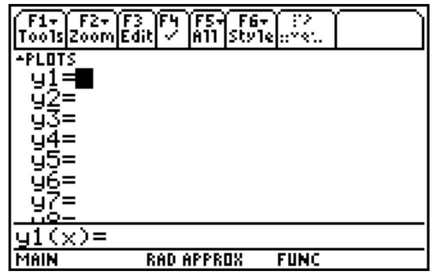
One Sided Limits

Name ______________________

Class ______________________

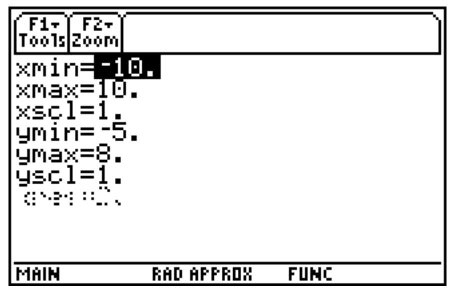
Set up – graphing piecewise functions that show discontinuity.

1) After turning on your device, go to the Y= screen by pressing [◆] [F1].



2) Turn the functions off or clear them; press **F1 > Clear Functions**.
Note: You can turn functions off by un-checking them using F4.

3) Turn Discontinuity Detection on. Press **F1 > Format** to find the option for Discontinuity Detection.



4) Set the window, using [◆] [F2], to the settings shown at the right.

5) Back on the Y= screen enter three piecewise functions.

At y1 press [ENTER]. Find **when(** in the CATALOG quickly by pressing [CATALOG] [.]. This shows the notation: **when(condition, true, false)**

For y1, type **when(x<1,1,a)|a=5**

The "such that" bar key ([|]) is to the left of the [7] key.

$$y1(x)=\begin{cases}1, x<1\\ a, x\geq 1\end{cases}|\,a=5$$

For y2, type **when(x<1,x+2,a*x^2)|a=5**

$$y2(x)=\begin{cases}x+2, x<1\\ a\cdot x^2, x\geq 1\end{cases}|\,a=5$$

For y3, type **when(x<2,2sin((x–1)π/2), a+3sin((x–4)π/2))|a=5**

$$y3(x)=\begin{cases}2\sin\left((x-1)\dfrac{\pi}{2}\right) & ,x<2\\ a+3\sin\left((x-4)\dfrac{\pi}{2}\right) & ,x\geq 2\end{cases}|\,a=5$$

6) Graph one function at a time by using [F4] to have only one function checked at a time.

On a graph screen examine both sides of where the discontinuity exists using F3 Trace.

7) For Problems 1 and 2 below, use [◆] [F4] to have table settings of tblStart = 0.98 and Δtbl = 0.01, to numerically examine the left and right-hand limits. Be sure to press [ENTER] to save changes before pressing [◆] [F5] to view the table.

For Problems 1, 2, and 3 estimate the limits graphically and numerically using trace and table.

Problem 1

$$y1(x)=\begin{cases}1, x<1\\ a, x\geq 1\end{cases} \mid a=5$$

$\lim\limits_{x\to 1^-} y1(x) \approx$ __________

$\lim\limits_{x\to 1^+} y1(x) \approx$ __________

Try other values for ***a*** in the graph of *y*1(x) to find what ***a*** makes $\lim\limits_{x\to 1} y1(x)$ exist. On the Y= screen, press [ENTER] when y1 is highlighted. Press ► and then backspace [←] to try different values for ***a***. Graph it to see if appear continuous.

a = __________

Problem 2

$$y2(x)=\begin{cases}x+2, x<1\\ a\cdot x^2, x\geq 1\end{cases} \mid a=5$$

$\lim\limits_{x\to 1^-} y2(x) \approx$ __________

$\lim\limits_{x\to 1^+} y2(x) \approx$ __________

Try other values for **a** in the graph of *y*2(*x*) to find what ***a*** makes $\lim\limits_{x\to 1} y2(x)$ exist.

a = __________

Show calculations of the left hand limit and the right hand limit to verify that your value for ***a*** makes the limit exist.

Problem 3

$$y3(x)=\begin{cases}2\sin\left((x-1)\frac{\pi}{2}\right) \quad , x<2\\ a+3\sin\left((x-4)\frac{\pi}{2}\right), x\geq 2\end{cases} \mid a=5$$

$\lim\limits_{x\to 2^-} y3(x) \approx$ __________

$\lim\limits_{x\to 2^+} y3(x) \approx$ __________

Try other values for **a** in the graph of *y3*(*x*) to find what ***a*** makes $\lim\limits_{x\to 2} y3(x)$ exist.

a = __________

Show calculations of the left hand limit and the right hand limit to verify that your value for ***a*** makes the limit exist.

Extension – Continuity

A function is continuous at $x = c$ if:

- $f(c)$ exists
- $\lim_{x \to c} f(x)$ exists, and
- $\lim_{x \to c} f(x) = f(c)$

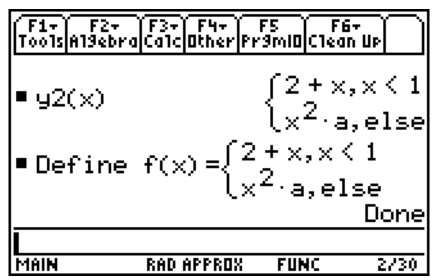
Use CAS to algebraically solve for ***a*** that makes

(a) $\lim_{x \to 1} y2(x)$ exist

(b) $\lim_{x \to 2} y3(x)$ exist

Then prove each function is continuous.

<u>Key press help:</u>



- Begin by pressing [HOME]. Clean Up the screen by pressing [2nd] [F1]. Choose **NewProb** and press [ENTER] to put this on the command line and [ENTER] to execute the command.
- Type **y2(x)** [ENTER]. The Define command is under the F4 menu. Type **Define f(x)=**, then up arrow to highlight the output from the previous line. Press [ENTER] on the highlighted piecewise function to copy it down to the command line.
- To solve a right sided limit, press [F3] > limit(. On the command line enter **limit(f(x),x,1,1)** [ENTER].
- Now, press [F2] [ENTER] to select **solve(**. Then up arrow to select the input from the previous line, press [ENTER]. Next type [=]. Up arrow to the input again and press [ENTER]. This time put a negative [(-)] in front of the last 1. Finally type [,] [alpha] [=] and close the parentheses. This method will enable you to quickly enter **solve(limit(f(x),x,1,1)=limit(f(x),x,1,–1),a)**.

