EXPLORATIONS

Activity 4

The Calcumites Are Coming!

## Introduction

This is a Calcumite. It may look cute to the casual observer, but it is a creature that threatens to take over the world. Mutant, evolutionary cousins of your graphing calculator, these calcumites are increasing in number and will perhaps one day control the whole world! Their motto is "you push my buttons and I'll push yours." Does that sound like a friendly critter to you? Our only hope lies in understanding the population dynamics of this creature; a formidable task that will be facilitated by our TI-73, the only weapon that may save us!
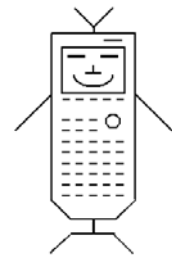
Problem

Understanding population dynamics as it relates to both humans and other organisms is important in understanding the balance of nature. Ideal growth of a population refers to a population whose growth is unimpeded in any way — not by predators, disease, overcrowding, or limited resources. In practice, ideal growth cannot continue indefinitely because there are limiting factors in nature. How can we mathematically model a population's growth when it is *ideal* and when it is *limited*?

Collecting the data — Part I

In the first part of this activity, you will determine the growth rate of an *ideal* population.

1. The following page shows six generations of calcumites. Make the following assumptions about calcumites:

    ♦ They mate when they are one year old.

♦ They mate once each year.

♦ They produce one pair (a male and a female) each time they mate.

♦ They never die.

2. The population for each year follows the sequence 1, 1, 2, 3, 5. To determine the next number in the sequence, carefully examine the diagram. Observe that all five pairs A, B, C, D, and E will all survive. In addition, 3 pairs, A, B, and D will have a pair of offspring, bringing the total pairs of calcumites for the next generation to 8.

	Year	**Number of Pairs**
	1	1
	2	1
	3	2
	4	3
	5	5
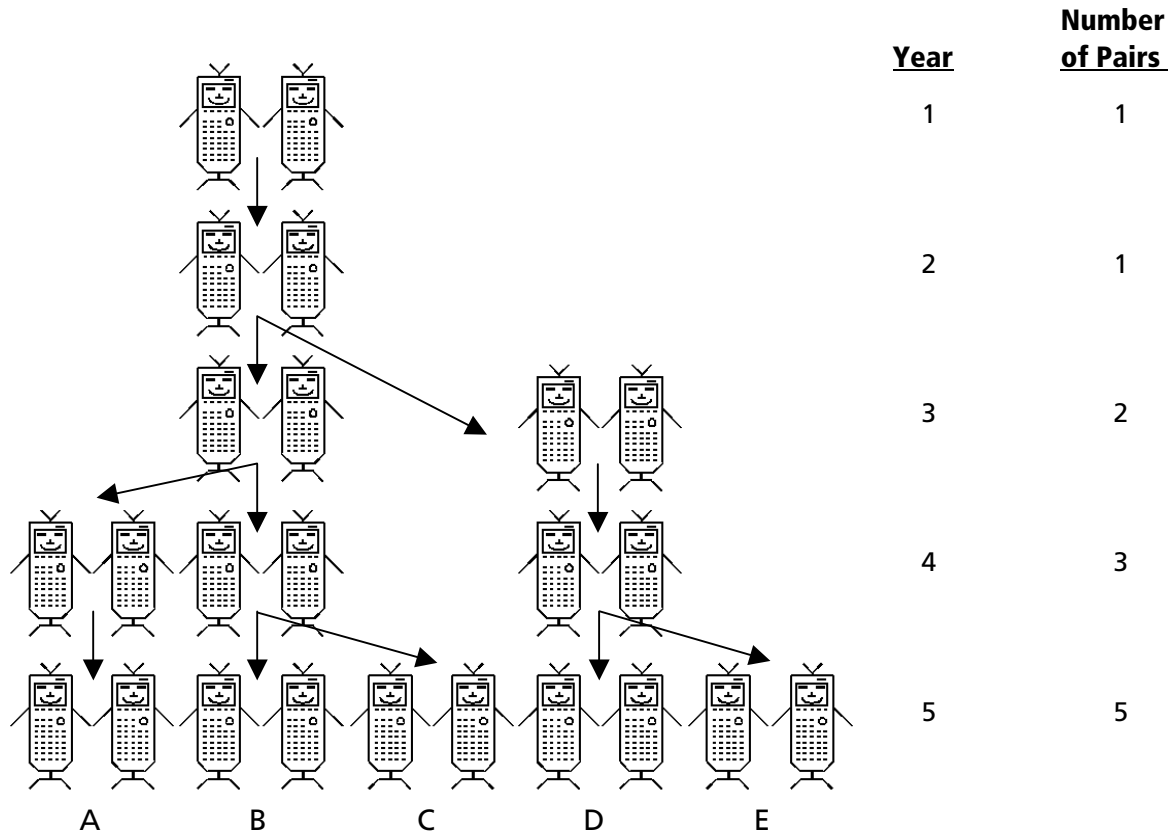
Determine the relationship of numbers in this sequence. (It may be easier to work backwards.) How is the 8 related to the two integers before it? How is the 5 related to the two integers before it?

Predict the numbers after 8. Fill in the chart entitled *Ideal Population Growth — Exponential* on the **Data Collection and Analysis** page for the first 10 generations of calcumites. Check your answer with your teacher before proceeding to the next section.

Setting up the TI-73

Before starting your data collection, make sure that the TI-73 has the STAT PLOTS turned OFF, Y= functions turned OFF or cleared, the MODE and FORMAT set to their defaults, and the lists cleared. See the Appendix for a detailed description of the general setup steps.

Entering the data in the TI-73

1. Press $\boxed{\text{LIST}}$.

2. Enter the generation numbers 1 through 10 in **L1**.

3. Enter the number of calcumite pairs in **L2**. (Make sure that the pairs of generation numbers and calcumite pairs match in each column.)

Setting up the window

1. Press $\boxed{\text{WINDOW}}$ to set up the proper scale for the axes.

2. Set the **Xmin** value by identifying the minimum value in **L1**. Choose a number that is less than the minimum.

3. Set the **Xmax** value by identifying the maximum value in each list. Choose a number that is greater than the maximum. **Do Not Change the ∆X Value.** Set the **Xscl** to **1**.

4. Set the **Ymin** value by identifying the minimum value in **L2**. Choose a number that is less than the minimum.

5. Set the **Ymax** value by identifying the maximum value in **L2**. Choose a number that is greater than the maximum. Set the **Yscl** to **5**.

Graphing the data: Setting up a scatter plot

In order to analyze the data, you will need to set up a scatter plot and model the data by graphing the results of an exponential regression.
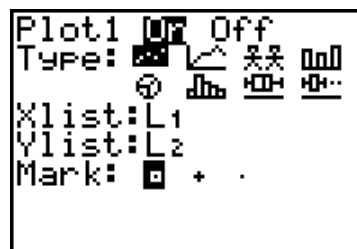
1. Press $\boxed{\text{2nd}}$ [PLOT]. Select **1:Plot1** by pressing **1** or $\boxed{\text{ENTER}}$.

2. Set up the plot as shown by pressing
[ENTER] [▼] [ENTER] [▼] [2nd] [STAT] **1:L1** [▼] [2nd] [STAT]
2:L2 [▼] [ENTER].

```
Plot1  On  Off
Type: ▨  ⬿  ⚹  ▥
      ☉  ▥  ▥  ▥
Xlist:L₁
Ylist:L₂
Mark: ▪  +  ·
```

3. Press [GRAPH] to see the plot.

4. The graph to the right shows the rate of calcumite population growth for three intervals of the plot shown in number 3:
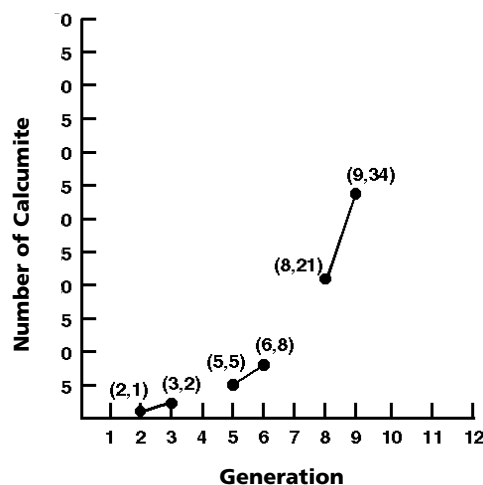
2 to 3 generations

5 to 6 generations

8 to 9 generations

Determine the growth rate for each of those time intervals by determining the *slope* of each of the plots, using the following formula for slope:

$$\text{Slope} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$
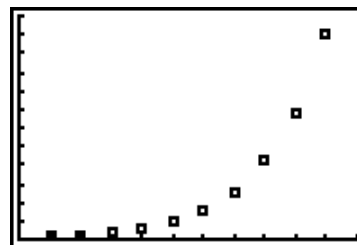
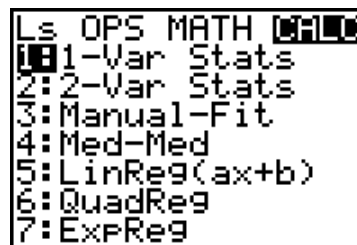**Calcumite Population Growth — Ideal**

Enter the slopes on the **Data Collection and Analysis** page.

Answer Part I questions 1 and 2 on the **Data Collection and Analysis** page.

You have examined linear models in previous activities. In a linear model, the slope does not change. Therefore, a linear model would not be appropriate here. What type of regression would be a suitable model for this data?

Analyzing the data: Finding an exponential regression

1. Press [2nd] [STAT] [◀] to move the cursor to the **CALC** menu.

```
Ls OPS MATH CALC
1▮1-Var Stats
2:2-Var Stats
3:Manual-Fit
4:Med-Med
5:LinReg(ax+b)
6:QuadReg
7:ExpReg
```

2. Select **7:ExpReg** by pressing **7.**

```
ExpReg
```

3. Press [2nd] [STAT] **1:L1** [,] [2nd] [STAT] **2:L2** [,].

```
ExpReg L₁,L₂,█
```

4. Press [2nd] [VARS]. Select **2:Y-Vars** by pressing **2**.

```
FUNCTION
1:Y₁
2:Y₂
3:Y₃
4:Y₄
5:FnOn
6:FnOff
```

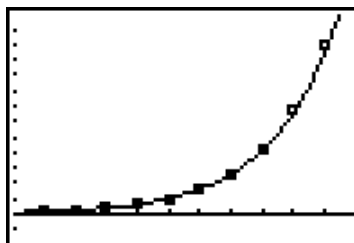**5.** Select **1:Y1** by pressing **1** or [ENTER].

```
ExpReg L₁,L₂,Y₁█
```

6. Press [ENTER] to calculate the exponential regression. The function is pasted into **Y1**.

```
ExpReg
 y=a*b^x
 a=.4882960783
 b=1.598311821

█
```

7. Press [GRAPH] to see a graph of the exponential regression model.

   The plot displayed shows the ideal calcumite population growth over a period of 10 generations.

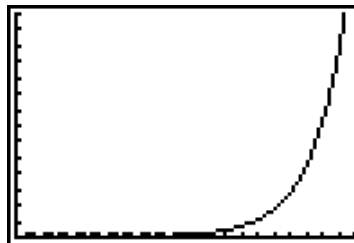8. Examine how large the population will become after 20 generations. To do this, the window must be expanded.

   Press WINDOW and set the window values as shown.

   Do NOT change the ΔX value.

```
WINDOW
 Xmin=0
 Xmax=21
 ΔX=.2234042553…
 Xscl=1
 Ymin=0
 Ymax=7000
 Yscl=500■
```

9. Press GRAPH to see the plot.

10. Find the population of calcumites after 20 generations by pressing TRACE ▼ to place the cursor on the graph of **Y1**.

   Type **20** and press ENTER to show the number of calcumite pairs (*y* value) after 20 generations of population growth.

```
Y1=.48829607834354*1.59_

X=20■
```

11. Record the number of pairs of calcumites after 20 generations on the **Data Collection and Analysis** page.

```
Y1=.48829607834354*1.59_

X=20 ⌐⌐⌐⌐⌐⌐ Y=5779.8087
```

It becomes apparent that a population cannot keep rising at this rate. Keep in mind that a one-year generation time for calcumites is somewhat long compared to many other organisms. For example, the generation time of a fruit fly is about two weeks. Also, keep in mind that some organisms produce many offspring, not just one pair. For example, tapeworms may produce 600 million eggs annually, but only a few survive. There are always factors that limit the growth of a population. If this were not true, the earth would soon be covered with fruit flies, tapeworms, or our imaginary calcumites!

Collecting the data — Part II

In the second part of this activity, you will investigate the growth rate of a *limited* population.

As the calcumite population grows and becomes denser, more of them die for various reasons. Some die from disease. (They are infected with a computer-like virus that attacks calcumites.) Some calcumites are eaten by TI-73 calculators. Others perish because they cannot find enough food and resources. The following chart shows 10 generations of calcumites whose growth is *limited*. Plot

the growth of this population and analyze the consequences of such limited growth.

Generation	1	2	3	4	5	6	7	8	9	10
Pair Number	1	1	2	3	5	8	10	11	12	12

Entering the data in the TI-73

1. Press [LIST].

    The generation data in **L1** is the same as in the study of ideal population growth and need not be changed.

2. Clear the data from **L2** by moving the cursor to the **L2** label and pressing [CLEAR] [ENTER].

    Enter the number of calcumite pairs for limited population growth into **L2**.

    (Make sure that the pairs of generation numbers and calcumite pairs match in each column.)

Setting up the window

1. Press [WINDOW] to set up the proper scale for the axes.

2. Set the **Xmin** value by identifying the minimum value in **L1**. Choose a number that is less than the minimum.

3. Set the **Xmax** value by identifying the maximum value in each list. Choose a number that is greater than the maximum. **Do Not Change the ΔX Value.** Set the **Xscl** to **1**.

4. Set the **Ymin** value by identifying the minimum value in **L2**. Choose a number that is less than the minimum.

5. Set the **Ymax** value by identifying the maximum value in **L2**. Choose a number that is greater than the maximum. Set the **Yscl** to **1**.

Graphing the data: Setting up a scatter plot

In order to analyze the data, you will need to set up a scatter plot and model the data by graphing a logistic function.

1. Press [2nd] [PLOT]. Select **1:Plot1** by pressing **1** or [ENTER].

2. Set up the plot as shown by pressing [ENTER] [▼] [ENTER] [▼] [2nd] [STAT] **1:L1** [▼] [2nd] [STAT] **2:L2** [▼] [ENTER].

   Note: *This step may be unnecessary if settings were not changed after you plotted ideal population growth.*

3. Press [GRAPH] to see the plot.

   Note: *The exponential regression model for your ideal population is still on the screen. You will use that model for comparison purposes.*
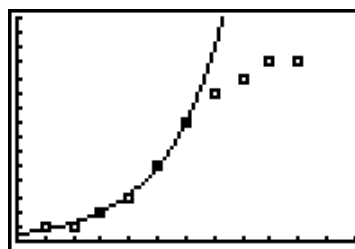
4. The graph to the right shows the rate of calcumite population growth for three intervals of the plot shown in number 3.

   2 to 3 generations

   5 to 6 generations

   8 to 9 generations
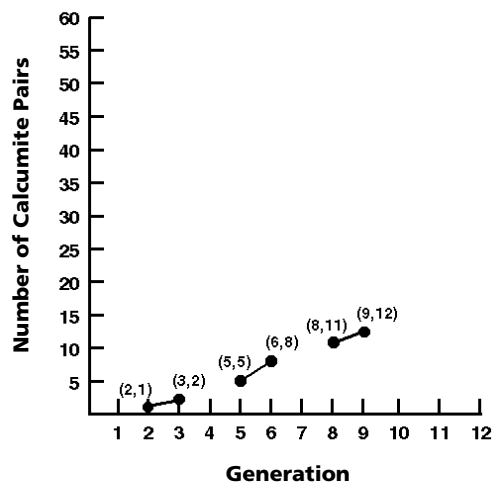
   Determine the growth rate for each of those time intervals by determining the *slope* of each of the plots, using the following formula for slope:

   $$\text{Slope} = \frac{(y_2 - y_1)}{(x_2 - x_1)}.$$

Calcumite Population Growth — Limited

Enter the slopes on the **Data Collection and Analysis** page.

Answer Part II questions 1 and 2 on the **Data Collection and Analysis** page.

## Analyzing the data: Finding a logistic regression

It is necessary to determine an appropriate regression model for this data. You know it is not linear since the slope changes. An exponential regression would not be appropriate, since the slope does not continually increase over the three intervals. What type of regression would be a suitable model for this data?

The function that best models this data is a logistic function. This is a complex equation (beyond the scope of your course), yet once plotted, you will be able to analyze it and learn how it most accurately shows population growth in nature.
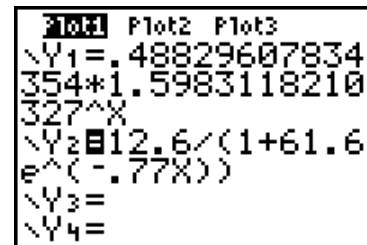
Using the data provided, the equation is as follows:

$$Y = 12.6 / (1 + 61.6e^{-0.77X})$$

1. Press ⌨Y=. Scroll down to **Y2**.
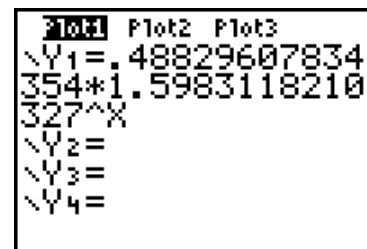
2. Type **12.6** ÷ (**1** + **61.6** ⌨MATH ◄ **4:e^(** (−) **.77** ⌨x)).

3. Press ⌨GRAPH to view a graph of the logistic function.
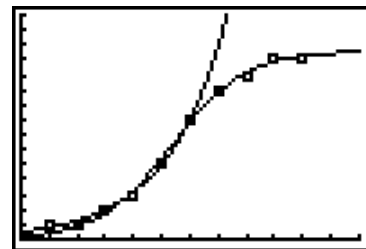
The logistic function is complex beyond the scope of your studies this year. You can, however, make a comparison between a population growing exponentially and one that is growing logistically.

It becomes obvious that the rate at which the population grows is changing; it starts slow, speeds up, and then slows down. This is different from what you observed with ideal population growth (exponential). In order to examine how large the population will become after 20 generations, it is necessary to expand the window.

4. Press WINDOW and set the window values as shown.

Do Not Change the ΔX Value.

```
WINDOW
 Xmin=0
 Xmax=22
 ΔX=.2340425531…
 Xscl=1
 Ymin=0
 Ymax=20
 Yscl=1█
```

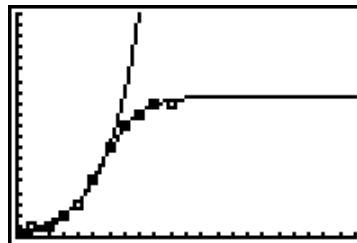**5.** Press GRAPH to see the enlarged generation growth graph.

> **Note:** *This plot levels off. The population size where it levels off is referred to as the carrying capacity.*

6. To find the carrying capacity, press TRACE ▲ so that the cursor is on the graph of the **Y2** logistic function.
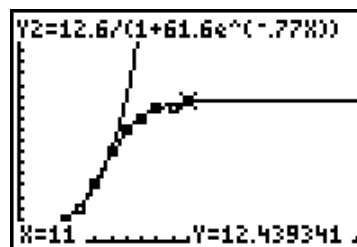
```
Y2=12.6/(1+61.6e^(-.77X))



X=11 ⌐⌐⌐⌐⌐⌐ Y=12.439341 ⌐
```

7. Type **20** and press ENTER to show the number of calcumite pairs (*y* value) after 20 generations of population growth.

```
Y2=12.6/(1+61.6e^(-.77X))



X=20.12766 ⌐ Y=12.599856 ⌐
```
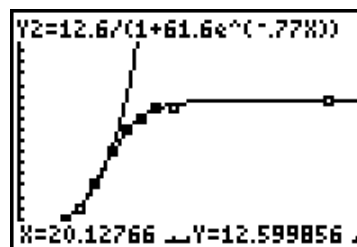
8. Record the approximate size of the population after 20 generations on the **Data Collection and Analysis** page.

9. To get an overview of how the two growth curves compare, reset the window of the calculator as shown.

Do Not Change the ΔX Value.

```
WINDOW
 Xmin=0
 Xmax=22
 ΔX=.2340425531…
 Xscl=1
 Ymin=0
 Ymax=50
 Yscl=1█
```

10. Press GRAPH to see the plot.

Answer Part II, Question 3 on the **Data Collection and Analysis** page.

Data Collection and Analysis

Name _____

Date _____

Activity 4: The Calcumites Are Coming!

Collecting the data

Generation	1	2	3	4	5	6	7	8	9	10
Pair Number	1	1	2	3	5					

Ideal Population Growth — Exponential

Generation	1	2	3	4	5	6	7	8	9	10
Pair Number	1	1	2	3	5	8	10	11	12	12

Limited Population Growth — Logistic

Ideal Population Growth	**Limited Population Growth**
Slope (Interval: 2 – 3 Generations) = _____	Slope (Interval: 2 – 3 Generations) = _____
Slope (Interval: 5 – 6 Generations) = _____	Slope (Interval: 5 – 6 Generations) = _____
Slope (Interval: 8 – 9 Generations) = _____	Slope (Interval: 8 – 9 Generations) = _____
Number of Calcumites after 20 Generations _____	Number of Calcumites after 20 Generations _____

Analyzing the data — Part I

1. What does the *slope* of a line on a population plot tell you?

_____

_____

2. Using the *slope* data, examine and describe the 2 – 3, 5 – 6, and 8 – 9 generation time intervals for the plot of *ideal* growth.

_____

_____

Analyzing the data — Part II

1. Describe the same generation time intervals for the plot of *limited* growth.

_____

_____

2.   Which do you think is a more realistic model in nature? Why?

_____

_____

3.   Which model - exponential or logistic - would be the better model to describe the following situations? Explain your answer in each case.

   a.   The velocity (speed) of a car going down an endless hill. (Graph velocity versus time.)

   _____

   _____

   b.   A virus spreading in your classroom. (Graph the number of people infected versus time.)

   _____

   _____

   c.   The number of TI-73 calculators sold. (Graph number sold versus time.)

   _____

   _____

Extension

Leonardo Fibonacci was an Italian mathematician who lived approximately 1175 – 1250. He is famous for discovering an interesting relationship between sequences of numbers that is now known as the *Fibonacci sequence*. The sequence starts with the following series of numbers: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, and so on. Starting with the third number, each number is equal to the sum of the two numbers preceding it. To study this interesting mathematical phenomenon, divide each number in the sequence by the number before it.

Use your knowledge of the natural world to find other species that contain examples of the Fibonacci sequence. Visualize the sequence by plotting your data. Notice that the data tends to stabilize around a certain number.

Teacher Notes

Activity 4

The Calcumites Are Coming!

Preparation

- ♦ The calcumite population examined in this activity is similar to the way in which Fibonacci examined a hypothetical rabbit population.

- ♦ Activity 8 will further examine Fibonacci sequences as applied to the spiral of a snail shell.

- ♦ While the exponential regression used to generate the plot of ideal population growth should be discussed, the logistic equation is beyond the scope of an introductory Algebra class. Yet, the general nature of the plot, with its changing slopes, can be studied to give the students an idea of how a population in nature often grows and stabilizes around the carrying capacity.

- ♦ It may be easier for some students to use manipulatives to better visualize the reproductive patterns of the calcumites. Here are a few suggestions:

  - – Make copies of the calcumites on the final page of this activity.

  - – Use coins or candy to simulate the calcumite populations.

Answers to Data Collection and Analysis questions

Collecting the data

Sample data:

Generation	1	2	3	4	5	6	7	8	9	10
Pair Number	1	1	2	3	5	*8*	*13*	*21*	*34*	*55*

Ideal Population Growth — Exponential

Generation	1	2	3	4	5	6	7	8	9	10
Pair Number	1	1	2	3	5	8	10	11	12	12

Limited Population Growth — Logistic

Ideal Population Growth

Slope (Interval: 2 – 3 Generations) = 1

Slope (Interval: 5 – 6 Generations) = 3

Slope (Interval: 8 – 9 Generations) = 13

Number of Calcumites
after 20 Generations   5780

Limited Population Growth

Slope (Interval: 2 – 3 Generations) = 1

Slope (Interval: 5 – 6 Generations) = 3

Slope (Interval: 8 – 9 Generations) = 1

Number of Calcumites
after 20 Generations   12

Analyzing the data — Part I

1. What does the *slope* of a line on a population plot tell you?

The slope of a line on a population plot describes the rate of population growth.

2. Using the *slope* data, examine and describe the 2 – 3, 5 – 6, and 8 – 9 generation time intervals for the plot of *ideal* growth.

For ideal growth, the rate of growth increases. Observe that the slope increases from 1 to 3 to 13 for the ranges examined.

Analyzing the data — Part II

1. Describe the same generation time intervals for the plot of *limited* growth.

For limited growth, the rate of growth starts slow, speeds up, and then slows down. Observe that the slopes are 1, 3, and 1 for the ranges examined.

2. Which do you think is a more realistic model in nature? Why?

The logistic model is more realistic because a population cannot increase in size, at a faster rate, forever. Limiting factors such as food availability, climate, disease, predation, and competition will eventually limit the size of any population.

3. Which model, exponential or logistic, would be the better model to describe the following situations? Explain your answer in each case.

a. The velocity (speed) of a car going down an endless hill. (Graph velocity versus time.)

Exponential: It will continuously accelerate down this hypothetically endless hill, assuming no air resistance.

b. A virus spreading in your classroom. (Graph the number of people infected versus time.)

Logistic: It will spread slowly at first as there are only a few infected individuals to pass it on. As more people are infected, they will quickly pass it on. As the number of uninfected individuals lessens, the likelihood of an infected individual contacting an uninfected individual decreases, and so the rate of spread slows down.

c. The number of TI-73 calculators sold. (Graph the number sold versus time.)

Logistic: The number of people purchasing the TI-73 calculators starts out slowly, increases as more people spread the good news, and slows down as few people are left who have not yet purchased them.

CALCUMITE CUT OUTS