# Transformations – Graphical Dilation

**Student Activity**

7 8 **9** 10 11 12

TI-Nspire™ | Investigation | Student | 50 min

## Teacher Notes:

How much do students really understand about transformations?

Vertical transformations are more intuitive than horizontal. Horizontal changes feel "backwards" because they act on inputs, students often stay at a rule-following level rather than understanding the underlying process; this is particularly noticeable when it comes to dilations where the conceptual chasm widens.

Multiple representations and dynamic technology can help students bridge the gap. "technology helped students coordinate algebraic and graphical views" (Koyunkaya, M. Y., & Boz-Yaman, B. 2023). This aligns with previous findings (Anabousy, A., Tabach, M., & Nachlieli, T. 2015/2016) that when using dynamic software, horizontal stretches/compressions were the hardest transformations to predict correctly. Students improved when they could manipulate sliders and see the immediate effect, reinforcing the role of interactivity.

Challenge: Ask students familiar with quadratic functions of the form: $y = a(x - h)^2 + k$ to explain the transformations, beyond "h shifts it right…"; all sorts of *interesting* reasons why the negative sign preceding the *h* translates in the positive x direction compared with the positive sign preceding the *k* translates in the positive y direction. Even more challenging is their explanation around dilations!

This activity focuses on dilations and begins with the more familiar geometry space. It provides a quick review of AC9M7SP03 where students will have worked with transformations of points on the Cartesian plane.

Concepts covered here help prepare students for higher level mathematics such as "invariance of properties under transformation, and the relationship between the determinant of a transformation matrix and the effect of the linear transformation on the area of a bounded region of the plane."
[VCE Specialist Mathematics]

## Australian Curriculum Standards

**AC9M7SP03**

Describe transformations of a set of points using coordinates in the Cartesian plane, translations and reflections on an axis, and rotations about a given point

**AC9M9SP02**

Apply the enlargement transformation to shapes and objects using dynamic geometry software as appropriate; identify and explain aspects that remain the same and those that change

**AC9M9A06**

Investigate transformations of the parabola $y = x^2$ in the Cartesian plane using digital tools to determine the relationship between graphical and algebraic representations of quadratic functions, including the completed square form, for example: $y = x^2 \rightarrow \; y = \frac{1}{3}x^2$ (vertical compression) …



Author: P. Fox

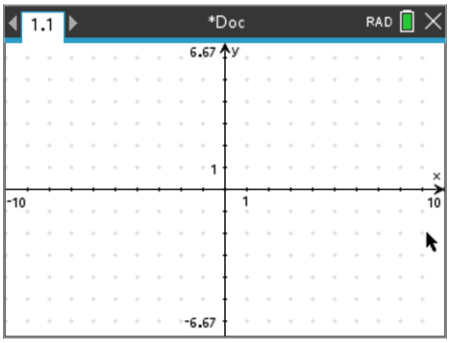
# Calculator Instructions: Transformations

Create a new TI-Nspire document and insert a Graphs application.

Displaying the grid will make it easy to keep the numbers simple.

menu > **View** > **Grid** > **Dot Grid**

To hide the Graph entry, press: esc



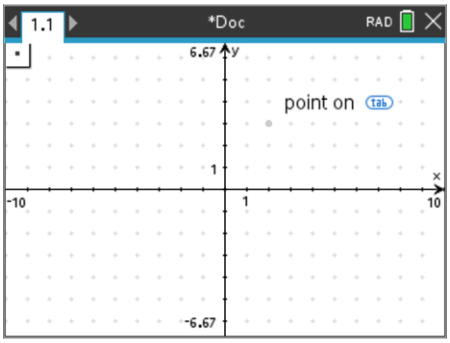

The keyboard shortcut to create a point in the Graphs application is to press **P**.
There are two options: Point or Point by Coordinates. Both options are used in this activity.

Place a point **on** the grid, "point on" appears as a prompt when the pen is close to the grid.

Once the point has been created, get the coordinates of the point:

ctrl + menu > **Coordinates & Equations**

Press esc to release the Point tool, then grab the point an move it around. The point should maintain integer coordinate values since the axis scale in both directions are provided in integer amounts.



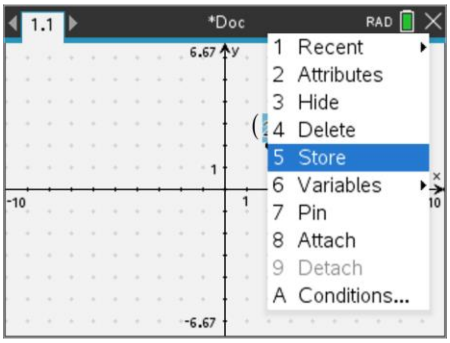

Hover the mouse over the abscissa (x – coordinate) then press:

ctrl + menu > **Store**

Store the abscissa as **xp**.

Repeat this process and store the ordinate (y – coordinate) as **yp**.

The coordinates will now appear bold. These values can be accessed and used in calculations in any other application within this problem.




Hover the mouse over the coordinate pair to see the variable link.

Create a new point using the keyboard shortcut, select:

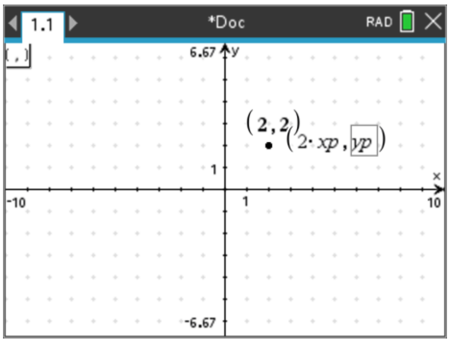
**Point by Coordinates**

For the abscissa, type: $2 \times xp$

For the ordinate, type: $yp$

**Note**: To navigate from the abscissa to the ordinate press: enter

This new point is referred to as "an image of point P".




Think about the image of point P just like you would a photographic image. The image can be enlarged or reduced in size stretching uniformly in both directions. A stretch in the horizontal or vertical direction only will change the proportion or form of objects in the image making them look taller or shorter, wider or narrower.



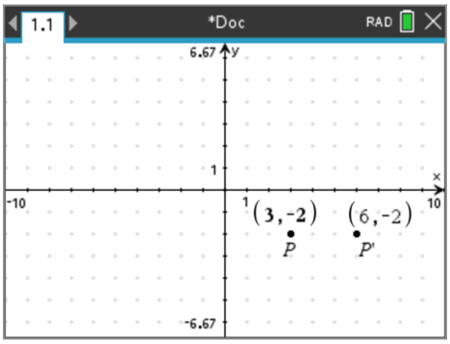
Author: P. Fox

Label the points: P and P′ where P′, pronounced “P prime” is the dilation of the original point P.

Move the mouse over the point then:

ctrl + menu > **Label**

The ‘prime’ notation can be accessed from: ?!▸




The coordinates of points P and P′ automatically move with their respective points. Sometimes this can make reading the coordinates difficult. The coordinates can be moved away from their respective points. Think of them as ‘magnetic’, once moved far enough away they will detach and remain stationary.

**Question: 1.**

Move point P horizontally.

a) Describe how the point P′ moves.

b) Are there any locations where the coordinates of P and P′ are the same?

c) Thinking about a photograph, how would you describe the objects in the image compared with the original?

**Answer**:

a) Point P′ is a dilation of point P ‘away’ from the y axis by a factor of 2, the abscissa of P′ is twice the value of the abscissa for P.

b) When P is of the form (0, $yp$), the coordinates of P and P′ are the same.

c) Objects in the image would look ‘wider’ than objects in the original image.

**Question: 2.**

Move point P vertically.

a) Describe how the point P′ moves.

b) Are there any locations where the coordinates of P and P′ are the same?

**Answer**:

a) The points move parallel to one another. Point P′ is still a dilation of 2 away from the y axis.

b) When P is on the y axis: (0, yp), the coordinates of P and P′ are the same.

**Question: 3.**

Edit the formula used for the abscissa of point P′ so that it is half the value of $xp$.

a) Describe how point P′ moves after this edit.

b) Are there any locations where the coordinates of P and P′ are the same?

c) Thinking about a photograph, how would you describe the objects in the image compared with the original?

**Answer**:

a) Point P′ is still a dilation of point P ‘away’ from the y axis but the factor is now ½.
The abscissa of P′ is half the value P.

b) When P is on the y axis: (0, yp), the coordinates of P and P′ are the same.

c) Objects in the image would look narrower than the original picture.

**Question: 4.**

Edit the formula used for the abscissa of point P′ so that it is equal to $xp$.
Change the formula for the ordinate so that it is twice the value of $yp$.

a) Describe how point P′ moves after this edit.

b) Are there any locations where the coordinates of P and P′ are the same?

c) Thinking about a photograph, how would you describe the objects in the image compared with the original?

**Answer**:

a) Point P′ is a dilation of point P 'away' from the x axis. The ordinate of P′ is twice the value of the ordinate for point P.

b) When P is on the x axis: (xp, 0), the coordinates of P and P′ are the same.

c) An object in the image will look taller than an object in the original picture.
Note: This is *similar* to making an object look thinner, the two transformations are not mutually exclusive.

**Question: 5.**

Edit the formula used for the abscissa of point P′ so that it is equal to: $2 \times xp$.
The formula for the ordinate remains as: $2 \times yp$.

a) Describe how point P′ moves after this edit.

b) Are there any locations where the coordinates of P and P′ are the same?

c) Thinking about a photograph, how would you describe the objects in the image compared with the original?

**Answer**:

a) Point P′ is a dilation of point P 'away' from both the x and y axes. The abscissa of P′ is twice the value of the abscissa for point P and the ordinate for point P′ is twice the value for the ordinate of point P.

b) When P is at the origin, the coordinates of P and P′ are the same: (0, 0).

c) An object in the image will look twice as large as an object in the original picture, however, it will be in the correct proportion.

The calculator's touchpad can be used as a touchpad by swiping or as single directional movement by clicking in the corresponding direction. (Arrow keys) The single direction movement is useful in menus.

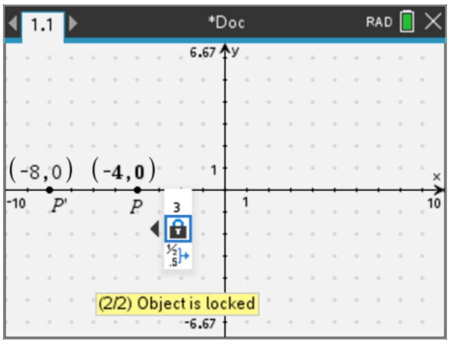
Place point P on the x axis.

Move the cursor over the ordinate for point P and press:

ctrl + menu > **Attributes**

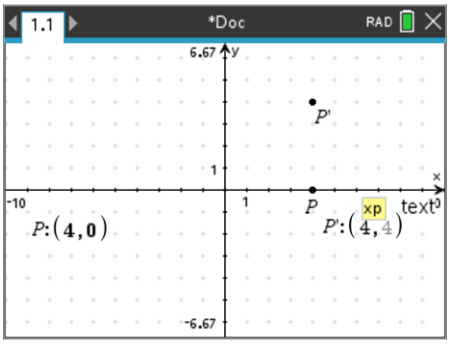
Navigate to the padlock. Move to the right to lock the ordinate to zero.

Try and move point P off the x axis.






Author: P. Fox




Our image (P′) is going to be linked or mapped to the abscissa only.

Edit the rule for P′ so that:

The abscissa is equal to: *xp*

The ordinate is equal to: *xp*

Drag P along the $x$ axis. Observe the movement of P′, the image of P.

It is possible to leave a temporary record of the path of P′.

[menu] > **Trace** > **Geometry Trace**

Click on the point P′, then drag P along the $x$ axis.

**Question: 6.**

What is the equation for the path of point P′ ?

**Answer**:

Point P′ moves along the path: $y = x$.

**Question: 7.**

Change the definition for P′: The abscissa changes to xp/2 while the ordinate remains as yp.

a) Thinking about dilations, what is happening to point P′ ?

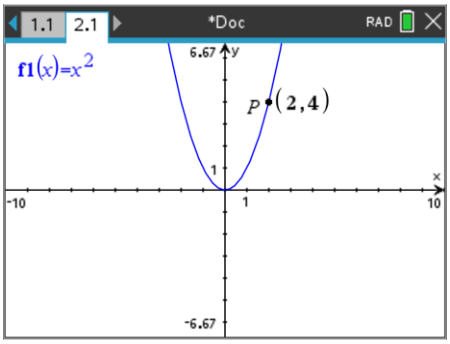
b) Change the colour of point P′ and do a geometry trace of its new path and determine the corresponding equation.

**Answer**:

a) Point P′ experiences a dilation by a factor of ½ towards the $y$ axis, it continues to move in a straight line.

b) Point P′ moves along the path: $y = 2x$.

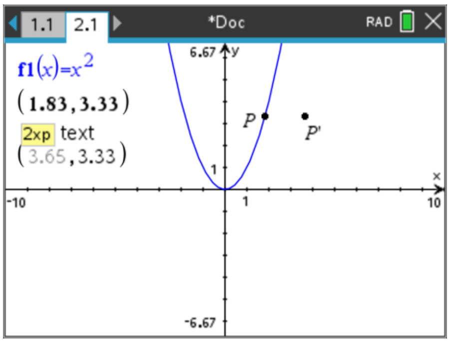
## Transformation Algebra




Insert a new problem into the current TI-Nspire document.

[doc▾] > **Insert** > **Problem**

- Insert a Graphs application and graph the function: $y = x^2$.
- Place a point on the graph. Display the coordinates and store them as xp and yp.
- Label the point: P




Insert a **Point by Coordinates** and set them as follows:

**(2×xp, yp)**

Label the point as: P′

Drag point P along the function and observe the path traced out by: P′.



Author: P. Fox

Point P(xp,yp)[1] is no longer free to move anywhere; it now moves along the function: $y = x^2$. The coordinates of P′ are defined by xp and yp, so the path of P′ can also be defined, the aim here is to determine the equation for that path.

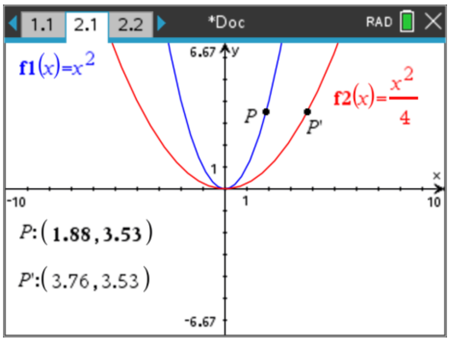
Point P′ will be defined as: P′($x'$,$y'$) & Point P as P($x$, $y$)

We need to find a rule that relates $x'$ to $y'$. Our relationships, as defined on the calculator are as follows:

| | Equation 1 | Equation 2 | Equation 3 |
|---|---|---|---|
| | $x' = 2 \times xp$ | $y' = yp$ | $y = x^2$ |
| **Step 1:** Changing the notation: | $x' = 2x$ | $y' = y$ | $y = x^2$ |
| **Step 2:** Express Eqn1 & Eqn2 in terms of $x$ and $y$ respectively: | $x' \div 2 = x$ | $y' = y$ | |
| **Step 3:** Substitute Eqn 2 into Eqn 3 | | | $y' = x^2$ |
| Substitute Eqn 1 into Eqn 3 | | | $y' = \left(\frac{x}{2}\right)^2$ |

With the relationship established, the 'prime' notation can now be removed and the function graphed:

$$y = \frac{x^2}{4}$$




- Point P lies on the graph with equation: $y = x^2$
- Point P′ lies on the graph with equation: $y = \frac{x^2}{4}$
- The relationship between P(xp,yp) and P′(x′,y′) is such that: $x' = 2 \times xp$
- The dilation factor is 2 units away from the y axis.

**Question: 8.**

Change the definition for P′: The abscissa changes to xp/2 while the ordinate remains as yp.

a) Thinking about dilations, what is happening to point P′ ?

b) Determine the relationship between (equation) $x'$ and $y'$.

c) What is the dilation factor for the graph?

**Answer**:

a) Point P′ is 'squeezed' towards the y axis or experiences a dilation of ½ towards the $y$ axis..

b) Equation: $y = 4x^2$

c) Dilation factor: ½ towards the y axis.

[1] Point P is expressed in terms of (xp, yp). This notation reflects the limitation of the digital platform rather than mathematical terminology. Assigning values to variable names such as $x$ and $y$ on the calculator means they will no longer treated as variables, however, it is important to maintain correct mathematical notation, written notes are not bound by such limitations.



Author: P. Fox

**Question: 9.**

Change the definition for P′: The abscissa changes to xp while the ordinate changes to: $2 \times yp$.

a) Thinking about dilations, what is happening to point P′ ?

b) Determine the relationship between (equation) $x'$ and $y'$.

c) What is the dilation factor for the graph?

**Answer**:

a) Point P′ is stretched away from the $x$ axis or experiences a dilation of 2 away the $x$ axis..

b) Equation: $y = 2x^2$

c) Dilation factor: 2 away from the $x$ axis.

**Question: 10.**

Change the definition for P′: Leave the abscissa as xp and change the ordinate to: $yp \div 2$.

a) Thinking about dilations, what is happening to point P′ ?

b) Determine the relationship between (equation) $x'$ and $y'$.

c) What is the dilation factor for the graph?

**Answer**:

a) Point P′ is squeezed towards the $x$ axis or experiences a dilation of 1/2 towards the $x$ axis..

b) Equation: $y = \frac{x^2}{2}$

c) Dilation factor: ½ towards the $x$ axis.

**Question: 11.**

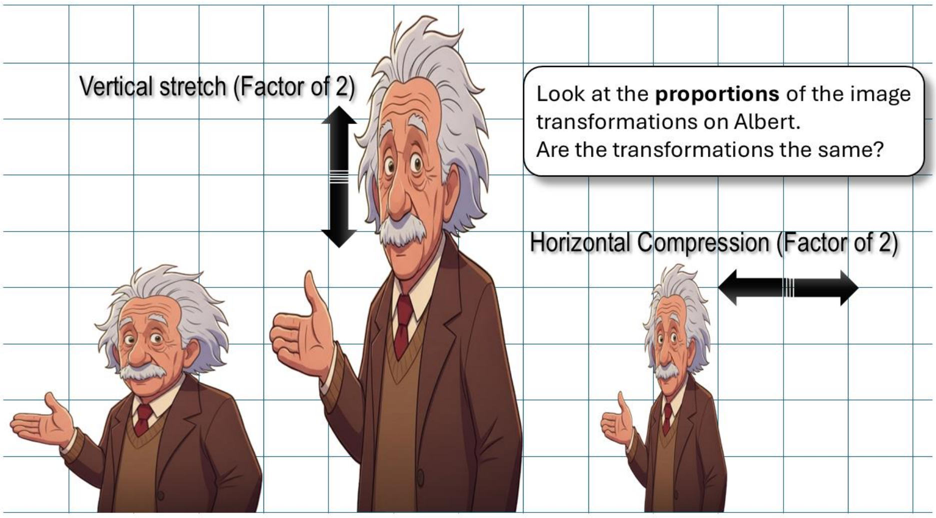
If the dilation factor for a quadratic is ½ towards the $y$ axis, what would it be equivalent to if referenced against the $x$ axis? Explain the similarities / differences and support your answer with appropriate equation(s).

**Answer**:

The equivalent transformation would be 4 from the $x$ axis. Equation: $y = 4x^2$

Differences / Similarities: The algebraic relationship between the variables is the same, so they are algebraically equivalent. The subtle differences are best explained through an image.

The original image (left) has been dilated by a factor of 2 as a vertical stretch and also by a factor of 2 by a horizontal compression, the aspect ratio or proportion of the two dilated images is the same.






Author: P. Fox