## Extension: Linear and Exponential Models

## Connecting Algebra 2 to Statistics

Name $\qquad$
You can transform an exponential function into a linear function by taking the logarithm of each side. Since linear models are easy to recognize, you can then determine whether an exponential function is a good model for the data.

1. Write the general form of an exponential function.
2. Take the Logarithm of each side.
3. Use the Product Property and Power Property to Expand the right side of your equation.
4. Then rewrite the equation in $y=m x+b$.

$$
y=a b^{x}
$$

$$
\log y=\log a b^{x}
$$

$$
\log y=\log a+(\log b) x
$$

$$
\log y=(\log b) x+\log a
$$

If $\log b$ and $\log a$ are constants, then $\log y=(\log b) x+\log a$ is a linear equation in slope-intercept form.
In Statistics, data is collected from many different sources. We have analyzed data that showed a linear pattern. However, some data show non-linear patterns. We can use our knowledge of exponential and logarithmic equations to transform data and determine if an exponential model is suitable for the data.

## Problem 1

Determine whether an exponential function is a good model for the values in the table.

| x | 0 | 2 | 4 | 6 | 8 | 10 |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| y | 3 | 5.1 | 8.6 | 14.5 | 24.5 | 41.4 |

Step 1: Enter the data into a Lists and Spreadsheet Page
Step 2: Graph the data on a Data and Statistics Page
Describe the direction, form, and strength of the relationship in the scatter plot.
The data appear to have a positive strong nonlinear pattern.

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Step 3: Now, go back to your Lists and Spreadsheet Page and create a new list. Call this list ly for log y. We are going to plot the $x$ values to the logarithm of the $y$ values. But first, you must find the logarithmic values for all the $y$ values in your table. As shown in the screen shot below, you must enter a formula in line below the label ly. Once you have created the new list, insert a new data and statistics page and plot the $x$ values against the log $y$ values (ly).

|  | $\mathrm{A}_{\mathrm{XV}}$ | $B^{\text {yv }}$ | Cly | D |
| :---: | :---: | :---: | :---: | :---: |
| - |  |  | $=\log (\mathrm{y} v, 10$ |  |
| 1 | 0 | 3. | 0.477121 |  |
| 2 | 2 | 5.1 | 0.70757 |  |
| 3 | 4 | 8.6 | 0.934498 |  |
| 4 | 6 | 14.5 | 1.16137 |  |
| 5 | 8 | 24.5 | 1.38917 | $\checkmark$ |
| $\left.C\right\|^{\mid y:=\log _{10}(\mathbf{y v})}$ |  |  |  |  |

Describe the transformation that occurred. Describe the form of the relationship. Add the regression line by pressing Menu, 4: Analyze, 6: Regression, 2: Show Linear ( $a+b x$ ). Write the regression equation below.

The data now appear to be in a linear positive strong correlation.
$y=.478426+.113872 x$

Step 4: Insert a Calculator page and press menu, 5: Statistics, 1: Stat Calculations, 4: Linear Regression $(a+b x)$. Does the linear regression show a strong correlation? Write your answer below along with the value of $r$ to justify your response.

The linear regression appears to be a strong relationship. The correlation rvalue is . 999998.

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Step 5: We are going to find the exponential model for our original data set. Now let's perform some Algebra on our equation from Step 3 to transform it from a linear equation to an exponential equation. Once we have found our exponential model algebraically, we will return to the original Data and Statistics Page and check our answer with the calculator generated exponential regression model. Once you are on the exponential scatter plot, press: Menu, 4: Analyze, 6: Regression, 8: Show Exponential. How does the Exponential equation found algebraically compare to that of the handheld's exponential model?

$$
\begin{aligned}
\log y & =.478426+.113872 x \\
10^{\log y} & =10.478426+.113872 x \\
y & =10^{.478426} 10.113872 x \\
y & =3.00903(1.29979)^{x}
\end{aligned}
$$

The model generated by the handheld is the same as the exponential model found algebraicallyby hand.

