystal are always traveling up along the vertical axis in the graph. You can animate the paths of both groups for the various questions to make sure students understand how they move. One way to do this is to place a finger on the starting points of each group and move up the vertical axis.

You can also use a graphing calculator in parametric mode. (You do not have to explain parametric equations to your students or show them how to do enter them on the graphing calculator to do the activity.) On your calculator, press MODE and use the settings shown below on the left. This sets your calculator to Parametric mode, and to draw all graphs simultaneously. Then press Y= and enter the equations shown below on the right.



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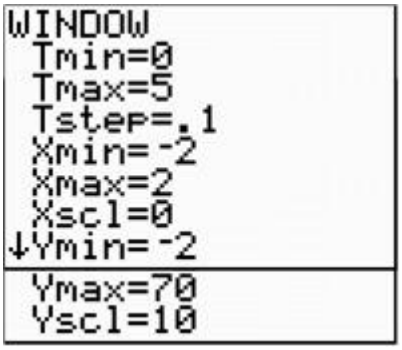
One advantage of using parametric equations is that you can control the position of the dots as a function of time. The X_{1T} equation controls the horizontal position of Buck and Crystal. It is set at 0 so the circle will move along the vertical axis. The Y_{1T} equation controls the vertical position of Buck and Crystal. When T = 0 they are at 10 and move 8 units up for every 1 unit of time. The X_{2T} and Y_{2T} equations control the position of Agent Edgerton. (This models what the students will be doing as they answer the questions in the activity.) Press [WINDOW] and use the settings shown below. Then press [GRAPH] to see the animation.



This animation shows the solution to Question 1. Small changes to the X_{2T} and Y_{2T} equations (and the window settings) will allow you to show animated solutions to Questions 3, 4, and 5. You may want to show the animation for Question 1 as an introduction and save the animations for the other questions after students have explored the activity.

There are many opportunities for students to make mistakes when working on Question 8. You may want to model how to begin to make the graph for Question 8 in front of the class and then have them finish the exploration. Some students might be confused on how the graph shows the time it takes Agent Edgerton to catch up to Buck and Crystal. Time is often shown as the independent variable on the horizontal axis. In this graph, time is still the independent variable but it is not shown on either axis. The time it takes for Agent Edgerton to catch up to Buck and Crystal is found by counting the number of 3 cm segments that are created before the two groups are at the same point.

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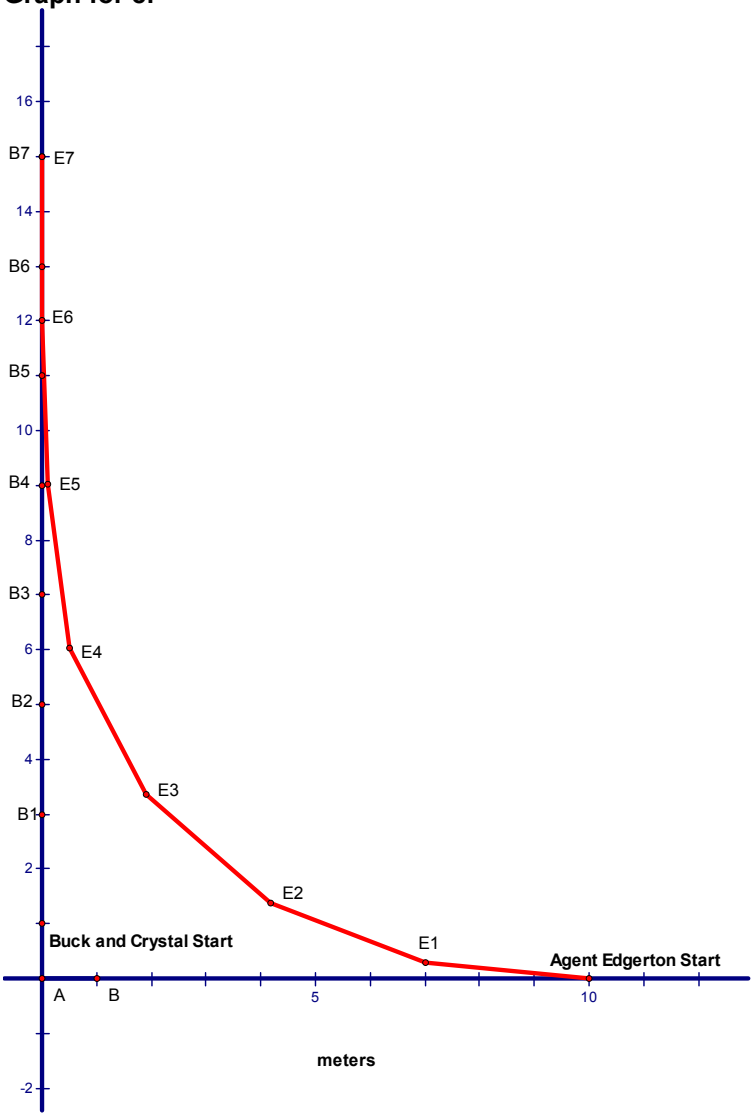
Student Page Answers:

1. *Answers will vary.* **2.** *5 hours* **3.** *1.25 hours* **4.** *Agent Edgerton will never catch Buck and Crystal. Agent Edgerton is traveling at the same speed as Buck and Crystal and will always be 10 miles behind.* **5.** *10.5 miles per hour* **6.** *Answers will vary.* **7a.** *Agent Edgerton needs to travel* $\sqrt{2041} \approx 45.18$ *miles , so it would take him about 4.5 hours to get to the point. Buck and Crystal would arrive in 5 hours. Agent Edgerton would get there first.* **7b.** *28 miles* **8.** *About 7 seconds. (See graph below)* **9.** *Agent Edgerton will never catch Buck and Crystal. Buck and Crystal travel due north while the direction of Agent Edgerton keeps changing. He is always south of them.*

Graph for 8.



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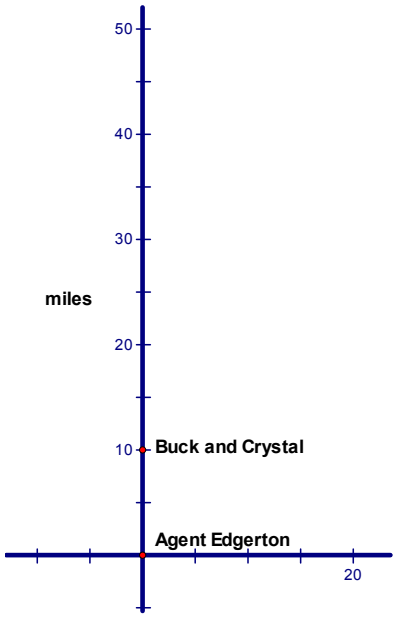
NUMB3RS Activity: The Chase

In "Spree, Part 1," Buck and Crystal are a couple traveling across the United States, committing crimes everywhere they go. They are being chased by Agent Edgerton. Charlie creates a map showing the places that Buck and Crystal have visited and hopes that he can use the map to predict where Buck and Crystal will go next. In this activity, you will begin to explore the mathematics of pursuit curves.

A Simple Chase

Suppose Buck and Crystal stole bicycles and began riding up a street. Assume that they ride 8 miles per hour. Agent Edgerton sees them and begins to chase them on his bike.

1. Suppose Agent Edgerton bikes at 10 miles per hour. Place an "x" on the graph to the right where you think he will catch up to Buck and Crystal. Buck and Crystal start at (0,10) and Agent Edgerton starts at (0,0). Assume that both groups travel up the vertical axis on the graph.



2. The table at the right shows the number of miles from Agent Edgerton's starting point that each group travels as the chase progresses. Complete the table. Then use the table to determine how many hours it will take for Agent Edgerton to catch up to Buck and Crystal.

Time (t) Duration of Chase (hours)	Buck and Crystal's Position on Vertical Axis after t hours (miles)	Agent Edgerton's Position on Vertical Axis after t hours (miles)
0	10	0
0.5	14	5
1	18	
1.5		
2		
2.5		
3		
3.5		
4		
4.5		
5		
5.5		
6		

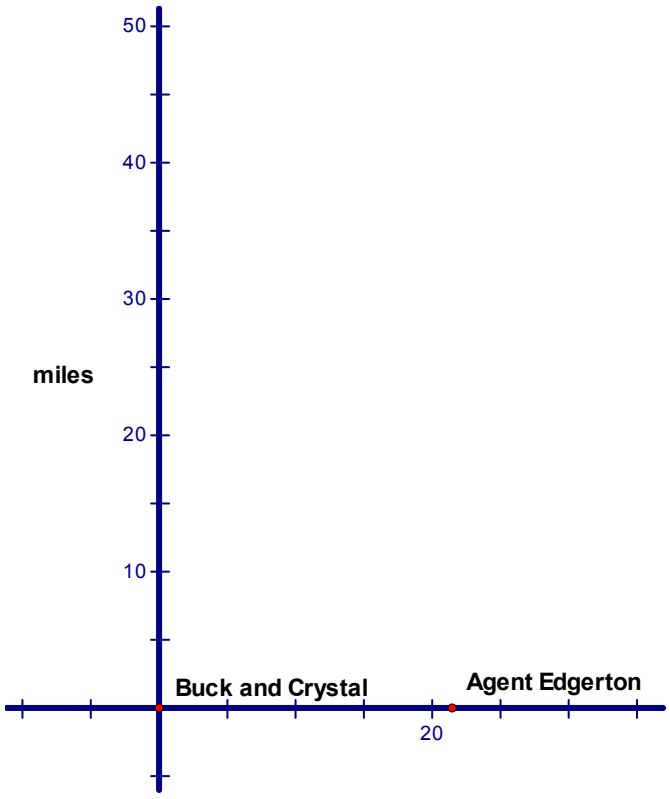
3. Suppose that both groups start at the same initial position as in Question 1, but this time, Agent Edgerton travels at 16 miles per hour. How many hours will it take Agent Edgerton to catch up to Buck and Crystal?

4. Answer the same question as in Question 3, but suppose that Agent Edgerton travels at 8 miles per hour. Explain your answer.

5. Find the speed that Agent Edgerton would have to travel if he catches Buck and Crystal after 4 hours.

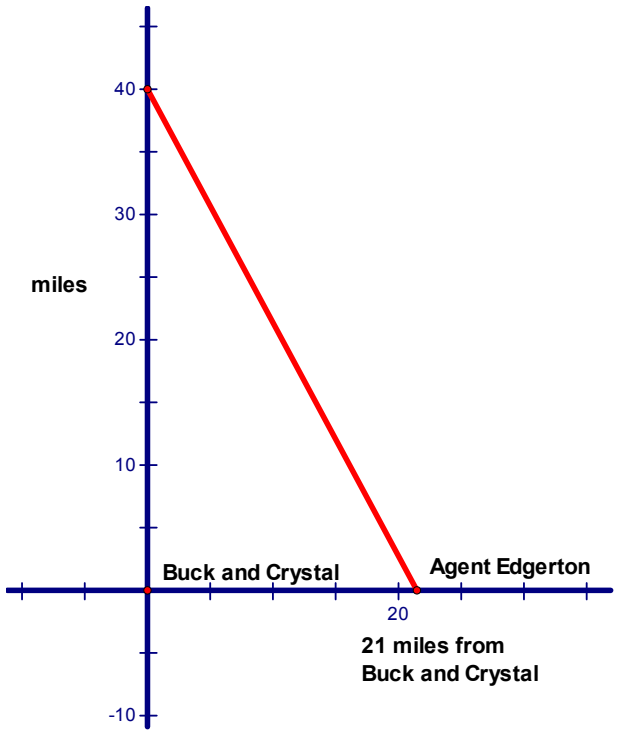
A More Complex Chase

A more complex chase scenario is shown in the graph below. Buck and Crystal will again travel up the road along the vertical axis at a speed of 8 miles per hour. Agent Edgerton begins 21 miles away and will travel at 10 miles per hour. Agent Edgerton decides that he will choose a point on the vertical street and ride in a line towards that point. He hopes that he will arrive at that spot at the same time as Buck and Crystal.



6. Place an "x" on the vertical axis where you think Agent Edgerton will meet Buck and Crystal at the same time.

7. Suppose Agent Edgerton chooses the 40 mile marker on the vertical axis as the point he will aim towards.



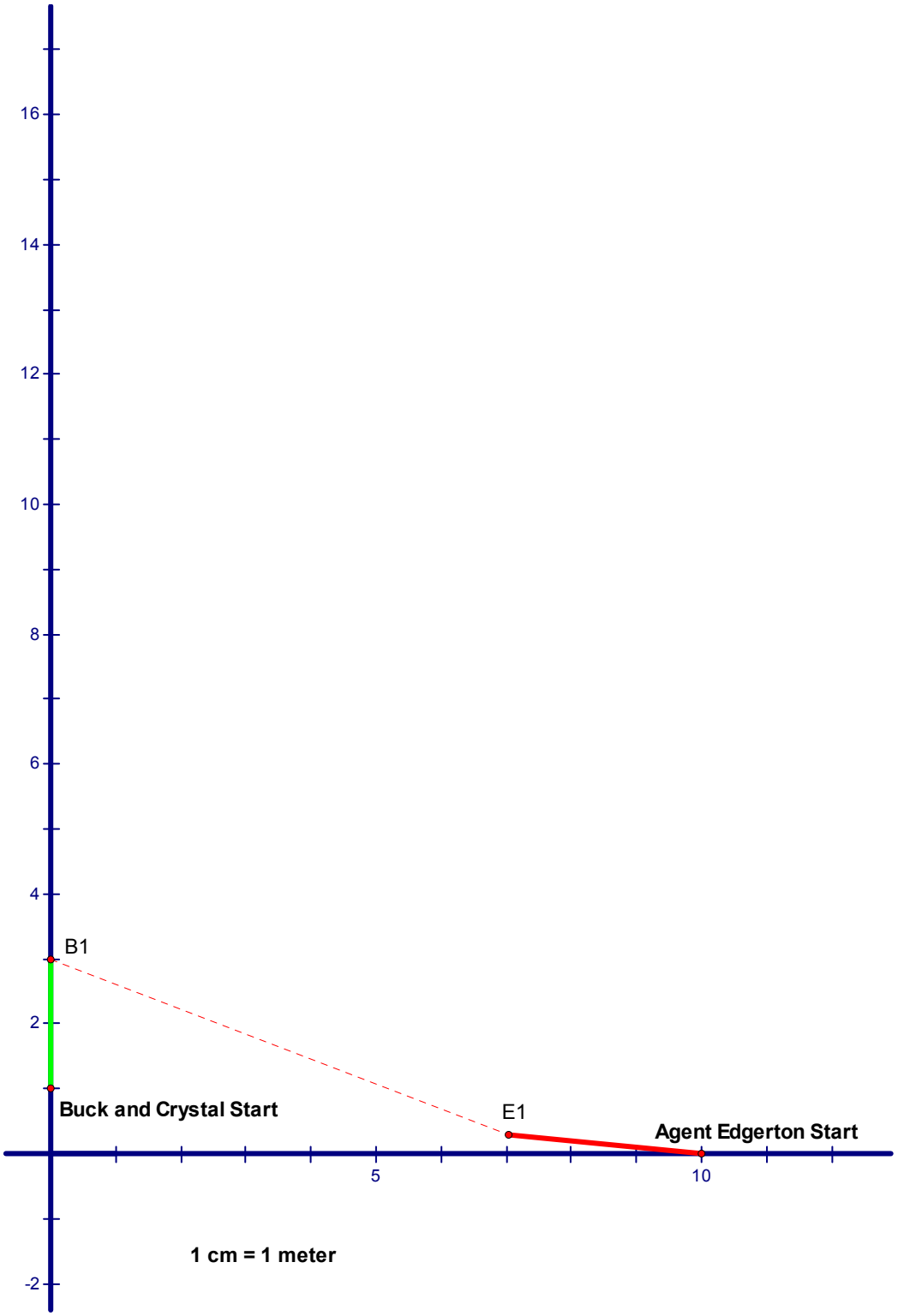
a. Who will get to the point first? Explain how you found your answer.

b. Find a point on the vertical axis where both groups would arrive at the same time. Explain the methods you used to find your answer.

Final Chase

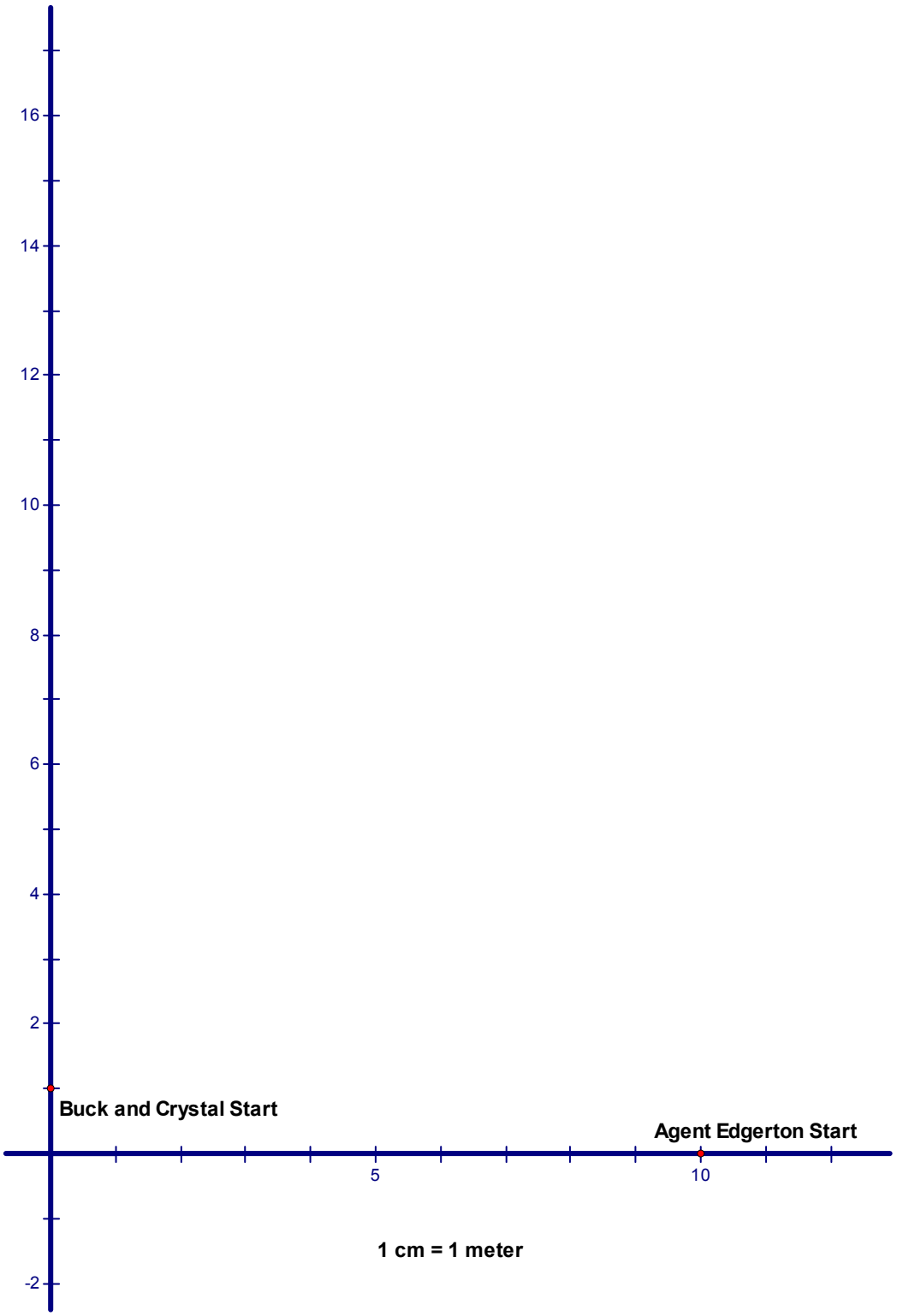
The last section of this activity is designed to introduce pursuit curves. If Agent Edgerton was actually chasing Buck and Crystal, he probably would not know the speed that they were traveling and he probably would not have the time to calculate the exact point he should aim towards. In a pursuit, the pursuer (Agent Edgerton) will make adjustments to his chase as he is chasing his target (Buck and Crystal).

Suppose that Agent Edgerton saw Buck and Crystal as shown in the diagram on the next page. Assume Buck and Crystal move along the vertical axis at a speed of 2 meters per second. Assume Agent Edgerton adjusts his path once per second (this assumption makes it possible to draw the curve using the methods described next). Suppose Agent Edgerton runs 3 meters in the direction that he currently sees Buck and Crystal in one second. The point E1 was drawn by placing a ruler along the starting points for both groups then measuring towards Buck and Crystal's starting point 3 centimeters from Agent Edgerton's starting point. (Each meter is represented by 1 centimeter on the graph.) Point E1 represents the position of Agent Edgerton after 1 second. Point B1 was found by measuring up the vertical axis 2 centimeters and represents the position of Buck and Crystal after 1 second. Find the point E2 by measuring 3 centimeters from E1 along the line between E1 and B1. Then find B2 by moving up 2 centimeters from B1. Repeat this process until Agent Edgerton catches up with Buck and Crystal. The curve you create on this graph is an approximation of a **pursuit curve**.



8. Approximately how long will it take for Agent Edgerton to catch Buck and Crystal?

9. Repeat the process again, but this time assume that Agent Edgerton travels at a speed of 2 meters per second. How many seconds will it take for Agent Edgerton to catch Buck and Crystal? Explain your answer.



The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extensions

Pursuit Curves

- An animated description of pursuit curves can be found at **http://curvebank.calstatela.edu/pursuit/pursuit.htm**.
- A more complex description of pursuit curves can be found at **http://mathworld.wolfram.com/PursuitCurve.html**.
- Try to find the speed that Agent Edgerton would have to travel so that he would catch Buck and Crystal at the 15 centimeter mark on the vertical axis (as shown in Question 8).
- Use parametric equations to model the path of the two groups for Questions 1, 3, 4, 5, and 7. Use your TI-83 Plus/TI-84 Plus graphing calculator to model these situations. To learn more about parametric equations, go to the Web site below. **http://mathworld.wolfram.com/ParametricEquations.html**

Challenge – Pursuit Curves

Questions 8 and 9 can also be solved using parametric equations. Try to find the equations that model the paths of the two groups.

Related Activities

A related activity at the NCTM Illuminations Web site includes an interactive Java applet that explores some variables that influence the path of an airplane chasing a storm. This can be found at **http://illuminations.nctm.org/ActivityDetail.aspx?ID=43**. A more detailed version of this activity can be found at **http://standards.nctm.org/document/eexamples/chap7/7.1/part2.htm**.

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