

## **NUMB3RS Activity: I...It...Itera...Iteration Episode: "Rampage"**

**Topic:** Iterative processes

**Grade Level:** 9 - 12

**Objective:** Examine random iterative processes and their outcomes

**Time:** about 45 minutes

**Materials:** TI-83/84 Plus graphing calculator, and the following calculator programs: BROWNIAN and CHAOS. To download these programs, go to <http://education.ti.com/exchange> and search for "6783."

### **Introduction**

In "Rampage," a suspect enters the FBI Bullpen and moves about, apparently trying to shoot people at random. Charlie examines a diagram of the shooter's path through the Bullpen and suggests it might resemble Brownian motion, the random movement of a dust particle suspended in a fluid. First observed through a microscope by Robert Brown around 1827, Brownian movement served as proof of the existence of atoms. In 1900, Louis Bachelier first described Brownian motion using mathematics.

Brownian motion is the movement caused by the collision of fluid molecules with a much larger dust particle. When observed through a microscope, the motion appears to be erratic and irregular similar to a random walk. Brownian motion is mathematically modeled using an iterative process in which each successive step is determined by a normal probability distribution with a mean of 0 and a standard deviation that may vary depending on the model.

A Cartesian coordinate system can be used to model this motion in two dimensions. Using the origin as the initial starting point, a random number is generated from a normal distribution (In this activity the mean is 0, and the standard deviation is 1.) indicating the change in the  $x$ -coordinate ( $h$ ). The change in the  $y$ -coordinate ( $v$ ) is found in the same manner. The segment from the origin to the point  $(0 + h, 0 + v)$  representing the first movement in the Brownian motion is drawn on the coordinate system. The process is repeated this time starting at the endpoint  $(0 + h, 0 + v)$ . The modeling continues through the desired number of iterations.

In the first part of this activity, students create a stochastic model (sequences generated by probabilistic rules) of Brownian motion using this iterative method. The students first calculate ten iterations by hand and then use the TI-83/84 Plus program BROWNIAN to view the plot of hundreds of iterations.

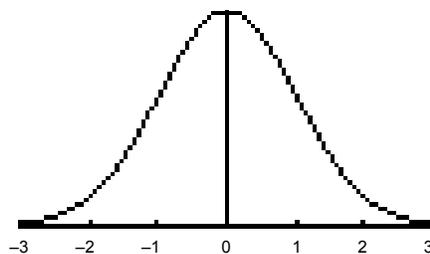
The resulting path appears to be very random, as students might expect. In the second part of the activity, students are introduced to the Chaos Game. This game also uses random iterative processes to generate points on a graph; however, the outcome is very predictable or deterministic. Again, the students first do ten iterations by hand. They then use the TI-83/84 Plus program CHAOS to view the plots of thousands of iterations.

In the Chaos Game, students begin with three points that represent the vertices of an equilateral triangle. First a point is selected as the starting point. Then, one of the vertices is selected at random. The point half way between the starting point and the chosen vertex is plotted. This new point is used as the starting point for the next iteration. The result is a deterministic model.

In the extensions for this activity, students examine the Sierpinski triangle and the Koch snowflake as simple fractals. As an option to these extensions, you may wish to have students work through NCTM's *Student Math Notes* (May 2002) entitled "Do It Again, Sam." To access this publication online you must be an NCTM member. It can be found at [http://my.nctm.org/eresources/archive.asp?journal\\_id=5](http://my.nctm.org/eresources/archive.asp?journal_id=5).

### **Discuss with Students**

1. If you drew a map of the path of someone who was asked to walk "randomly" about the room, would you expect to find any patterns in the path?
2. When two fair six-sided dice are rolled and the total number of dots is counted, sums from 2 to 12 can be represented. Is the probability of the numbers on the dice summing to 12 the same as the probability of the numbers on the dice summing to 7? Explain.
3. In a normal probability distribution, some events occur with greater probabilities than others. The graph below shows a normal probability distribution with a mean of 0 and a standard deviation of 1. It shows how the probabilities of generating random numbers between  $-3$  and  $3$  are distributed. Between which two numbers would you expect the greatest probability to occur, 0 and 1, 1 and 2, or 2 and 3? Explain.

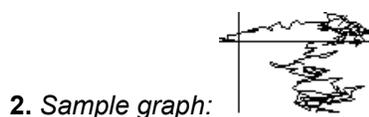
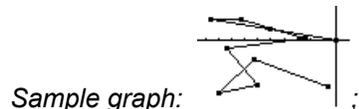


**Discuss with Students Answers:**

1. Answers may vary. No. If the path is random I would not expect any patterns.
2. No. The probability of showing a 7 is greater because there are more ways for two dice to total 7 than to total 12.
3. Between 0 and 1. There is more area under the curve between these two values.

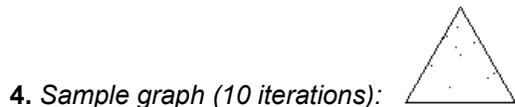
**Student Page Answers:**

1. Answers will vary: Ten points in a Brownian motion are  $(0, 0)$ ,  $(-1.7, 0.6)$ ,  $(-3.2, 1.1)$ ,  $(-2.4, 1.1)$ ,  $(-0.8, 0.1)$ ,  $(-2.8, -0.4)$ ,  $(-2.0, -2.3)$ ,  $(-3.0, -2.7)$ ,  $(-2.1, -1.0)$ ,  $(-0.2, -2.4)$



There does not appear to be a recognizable pattern.

3. Sample response: There might be an obvious location not visited by the shooter, possibly indicating a conscious effort to avoid a particular person.



There does not appear to be any recognizable pattern.

5. Sample graph (5,000 iterations):  Yes, there is a pattern that is consistent in all the plots.
6. It may be called deterministic because the resulting model can be determined even though it is created by random event.

Name: \_\_\_\_\_

Date: \_\_\_\_\_

### NUMB3RS Activity: I...It...It...Iteration

In "Rampage," a shooter gets into the FBI Bullpen and tries to shoot several people at random. Don asks Charlie to look at a diagram of the path the shooter followed, to see if there might be any clues as to who the shooter was after or if someone in the office was involved with the shootings. Charlie mentions that the path looks very random and is similar to Brownian motion – the movement of a dust particle suspended in fluid. The movement is caused by the collision of fluid molecules with the much larger dust particle.

1. Brownian motion can be modeled on a rectangular coordinate system using an **iterative process**. This means that each step is based on the outcome of the previous step(s). Brownian motion also uses a **normal probability distribution** with a mean of 0 and a standard deviation of 1 to determine what happens at each step. This means that some events occur with a greater probability than others following the rules of a normal distribution.

Fill in the table below using the **randNorm(** command on a TI-83/84 Plus calculator. Note: to find **randNorm(** press **MATH**, go to the **PRB** menu, and select **6:randNorm(**. Find the next x-coordinate by entering **randNorm(0,1)** on your calculator and adding the value rounded to the nearest tenth to the previous x-coordinate in the table. Repeat the same step to find the next y-coordinate. Continue finding the rest of the x- and y-coordinates in the table using the previous coordinates and the **randNorm(** command. Then, create the start of a model of Brownian motion by connecting the points from the table in order on a coordinate grid.

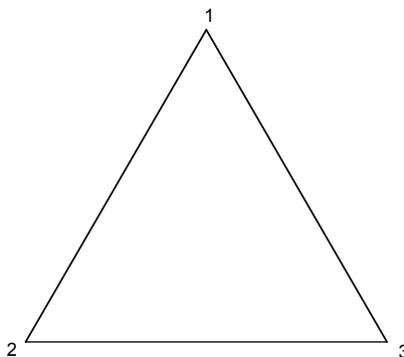
Pt	x-coord.	y-coord.	Pt	x-coord.	y-coord.
1	0	0	6		
2			7		
3			8		
4			9		
5			10		

2. In Question 1, you graphed the first ten steps of a Brownian motion. However, this motion continues forever. To get a better model, many more steps in the process should be performed. This can be done using the programming capabilities of your calculator. Obtain a copy of the BROWNIAN program from your teacher. Run the program and plot a model of Brownian motion containing at least 200 segments. Compare your model to others in the class. Does there appear to be a recognizable pattern that can be found in the models?

3. If the shooter's movements through the FBI Bullpen were not random as is Brownian motion, how might this show up in the graph?
4. Other stochastic processes (those based on probabilities) can generate some unexpected results. One such example is often referred to as the Chaos Game. One version of the Chaos Game begins by plotting the vertices of an equilateral triangle. A starting point is selected and plotted on the same plane as the equilateral triangle. The point halfway between the starting point and a randomly selected vertex is plotted. This becomes the new starting point.

Using this new point as the starting point, one of the vertices of the triangle is again randomly selected. The point half way between the new starting point and the selected vertex is plotted. This process is repeated using the last point plotted as the new starting point.

On the equilateral triangle below, plot the first 10 points in the Chaos Game using the command **randInt(1,3)** to generate the numbers 1, 2, or 3 as the choice of vertices to use in each step. Note: to find **randInt(**, press **MATH**, go to the **PRB** menu, and select **5:randInt(**.



Compare your plot to those of others in the class. Does there appear to be a recognizable pattern in the Chaos Game?

5. In Question 4, you graphed the first ten points in the Chaos Game. However, this game can continue forever. To get a better idea about what happens when the game is continued, obtain a copy of the CHAOS program from your teacher. Run the program for at least 1,000 points. Compare your resulting model to others in the class. Does there appear to be a recognizable pattern that can be found in the models?
6. The Chaos Game is an example of a **deterministic process**. Why do you think it has been given this name?

*The goal of this activity is to give your students a short and simple snapshot into a very extensive math topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.*

## Extensions

### Extension 1

The resulting figure created from the Chaos Game is a Sierpinski triangle. Named for Warclaw Sierpinski, this triangle is one of the most common examples used when discussing **fractals**.

1. What are fractals and what makes the Sierpinski triangle a primary example of a fractal?
2. There are several methods for creating a Sierpinski triangle. One involves removing pieces of a triangle rather than plotting points as in the Chaos Game. With the help of the Web site below, investigate this method and create several iterations of this process.

<http://math.rice.edu/~lanius/fractals/>

3. As the iterations that create a Sierpinski triangle continue, less and less area remains. Determine a function that could be used to calculate the area remaining in the Sierpinski triangle given the number of iterations. For help see the Web site above.

### Extension 2

Many geometric objects created by an iterative process similar to that which models Brownian motion and the Sierpinski triangle are fractals. There are many simple fractals with basic characteristics that can be easily identified after only a few steps. One example is the Koch snowflake.

1. Research the steps necessary to create the Koch snowflake.
2. Create the first 3 iterations of a Koch snowflake. This can be easily done on isometric graph paper. For help, see the Web site below.

<http://math.rice.edu/~lanius/frac/koch.html>

3. As the number of iterations of the Koch snowflake increases, the length of each segment decreases. Determine a function that could be used to determine the length of each segment given the iteration number. Assume the equilateral triangle that you start with has sides of 1 unit each.
4. As the number of iterations increases the area enclosed by the snowflake appears to increase. If possible determine the area that the snowflake encloses as the number iterations increases without bounds. If not possible, explain why it is not possible to determine the area.
5. As the number of iterations increases the perimeter of the snowflake appears to increase. If possible determine the perimeter of the snowflake as the number iterations increases without bounds. If not possible, explain why.

### Other Resources

- For more information on Brownian motion, see the Web sites below.  
<http://classes.yale.edu/fractals/RandFrac/Brownian/Brownian.html>  
[http://xxx.ims.res.in/PS\\_cache/physics/pdf/0412/0412132.pdf](http://xxx.ims.res.in/PS_cache/physics/pdf/0412/0412132.pdf)
- For a Java applet with the Chaos Game, see the Web site below.  
<http://math.bu.edu/DYSYS/applets/chaos-game.html>
- The topic of random walks is related to Brownian motion. To learn about random walks, see the NUMB3RS activity **Walkabout**, which accompanies the episode "Mind Games." This activity can be downloaded for free from the Web site below.  
[http://www.cbs.com/primetime/numb3rs/ti/activities/Act2\\_Walkabout\\_MindGames\\_final.pdf](http://www.cbs.com/primetime/numb3rs/ti/activities/Act2_Walkabout_MindGames_final.pdf)