

Here's Looking At Euclid

ID: 8611

Time required
45 minutes

Activity Overview

In this activity, students use three different methods to calculate the greatest common divisor and least common multiple of two numbers. First they use prime factorization, a method they have likely seen before. Second, they apply Euclid's algorithm, an iterative process for finding the greatest common divisor that was developed by the "father of geometry" around 300 B.C., in conjunction with a formula for the least common multiple given the greatest common divisor. In order to use the algorithm, they must first grasp the remainder operation, which they explore via a table of values. Lastly, the students check their answers with the calculator's GCD and LCM functions. For each method, students are first guided through an example, then prompted to complete several calculations on their own.

Topic: Polynomial Functions

- *Greatest Common Divisor*
- *Least Common Multiple*
- *Euclid's Algorithm*

Teacher Preparation and Notes

- *This activity is designed for use in an Algebra 2 classroom, or as an extension for Algebra 1. It is best used to review factorization, GCD, and LCM. The algebraic approach prepares students for similar operations over other fields, such as factoring monomials and polynomial division.*
- *Prior to beginning the activity, students should know the definitions of greatest common divisor and least common multiple.*
- *This activity is intended to have students explore **individually and in pairs**, (with the teacher providing assistance when needed). However, an alternate approach would be to introduce each method in a whole-class format, then have students apply the method individually or in pairs.*
- *The TI-Nspire document titled HeresLookingAtEuclid_Soln shows the expected results of working through the activity.*
- **To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to <http://education.ti.com/exchange> and enter "8611" in the search box.**

Associated Materials

- *HeresLookingAtEuclid_Student.doc*
- *HeresLookingAtEuclid.tns*
- *HeresLookingAtEuclid_Soln.tns*

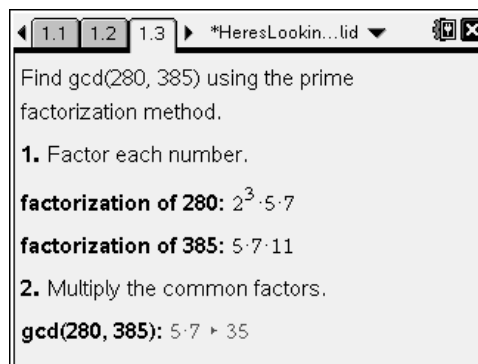
Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

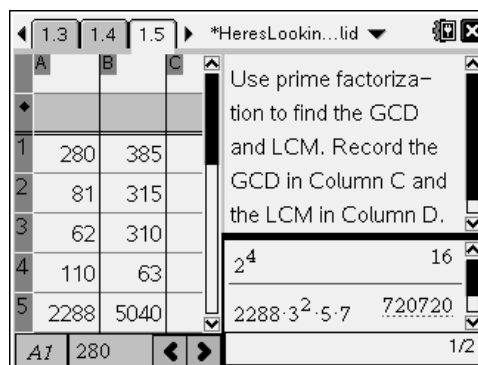
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Problem 1 – Prime factorization method

On page 1.3, students use the **Evaluate Selection** command from the Actions menu to step through the process of finding the GCD of two numbers from their prime factorizations. Guide students in evaluating the expression **factor(280)** by holding ⇧ while pressing ◀ or ▶ to highlight the expression, then choosing **MENU > Actions > Evaluate Selection**. Students will then factor 385 and multiply common factors. On page 1.4, students use a similar process to find the LCD.



Students are prompted to apply the process on their own to find the GCD and LCM of each pair of numbers in the table and record their answers in Columns C and D. (Press ctrl + tab to move between panes in the window.)



Students may have trouble finding the GCD of 110 and 63 because they have no common factors. Remind them that every number has at least two factors: 1 and itself. Numbers that have no common factors other than 1 are relatively prime.

Lead the class to draw some conclusions based on their experience with the prime factorization method, asking questions like: What does it mean if **factor(x) = x**? What is the GCD of two numbers that are relatively prime? What is the LCM of two numbers that are relatively prime?

Problem 2 – Euclid’s algorithm

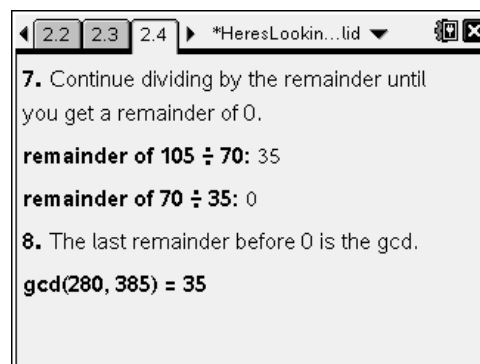
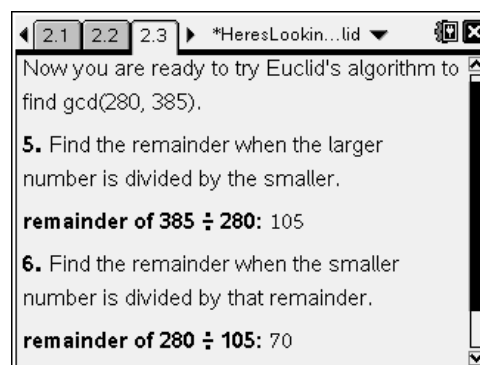
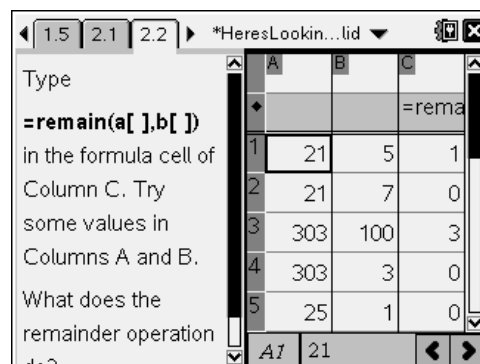
The prime factorization method allows students to clearly see the details underlying the GCD and LCM calculation of a and b , but it is cumbersome when a and b are large numbers. In this problem, students will explore Euclid’s algorithm for calculating the GCD. The algorithm is based on the **Remainder** operation, also known as modulus or mod. Entering **remain(a, b)** will return the remainder when a is divided by b .

After allowing students discover the remainder operation for themselves by examining its behavior in the spreadsheet, test their understanding by asking them to predict the output of operation on a pair of numbers. Then discuss the operation in more detail. *Is the remainder operation commutative? (Does the order of the numbers matter?) Is the remainder operation a function? Why or why not?*

Next, students are guided through the execution of the algorithm. As they scroll down the page, they again use the **Evaluate Selection** command to evaluate each expression that follows the bold text.

Students may need help at first deciding which numbers to use in each subsequent iteration of the algorithm. Explain that the goal of the algorithm is to reach 0, so the numbers should be smaller each time. The smaller number takes the place of the larger number, and the remainder takes the places of the smaller number each time the process is repeated.

Could the algorithm continue once a remainder of 0 is reached? Why or why not?



Students are prompted to apply the algorithm on their own to find the GCD of each pair of numbers in the table, and record their answers in Column C.

Discuss how Euclid's algorithm performs in special cases, such as when a and b are relatively prime, or when b is a multiple of a . Optionally, you can also discuss why this form of Euclid's algorithm does not work if a or b is 0.

	A	B	
1	16	108	
2	456	759	
3	136	232	
4	567	440	
5	976	488	
A7	16		

Use Euclid's algorithm to find the GCD. Record the GCD in Column C.	
remain(9,5)	4
remain(5,4)	1
remain(4,1)	0
remain(976,488)	0

Students are given a formula for calculating the LCM when the GCD is known and are prompted to apply the formula.

Discuss the relationship between the formula for the LCM and the prime factorization method. The GCD is the product of the common factors, so multiplying a by b divided by the GCD is the same as multiplying a by the factors of b that are not common.

9. Find the $\text{lcm}(280, 385)$ using the formula:

$$\text{lcm}(a, b) = \frac{a \cdot b}{\text{gcd}(a, b)}$$

$$\text{lcm}(280, 385) = \frac{280 \cdot 385}{\text{gcd}(280, 385)}$$

$$= \frac{107800}{35} = 3080$$

Returning to page 2.4, students will use the formula to find the LCM of each pair, and record their answers in Column D.

Students will use a third and final method of calculating the GCD and LCM to check their answers: the calculator's **gcd()** and **lcm()** functions.

10. Return to page 2.5. Use the formula to find the LCM of each pair. Record the LCM in Column D.