Inflection Points
ID: 9993

Time required
45 minutes

Activity Overview
Students will investigate points of inflection on a function and its first and second derivatives, and discover how they relate to each other.

Concepts
- *Concavity*
- *First and second derivatives*
- *Points of inflection*

Teacher Preparation and Notes
This investigation offers an approach to show students how they can find points of inflection on the graph of a function and its first and second derivatives.

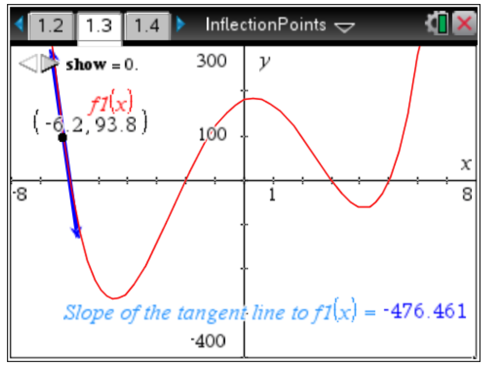
- *This activity could be used in Calculus as an introduction to what inflection points are and how to find them. Students should already know the definition of concavity and the concept of first and second derivatives. Students should know how to move points around on a graph.*
- *This activity is intended to be* ***teacher-led*** *at first and then students may be able to work in small groups, with partners, or individually. You may use the following pages to present the material to the class and encourage discussion. Students will follow along using their handhelds. The majority of the ideas and concepts are only presented in this document, so be sure to cover all the material necessary for students' total comprehension.*
- *The student worksheet InflectionPoints_Student.doc is intended to guide students through the main ideas of the activity, while providing some more instruction on how they are to perform specific actions using the tools of the TI-Nspire™ handhelds.*
- *Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.*
- ***To download the student TI-Nspire™ document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "9993" in the keyword search box.***

Associated Materials
- *InflectionPoints_Student.doc*
- *InflectionPoints.tns*

Suggested Related Activities
To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

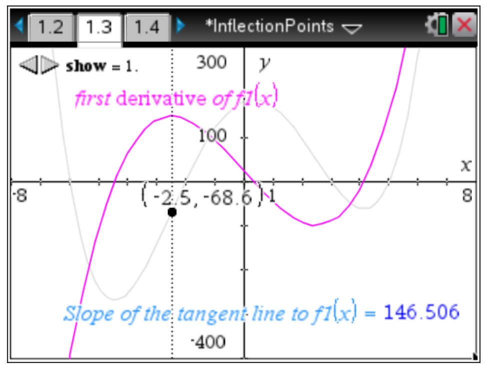
- *Higher Order Derivatives (TI-Nspire™ CAS technology) — 9325*
- *Concavity (TI-Nspire™ technology) — 16089*
- *Helicopter Bungee Jump (TI-Nspire™ CAS technology) — 11761*

Problem 1 – Finding points of inflection graphically



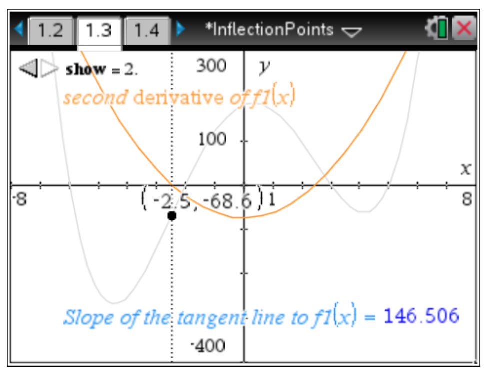
On page 1.3, the graph of **f1**(*x*) is shown. On the graph, students will move the tangent line and approximate the point where the concavity changes. Have a class discussion about where this would happen in relation to the slope of the tangent line. When the rate is increasing, the graph is concave up and when it is decreasing, the graph is concave down. Students should have some knowledge of concavity before this activity. Concavity should change around $x = -2.5$ and $x = 2.5$.

After students have presented their hypotheses, they need to define these points as points of inflection.



Students are given a graph of the first derivative of **f1**(*x*) on page 1.3 by clicking the 'show' minimized slider. Students need to use the points they found previously and find them on the graph of the first derivative. Students should see that the points are at the local minimum and maximum.

Discuss with the class that these points would occur where the slope changes from decreasing to increasing or vice versa so that would create a minimum or maximum point on a slope graph.



Next, students are given a graph of the second derivative **f1**(*x*) on page 1.3 when 'show=2' and are asked to see where the points of inflection show up on this graph. They occur at the *x*-intercepts.

Students are asked to summarize their findings:

- Points of inflection on the graph of a function occur where the graph changes concavity.
- Points of inflection for a function can be found on the graph of the first derivative of that function at the local minimum and maximum points.
- Points of inflection for a function can be found on the graph of the second derivative of that function at the *x*–intercepts. Students should be aware that *x*–intercepts of the second derivative are only *candidates* for inflection points. Every inflection point is an *x*–intercept of the second derivative, **but** not every *x*–intercept of the second derivative is an inflection point.

TI-Nspire™ Navigator™ Opportunity: *Class Capture*

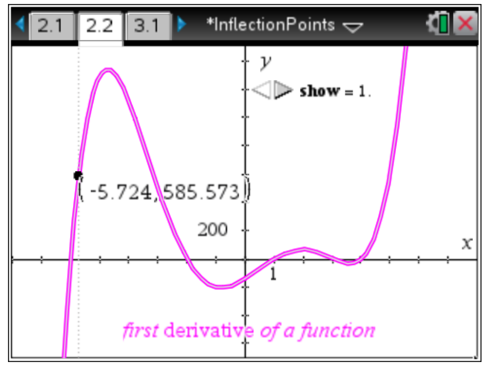
See Note 1 at the end of this lesson.

TI-Nspire™ Navigator™ Opportunity: *Quick Poll*

See Note 2 at the end of this lesson.

Problem 2 – Test your knowledge

On page 2.2, students are given a graph of the first derivative of a function and asked to find the points of inflection for the function. Students should use the point on the graph to find local minimum and maximum points. Points of inflection can be found at x = –4.727, –0.801, 1.946, and 3.582.



Students are then asked to verify their solutions using the graph of the second derivative ('show=2').

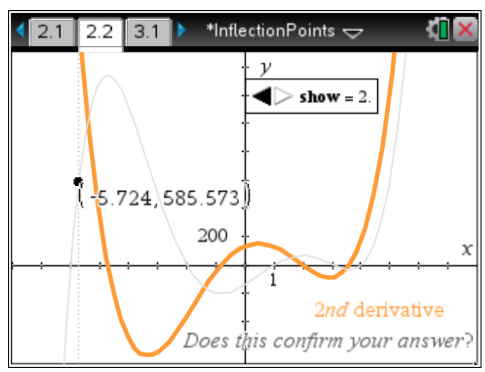
TI-Nspire™ Navigator™ Opportunity: *Class Capture*

See Note 3 at the end of this lesson.

TI-Nspire™ Navigator™ Opportunity: *Quick Poll*

See Note 4 at the end of this lesson.

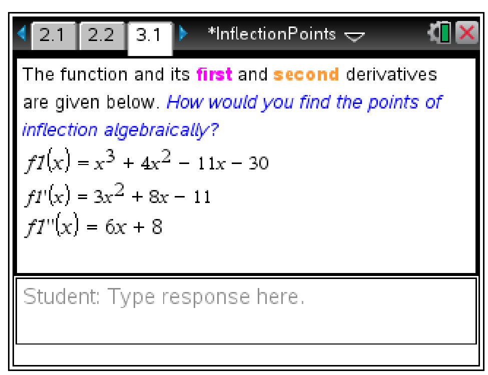
Problem 3 – Finding points of inflection algebraically

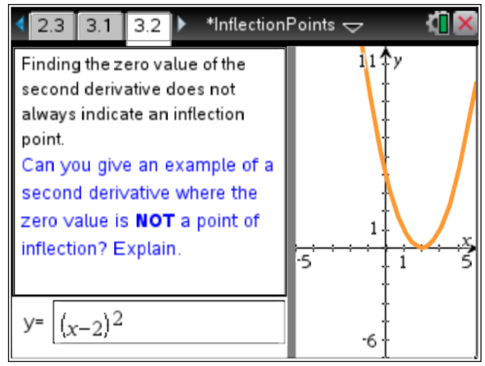


On page 3.1, students are given the function **f1**(x), along with its first and second derivatives. This is an opportunity to discuss with students the best way to algebraically find points of inflection. For the original function, that would be quite difficult.

For a first derivative, students would need to find the local maximum or minimum points which would be easy with a quadratic but much harder for anything else.

Using the second derivative, set the derivative equal to zero and solve for x. Students will most likely find that using the second derivative will be the easiest. The inflection point for this graph is at $x = -\frac{8}{6}$.





Care must be taken to test the result of simply solving the second derivative set equal to zero. As noted before, these points are only *candidates* for inflection points. For some functions, the point where the second derivative equals zero does NOT produce a point of inflection, especially in functions of degree greater than three.

For example, the basic quartic, $y = x^4$, has zeros for the function and for the first and second derivatives at $x = 0$. However, $y = x^4$ does not change concavity at $x = 0$ so the point (0, 0) is NOT an inflection point.

Similarly, if the second derivative of a function is $y = (x - 2)^2$, it can be seen from the graph that the y-values do not change from positive to negative or negative to positive even though $x = 2$ is a zero of the second derivative.

TI-Nspire™ Navigator™ Opportunities

Note 1
Problem 1, *Class Capture*
Compare the values the students find for their inflection points. Have a discussion where the students give their observations.

Note 2
Problem 1, *Quick Poll*
Use the Question application questions on page 1.5, 1.6, and 1.7 as Quick Polls to check for understanding. You may wait to discuss the answers until all 3 of the questions are answered. The questions are set up for Self-Check so the questions can be given as homework and students can check their work at their own pace. Use the **Teacher Tool Palette** to change to Exam mode

Note 3
Problem 2, *Class Capture*
Compare the values the students find for their inflection points.

Note 4
Problem 2, *Quick Poll*
Use the Question application on 3.1 and 3.2 and discuss responses.

