EXPLORATIONS

Activity 9

Computing by Degrees!

Teacher Notes

Concepts

- Sine ratio
- Cosine ratio
- Tangent ratio
- Degrees, minutes, seconds
- Inverses of trigonometric functions
- Angles of elevation
- Angles of depression

Calculator Skills

- Trig functions: [SIN], [COS], [TAN]
- Inverse trig functions: [2nd] [SIN^{-1}], [2nd] [COS^{-1}], [2nd] [TAN^{-1}]

Materials

- TI-30X IIS
- Student Activity pages (p. 87 - 88)
- Transparencies

Objective

- In this activity, students will use the calculator to solve trigonometry problems using sine, cosine, and tangent. They also will use the calculator to find inverses of these trigonometric functions.

Topics Covered

- Applying trigonometry to problem situations involving triangles
- Exploring real-world phenomena using the sine, cosine, and tangent functions

Introduction

Have you ever wondered how to determine the height of some tall building or object? Perhaps you thought about taking a long rope to the top of the building and dropping the end of the rope until it touched the ground. Then you could retrieve the rope and measure it to find the height of the building! The Canadian National Tower in Toronto is the tallest freestanding structure in the world. Could you find the height of this tower without measuring?

Investigation

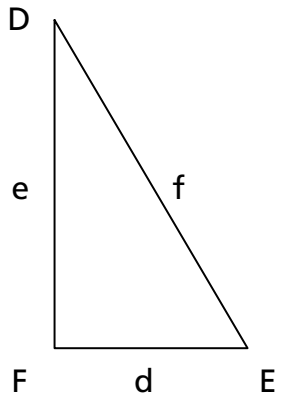
1. Explain that if you know the measures of an angle and the length of a side (leg or hypotenuse) in a right triangle, you can find the lengths of the other sides.
2. Display Transparency A and discuss the following relationships for the trigonometric functions.

$$\text{sine of the angle} = \frac{\text{length of the side opposite the angle}}{\text{length of the hypotenuse}}$$

$$\text{cosine of the angle} = \frac{\text{length of the side adjacent the angle}}{\text{length of the hypotenuse}}$$

$$\text{tangent of the angle} = \frac{\text{length of the side opposite the angle}}{\text{length of the side adjacent the angle}}$$

3. Tell the students that in a right triangle ΔDEF, the measure of $\angle E = 68°$ and the measure of d = 15 inches. Find the measure of e.



4. Show that the ratio relating d, e, and the measure of $m\angle E$ is the tangent.

$\tan \angle E = \frac{e}{d}$ $\quad$ $\tan 68° = \frac{e}{15}$ $\quad$ *15 tan 68° = e*

5. Use the overhead calculator to find the measure of side e. For each problem in Activity 9, the angle mode must be degrees; therefore, have the students do that first.

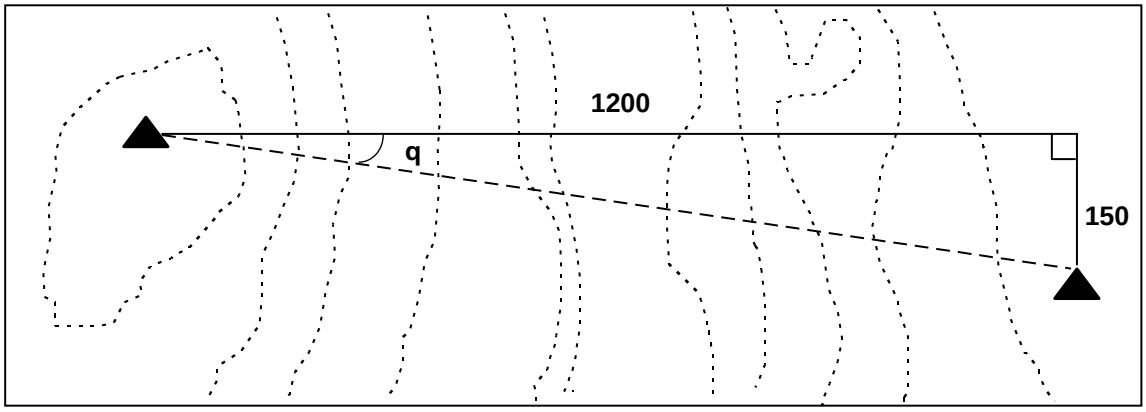
Press:	The calculator shows:
[DRG]	DEG RAD GRD DEG
[ENTER]	DEG
15 [TAN] 68 [)] [ENTER]	15 tan (68) 37.1263028 DEG

6. Explain that sometimes angle measures must to be expressed in more precise units. For example, angle measures may be expressed in decimal degrees, like 74.56°, or in degrees-minutes-seconds display. In fact, 74.56° may be expressed as 74°33’36”.

   Show the students how to use the calculator to find the sin(74°33’36”) in decimal form.

Press:	The calculator shows:
[SIN]	sin(DEG
74 [°’”]	o ’ ” r g DEG
[ENTER]	sin(74° DEG
33 [°’”] [▶] [ENTER] 36 [°’”] [▶] [▶] [ENTER]	←(74°33’36” DEG
[)] [ENTER]	sin(74°33’3 0.963909776 DEG

7. Show the students how to use the inverse functions to determine the measure of an angle. For example, two markers are placed on a contour map with different elevations. You want to find the angle of depression between the two markers.



8. Display Transparency B—a geometric model of the information read from the map. Then identify a trigonometric ratio appropriate to the situation, write the corresponding equation, and use a calculator to obtain the answer.

   $\tan(q) \frac{150}{1200}$

   $\tan(q) = 0.125$

   $q = \tan^{-1}(0.125)$

Press:	The calculator shows:
2nd [TAN-1] 0.125) ENTER	tan -1(0.125) 7.125016349 DEG

   The angle of depression is 7.125016349 degrees.

Wrap-up

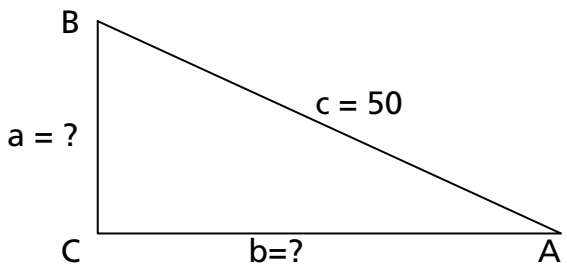
Generalizations of the ratios of right triangle trigonometry give rise to both trigonometric and circular functions. These functions are mathematical models for many periodic phenomena in the real world. Be sure that all of your students know how to solve the right triangle problems so that applications they will study at higher levels of mathematics make sense.

Extensions

- Joshua is flying a kite with 135 feet of string out. The angle of elevation of the kite varies from 45 degrees to 60 degrees during the flying time. What is the range of heights the kite will fly during this time?
- Revisit the Canadian National Tower problem at the beginning of this activity. Have students work in pairs to determine a method for finding the height of the Tower without measuring.

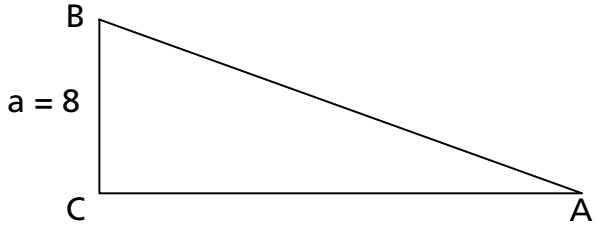
Solutions

1. Find the measure of side a and side b in the right triangle below given that the measure of angle A is 24 degrees.



Side a = 50sin(24°) = 20.34 and side b = 50cos(24°) = 45.68

2. Find the measure of the hypotenuse in the right triangle below given that the measure of angle B is 75 degrees.



The measure of the hypotenuse is 30.9

3. Given a right triangle DEF with the following angle measures and side measures:

   Measure of angle D = 23°45'10''

   Measure of angle E = 90°

   Measure of side ED = 15

   Draw a sketch of the triangle and predict if EF > ED or EF < DE.

   EF < DE

   a. Find the measure of angle F: *LF=90°–23°45'10 = 66.2472° or 66°14'50")*

   b. Find the measure of side EF: *EF = 15tan (23°45'10") = 6.6*

   c. Find the measure of hypotenuse DF: *DF = 15 ÷ cos (23° 45'10") = 16.39*

4. A jet plane is descending from 6000 feet above ground into DFW airport. The angle of depression between the pilot's line-of-sight and the control tower is 5 degrees. How many miles (ground distance) is the plane from the control tower? (Hint: 1 mile = 5280 feet)

   6000 *÷ tan (5°) = 68580.31 ft.* = 12.99 miles*)*

5. One of the tallest mountains in Alaska is 15,308 feet above sea level, just 14.25 miles from the Pacific coast. What is the angle of elevation from the summit of the mountain to the shoreline of the Pacific coast? (Hint: 1 mile = 5280 feet)

   (the angle of elevation is $\tan^{-1}\left(\frac{2.9}{14.25}\right) = 11.5°$*)*

6. What is the measure of the angle of depression from a tower that is 120 feet tall to a point that is 525 feet away from the tower on level ground?

   (The angle of depression is 12.9°)

Student Activity 9

Name _____

Date _____

Applying Trigonometric Models—Computing by Degrees!

Objective: *In this activity, you will use the calculator to solve trigonometry problems using sine, cosine, and tangent. You also will use the calculator to find inverses of these trigonometric functions.*

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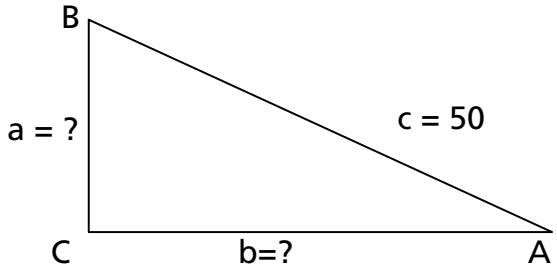
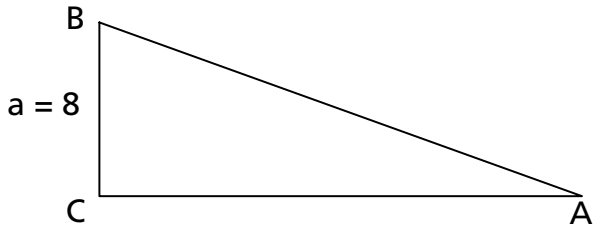


Side a = Side b =

2. Find the measure of the hypotenuse in the right triangle below, given that the measure of angle B is 75 degrees.



Hypotenuse =

3. Given a right triangle DEF with the following angle measures and side measures:

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   a. Find the measure of angle F:

   b. Find the measure of side EF:

   c. Find the measure of hypotenuse DF:

4. A jet plane is descending from 6000 feet above ground into DFW airport. The angle of depression between the pilot's line-of-sight and the control tower is 5 degrees. How many miles (ground distance) is the plane from the control tower? (Hint: 1 mile = 5,280 feet)

5. One of the tallest mountains in Alaska is 15,308 feet above sea level, just 14.25 miles from the Pacific coast. What is the angle of elevation from the summit of the mountain to the shoreline of the Pacific coast? (Hint: mile = 5280 feet)

6. What is the measure of the angle of depression from a tower that is 120 feet tall to a point that is 525 feet away from the tower on level ground?

TRANSPARENCY A
Trigonometric Functions

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$$\text{Cosine of the angle} = \frac{\text{length of the side adjacent the angle}}{\text{length of the hypotenuse}}$$

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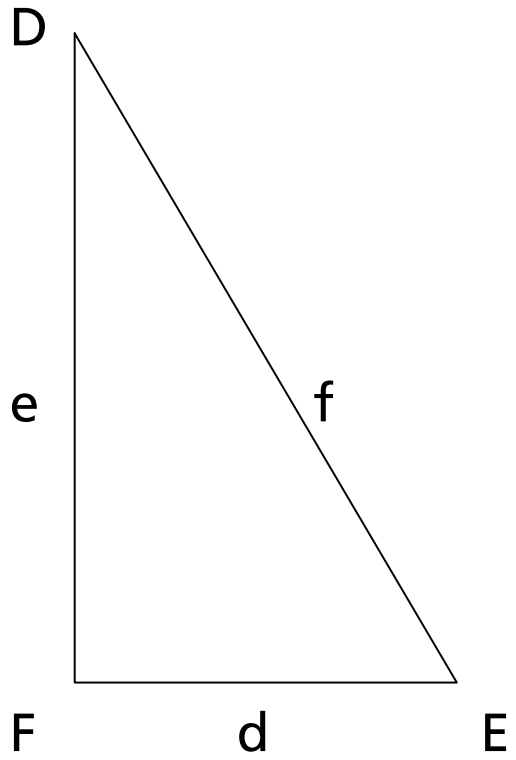
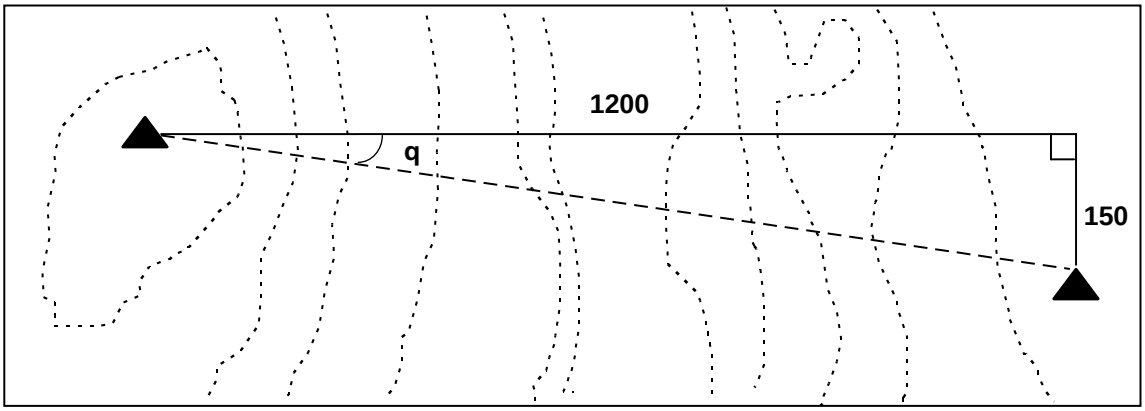


TRANSPARENCY B
Geometric Model



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