What's the Angle? - Discoveries About Convex Polygons

Geometry Major Topics: Interior and Exterior Angles of Convex Polygons

NCTM Principles and Standards:
     Content Standards

Geometry:

Analyze characteristics and properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

Analyze properties and determine attributes of two- and three-dimensional objects.

Explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them.

Establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others.

Apply transformations and use symmetry to analyze mathematical situations.

Use various representations to help understand the effects of simple transformations and their compositions.

Objectives: This lesson has students discover properties of the interior and exterior angles of convex polygons. Using The Geometer's Sketchpad Application for the TI-89 Calculator, students construct various convex polygons and discover properties concerning the sums of their interior and exterior angles. The students are also asked to justify their findings. This lesson can be used as a teacher directed activity or can be given to the students as a discovery activity. If the students are familiar with The Geometer's Sketchpad Application for the TI-89, this activity should take one 50 minute class period.

**Materials:**   TI-89 Graphing Calculator, The Geometer's Sketchpad Application

Prerequisites: Students should have a working knowledge of The Geometer's Sketchpad Application for the TI-89 and know the sum of the measures of the interior angles of any triangle is 180 degrees.

Procedure:

Activity 1:   Open The Geometer's Sketchpad application on the TI-89 by pressing the APPS button and choosing Sketchpad. Press F1 and choose **A:Preferences**. Press F2:Units and set the **Angle Unit** to degrees, the **Angle Precision** to **1:1**, and the **Other Precision** to **1:1**. This will round all angle measures to the nearest degree and any calculations to the nearest whole number.

Press F8 to activate the Toolbar and use the cursor controls to highlight the **straightedge** tool.   Using the **segment** tool, construct and label a convex quadrilateral ABCD on the right side of the screen by constructing 4 segments that intersect only at their endpoints.
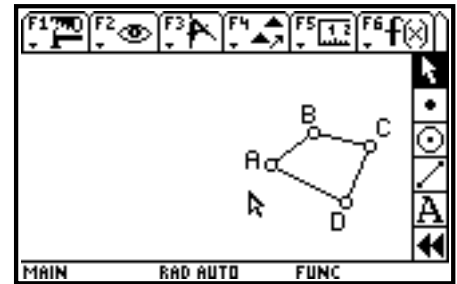
Press F8 and choose the **pointer** tool. Deselect the last segment constructed. Since we wish to measure an interior angle of the quadrilateral, select vertices A, B, and then C of the quadrilateral. Remember the second vertex selected will be the vertex of the angle measured. Press F5 and choose **5:Angle**. Angle ABC will be measured.
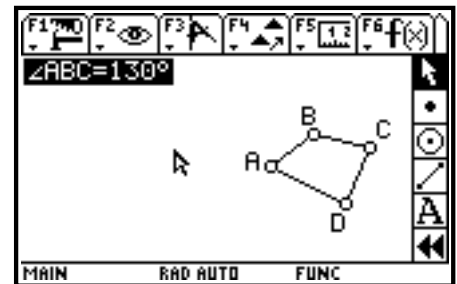
In a similar manner, measure the remaining three interior angles of the quadrilateral, remembering each time to deselect the angle measure before measuring the next angle. Press F5 and choose **C:Calculate**. Press F1 to show the four angle measures. Your screens should be similar to the ones below.
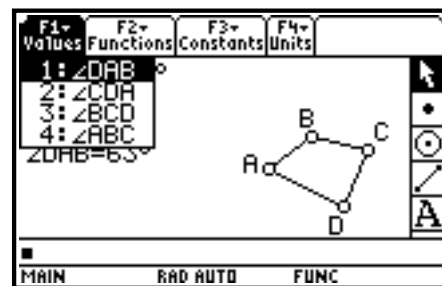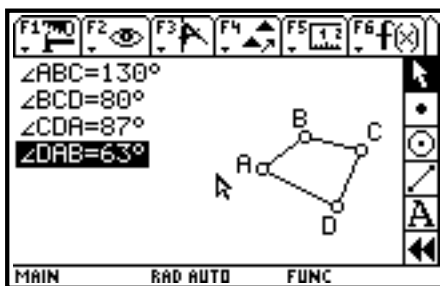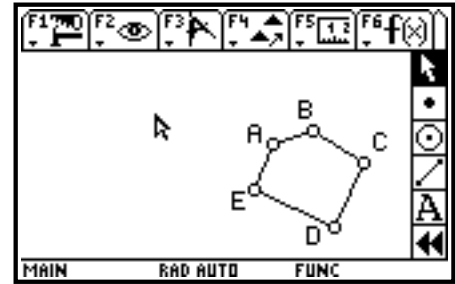
Find the sum of these four interior angles and drag the result to an appropriate location on the screen. Press F8 and use the **pointer** tool to drag the vertices of the quadrilateral to various locations on the screen. Observe the "changes" in the angle measures and the sum. Make a conjecture about the sum of the measures of the four interior angles of a convex quadrilateral and justify this conjecture.
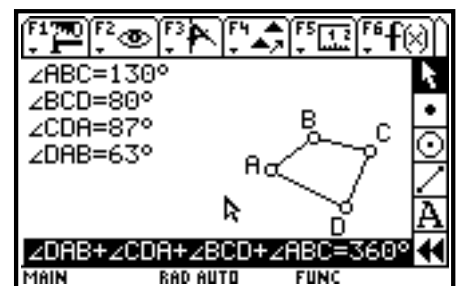
Teacher Note: The conjecture should be "the interior angles of a convex quadrilateral have a sum of 360 degrees." This can be justified by constructing one of the diagonals, such as segment BD. This divides the quadrilateral into two triangles. Each triangle has three angles whose measures have a sum of 180 degrees. Adding the measures of these six angles gives the sum of the measures of the four angles of the quadrilateral.

Press F1 and choose **5:Select All**. Press F1 and choose **3:Delete**. This will clear the screen. Press F8 to activate the Toolbar and use the cursor controls to highlight the **straightedge** tool.   Using the **segment** tool, construct and label a convex pentagon ABCDE on the right side of the screen by constructing 5 segments that intersect only at their endpoints.

As in the quadrilateral activity, measure the five interior angles of the pentagon and find their sum. Drag the vertices of the pentagon to various locations on the screen. Notice the "changes" in the angle measures and the sum. Make a conjecture about the sum of the measures of the five interior angles of a convex pentagon and justify this conjecture.

Teacher Note: The conjecture should be "the interior angles of a convex pentagon have a sum of 540 degrees." This can be justified by constructing two of the diagonals from the same vertex, such as segments AC and AD; the pentagon is divided into three triangles. Each triangle has three angles whose measures have a sum of 180 degrees. Adding the measures of these nine angles gives the sum of the measures of the five angles of the pentagon.

Clear the screen and construct and label a convex hexagon ABCDEF on the screen. Measure the six interior angles of the hexagon and find their sum. Drag the vertices of the hexagon to various locations on the screen. Observe the "changes" in the angle measures and the sum. Make a conjecture about the sum of the measures of the six interior angles of a convex hexagon and justify this conjecture.
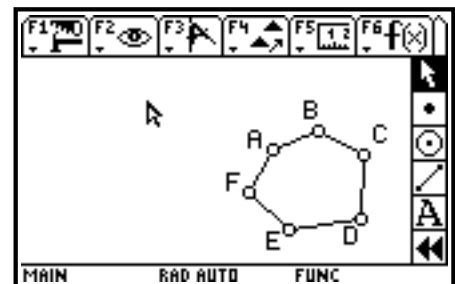
Teacher Note: The conjecture should be "the interior angles of a convex hexagon have a sum of 720 degrees." This can be justified by constructing three of the diagonals from the same vertex, such as segments AC, AD and AE. The hexagon is divided into four triangles. Each triangle has three angles whose measures have a sum of 180 degrees. Adding the measures of these twelve angles gives the sum of the measures of the six angles of the hexagon.
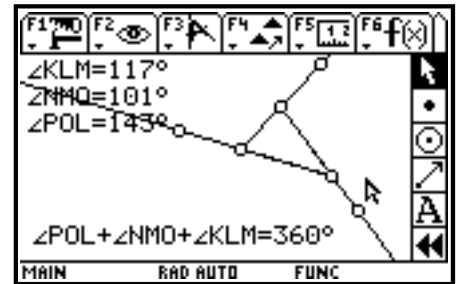
Make a conjecture about the sum of the interior angles of an n-sided convex polygon.

Teacher Note: The sum of the interior angles of an n-sided convex polygon can be found by the formula Sum = (n-2)*180, in which n is the number of sides of the polygon. This formula can be justified by the fact that any n-sided polygon can be divided into n-2 triangles by drawing diagonals from one of the vertices, and each triangle has three angles whose measures have a sum of 180 degrees.

Activity 2: Clear the screen as in Activity 1. Press F8 to activate the Toolbar and use the cursor controls to highlight the **straightedge** tool. Using the **segment** tool, construct a triangle on the right side of the screen by constructing 3 segments that intersect only at their endpoints. Press F8 and use the cursor controls to highlight the **straightedge** tool. Using the **ray** tool, overlay a ray on each of the sides of the triangle; place the endpoint of the ray on a vertex of the triangle, with the ray passing through another vertex of the triangle.
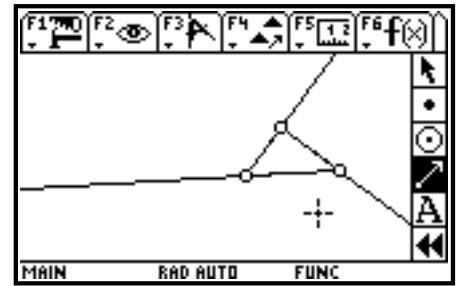
Press F8 to activate the Toolbar and use the cursor controls to highlight the **point** tool. Construct a point on each of the rays that is in the exterior of the triangle. Measure the three exterior angles of the triangle and calculate their sum. Drag the vertices of the triangle to various locations on the screen. Observe the "changes" in the angle measures and the sum.

Clear the screen. Construct a convex quadrilateral and overlay rays on the sides of the triangle. Construct points on each of these rays that are in the exterior of the triangle. Measure the four exterior angles of the triangle and calculate their sum. Drag the vertices of the triangle to various locations on the screen. Observe the "changes" in the angle measures and the sum.
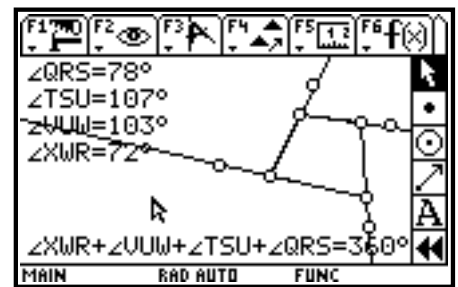
Clear the screen. Repeat the above activities using a convex pentagon and a convex hexagon. Make a conjecture about the sum of the measures of the exterior angles of any n-sided convex polygon.

Teacher Note: The conjecture should be "the sum of the measures of the exterior angles of an n-sided convex polygon, considering one exterior angle at each vertex, is always 360 degrees."

To establish why this might be true, clear the screen and construct a convex pentagon as an example of a convex polygon. Overlay rays on each of the sides of this pentagon, thus creating exterior angles. Press F8 and choose the **dilation** tool. Move the cursor to one of the vertices and press ENTER. This will select that vertex as the center of dilation. Select each of the rays of the pentagon and then drag, which will actually dilate the entire pentagon toward the center of dilation vertex. As the pentagon dilates toward the center of dilation, it should be evident why the sum of the measures of the 5 exterior angles is 360 degrees.

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