We are excited that you will be using these interactive investigations to assist your students in exploring and learning about Transformational Geometry. They are designed so that students investigate and discover the mathematics in under 15 seconds on any one of these TI-Nspire™ learning platforms:

- TI-Nspire™ CX handheld
- TI-Nspire™ CX Teacher or Student Software
- TI-Nspire™ App for iPad®

To obtain the optimal results for your students, follow these suggestions:

- **Always do the "Tour" activity (Lesson 0) first**, before doing any of the other lessons. The Tour contains information that is needed to understand how to do the other lessons. Follow the recommended sequence: Tour first (Lesson 0), then Lesson 1, Lesson 2, etc.

- Transformational Geometry experts recommend investigations of transformations first without a grid and also without numerical properties. Then introduce numerical properties such as side lengths, angle measures, coordinates, as needed, with or without a grid.

- The best approach using these activities in the classroom is to do part of each activity as a whole class, then have the students collaborate on other parts of the activity. **See the Suggested Lesson Plans beginning on page 4.**

- These activities are designed as a thorough first exposure to geometric transformations in middle grades and high school. Students should PLAY – INVESTIGATE – EXPLORE – DISCOVER using this technology. Also encourage students to make conjectures both verbally and handwritten. Use other paper and pencil activities, including compass and straightedge.

- Ensure that your students realize that observing something occur repeatedly does not "prove" it. We need to show that it works for **all** possibilities. To disprove requires only one counter example.

- The use of the **Think-Pair-Share** protocol works well with these activities. First students should do the exercise individually and make their own conjectures. They then share their ideas with a partner or group. Finally, the students share their conjectures with the class. The activities work best when students **read and follow the directions**.

- These lessons are created to investigate one or two concepts at a time. To investigate the concepts in a different order, create your own lessons utilizing this technology using the Options menu. **See the Teacher Tip on the bottom of page 3 of this document.**

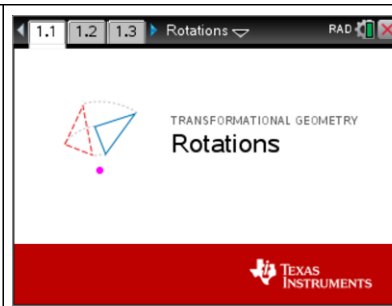
- To investigate the geometric transformations more deeply, use the other TI-Nspire activities found at the MathNspired.com website. These are located within the Geometry section and the Transformational Geometry subsection.

- The directions are written primarily for the TI-Nspire handheld. If using the computer software, use the mouse to select and move objects. If using the iPad app, tap on the appropriate icons and figures to move objects.

About the Rotations "Lesson Bundle"

In the Rotations "lesson bundle", students will explore rotations and their properties. Throughout the lessons, students are encouraged to make observations related to their investigations, leading them to discovering the properties of rotated figures. This lesson bundle includes the following lessons:

- **Lesson 0: Rotations Tour.** Explore the defining properties of rotations and learn how to use the rotations files. ***This must be done first before doing the other lessons.*** Page 5.
- **Lesson 1: Rotation Point.** Investigate the point of rotation and discover how to rotate a triangle by hand. Page 9.
- **Lesson 2: Angles & Sides.** Explore the relationship between measures of corresponding angles and lengths of corresponding sides of rotated triangles. Page 13.
- **Lesson 3: Perimeters & Areas.** Explore the relationship between the perimeters and areas of rotated triangles. Page 16.
- **Lesson 4: Grid & Coordinates**. Investigate the coordinates of vertices of triangles that have been rotated and look for patterns. Represent the coordinates algebraically. Page 20.
- **Lesson 5: Grid and Coordinates 2.** Continue to investigate the coordinates of vertices of rotated triangles and look for patterns. Page 26.
- **Lesson 6: Distance to Vertices.** Investigate the distances from the point of rotation to each of the vertices of rotated triangles and look for patterns. Page 30.
- **Lesson 7: Corresponding Sides.** Investigate the corresponding sides (not their lengths) and look for patterns. Page 34.
- **Lesson 8: Self-Assessment**. Summarize, review, explore and extend ideas about rotations. Page 38.



Tech Tips:

- This activity includes screen captures from the TI-Nspire CX handheld. It is also appropriate for use with the TI-Nspire family of products including TI-Nspire software and TI-Nspire Apps. Slight variations to these directions may be required if using other technologies besides the handheld.
- Watch for additional Tech Tips throughout the activity for the specific technology you are using.
- Access free tutorials at http://education.ti.com/calculators/pd/US/Online-Learning/Tutorials

Lesson Files:
Student Activity
Lesson dependent

TI-Nspire document
Rotations.tns

Math Objectives

Students will:

- verify experimentally the properties of rotations
- learn to identify and perform rotations
- understand the effects that a rotation has upon a triangle by easily manipulating the triangle
- explore the relationship between the corresponding parts of the pre-image and image triangles
- explain how underlying properties relate to rotation
- discover how to rotate figures by hand

Vocabulary

- Rotation
- Image/Pre-Image
- Corresponding sides
- Corresponding angles
- Horizontal
- Vertical
- Slope (m)
- Notation: $\overline{AB}$ represents the segment, AB represents its length.
- Area
- Perimeter
- Parallel lines
- Perpendicular lines

TI-Nspire™ Navigator™

- Use Class Capture to monitor students' use of the TI-Nspire document.

Activity Materials

Compatible TI Technologies : TI-Nspire™ CX Handhelds, TI-Nspire™ Apps for iPad®, TI-Nspire™ Software

Teacher Tip: These lessons are created to investigate one or two concepts at a time. To investigate other concepts, press the icon or the shortcut key O to open the Options menu. Use the tab key or the directional arrows (▲ ▼ ◀ ▶) to navigate through the list. Use the space bar ␣ to select or un-select the options.
In this way, you can ***create your own investigations***.

Suggested Lesson Plans

Working in groups of four, students should discuss what they discover doing these activities.

<u>Lesson 0</u> Do page 1 with the class as they work in groups of four. Continue with page 2 through exercise 5 with the class and encourage discussions. Lead the students as they do exercises 6. Then in each group of four, have each student pick a different part of exercise 7: i, ii, iii, or iv. Next have each group finish exercises 7 and 8. When students are finished, discuss the answers to exercises 7 and 8 with the class.

<u>Lesson 1</u> Do exercises 1 and 2 with the class. Have the students do exercises 3 and 4 in groups and discuss the results. Do 5a with the class. Have the students finish 5 and 6 in groups. When students are finished, discuss the solutions to exercises 5 and 6 with the class.

<u>Lesson 2</u> Do exercises 1 and 2 with the class. Then in each group of four, have each student pick a different part of exercise 3: i, ii, iii, or iv. Have students complete exercises 3 a – f in groups. Students should then finish exercises 4 through 6. Discuss the answers with the class.

<u>Lesson 3</u> Do exercises 1 and 2 a – d with the class. Then in each group of four, have each student pick a different part of exercise 2e: i, ii, iii, or iv. Have the students complete exercise 2 e – h in groups, and then discuss the answers. Next have students do exercises 3 and 4 in groups and discuss the answers.

<u>Lesson 4</u> Do exercises 1 and 2 with the class. Then have the students do exercises 3, 4, and 5 and discuss the answers in their groups and with the class as needed. Do exercise 6 (the summary) with the class. Do exercise 7a with the class. Assign the remaining exercises to be completed and do other examples as needed. Discuss the answers when finished.

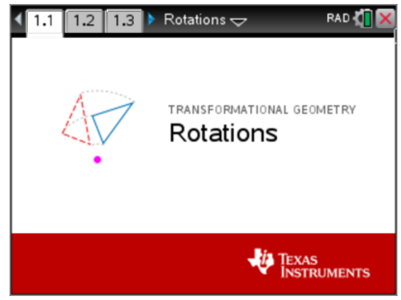
<u>Lesson 5</u> Have the students do exercise 1 on their own, which is a review of Lesson 4. They should use the grid to visualize the answers better. If students require assistance, allow them to help each other in groups. Then have the students do exercises 2 and 3 as a group. This is a way to check their answers to exercise 1 using the technology. Assign exercises 4 and 5 and discuss the results with the class.

<u>Lesson 6</u> Do exercises 1 through 3 with the class. Have students do exercise 4. Then in exercise 5, students are asked to pick a different part of exercise 5: i, ii, iii, or iv. Have students finish 5 through 7 and then discuss the answers with the class.

<u>Lesson 7</u> Do exercise 1 and 2i with the class. Have the students do 2 a – c together in groups. Have each student pick a different part of exercise 2d: i, ii, iii, or iv. Then finish 2 d – j. Assign exercises 3 and 4 and then discuss the results with the class.

<u>Lesson 8</u> Do exercises with the class as you think necessary. Assign any or all exercises and then discuss the answers and solutions with the class.

Lesson 0: Rotations Tour



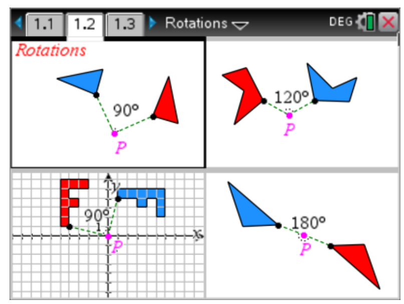
In this activity, you will investigate the defining properties of the transformation known as a rotation. You will also learn how to easily and quickly maneuver within all the Rotations activities using shortcut keys or the tab key.

Open the document: *Rotations.tns.*

PLAY INVESTIGATE EXPLORE DISCOVER

Move to page 1.2. (ctrl ▸)

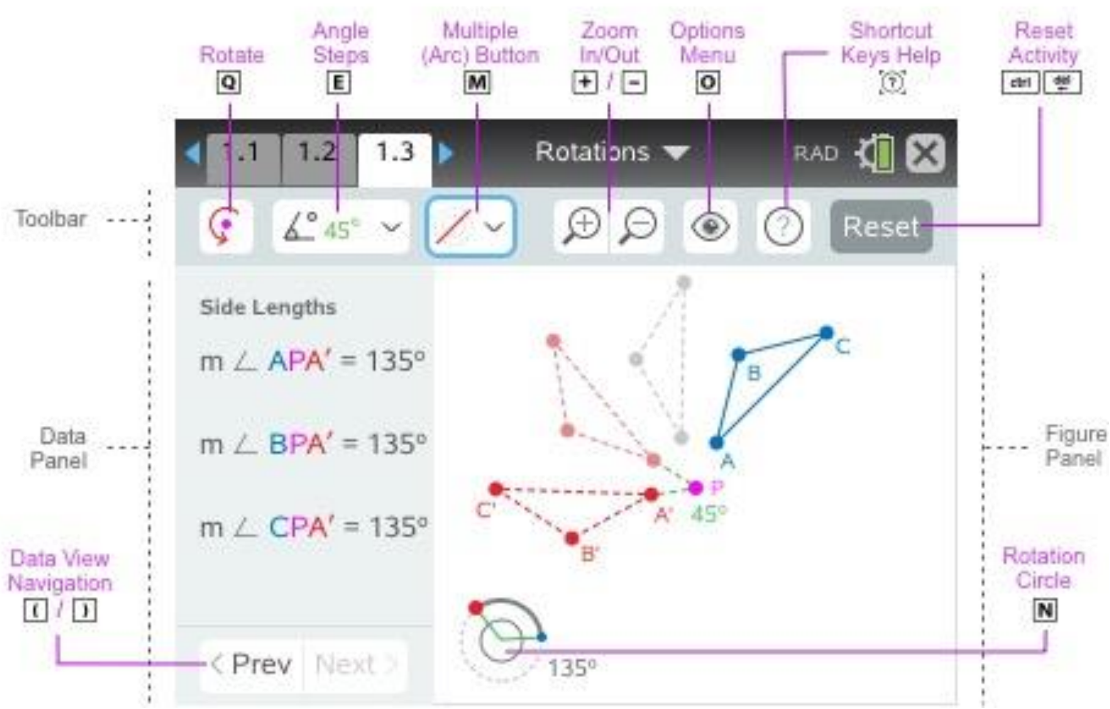
On the handheld, press ctrl ▸ and ctrl ◂ to navigate through the pages of the lesson.
(On the iPad®, select the page thumbnail in the page sorter panel.)



1. What do the 4 parts of the screen have in common?
   Make two conjectures.
   A ***conjecture*** is an opinion or conclusion based upon what is observed. Quickly discuss with your group.

   <u>Sample answer(s):</u> One figure is red, the other blue. Each has a pink point P. Each has an angle with P as the vertex. The figures are the same size and same shape, i.e. congruent.

Move to page 1.3. (ctrl ▸) Look at the figure below of an overview of the main rotations page and shortcut keys. ***Especially notice what the shortcut keys Q, +, and – represent.***



Navigating to and Selecting Screen Options or Objects

Handheld Tech Tip:
To choose an option or object, use any of the following 3 methods:

- Use the touchpad to move the pointer over the option or object and press the center of the touchpad ( ) to select (**click**) it.
- Use tab to move to the next option or object on the screen and use ⇧shift tab to go to the previous option or object.
- Use a **shortcut key** (ex: A for vertex A, Q to Rotate, etc.). Letters A, B, C are located at the bottom of the handheld.

Use the method that works best for you: **click**, **tab** or **shortcut key**.

iPad Tech Tip:
To choose a command or object, tap the icon or the object.

2. On the handheld, press the tab key (tab) multiple times and notice each of the icons and points as they are highlighted. To go in the opposite direction, press ⇧shift tab. Investigate.

3. Shortcut keys provide a fast way to perform actions and/or select objects on the screen on the handheld. A list of all shortcuts can be found in the Shortcut Keys Help menu (click on ? or press ctrl trig). **Look at this list now.** Use as needed. Press enter or esc to close the Shortcut Keys Help menu.

4. Start by rotating ΔABC about point P through an angle of 45°.

   To rotate ΔABC, press the Rotate key (click on or press Q).

   Zoom in (+) or out (−) as needed. Observe what happens on the screen.

   Blue ΔABC is called the pre-image and red $\Delta A'B'C'$ is called the image.
   $\Delta A'B'C'$ is read "triangle A prime, B prime, C prime."

5. To move and grab a vertex, press the letter key that corresponds to the vertex such as A (A), and use the directional arrows (▲ ▼ ◀ ▶) on the touchpad to move vertex A.
   Grab and move point A to play, explore, and discover ideas and investigate patterns.

Note: You can also use the **tab** key or **click** on the vertex that is needed.
(On the iPad®, tap the desired point and move it.)

a. What appears to be the relationship between ΔABC and $\Delta A'B'C'$? Discuss in your groups.

Sample Answer: They appear to be congruent.

b. Grab and move vertex B (B). Grab and move vertex C (C). Observe.

c. Discuss with your partner or group: what appears to be true about the pre-image and its image? Write your conjecture below. A ***conjecture*** is an opinion or conclusion based upon what is observed.

Sample Answer: When a triangle is rotated about a point 45°, its image is congruent to the original (pre-image) triangle.

6. Press menu to open the menu.

(On the iPad®, tap on the wrench icon to open the menu.)
Press 1 (1: Templates), 1 (1: Tour).

Rotate ΔABC about point P through an angle of 45° (click on or press Q).

Zoom in (+) or out (−) as needed.

a. Notice the blue pre-image triangle and the red image triangle.
Observe the 45° angle at point P and on the Rotation Circle at the bottom of the screen.

b. Rotate ΔABC about point P an additional 45° (click on or press Q).
Notice the new "ghosted" pre-image, the 45° angle at point P, and the measure on the Rotation Circle. Write that measure here: **90°**

c. Continue to rotate ΔABC about point P in steps of 45°. Observe what you see on the screen, particularly the location of the triangles and the measures displayed on the Rotation Circle.

Stop when the measure displayed is 360°.

7. Reset the page. Press Reset (ctrl del).
Change the angle of rotation and discover another way to rotate the triangle.
To change the stEp size of the angle of rotation, click on 45° or press E.
Use the directional arrows (▲ ▼ ◀ ▶) to move to the stEp size that you want, in this case 60°.
Press the space bar (␣) to select that measure and close the menu.

a. Another way to rotate the figure about point P is to either click on the red point on the Rotation Circle or just press the letter [N]. Use the directional arrows (▲ ▼ ◀ ▶): the up and right arrows rotate one direction, while the left and down arrows rotate the other direction.
Play – Investigate – Explore – Discover.
Rotate in both directions several times. Observe the pre-image and image triangles.

b. Each person in the group should select a different angle for the stEp size (click 45° or press [E]). i) 30^o ii) -30^o iii) -60^o iv) 15^o
Use the directional arrows (▲ ▼ ◀ ▶) to move to the box of the angle measure desired.
Press the space bar ([space]) to select that measure and to close the menu.

Select the red point on the Rotation Circle (click on the red point or press the letter [N]).
Use the directional arrows (▲ ▼ ◀ ▶) to rotate the triangle through several angles.
The up and right arrows rotate one direction, the left and down arrows rotate the other direction.
Explore and investigate further by grabbing and moving each of the three vertices ([A], [B], [C]) and notice how that affects the pre-image and image triangles.

Discuss in your groups what you observe.

<u>Sample Answer:</u> The pre-image triangle and the image triangle appear to be congruent.

c. Reset the page. Press Reset ([ctrl] [del]).
Open the Options menu (press or [O]).
Use the directional arrows (▲ ▼ ◀ ▶) to move to the box next to “Historical Images”.
Press the space bar key ([space]) to put a check mark in the box. Press [enter] or [esc].

Rotate ΔABC about point P through an angle of 45^o, using the Rotation Circle ([N]) or ([Q]).

Continue to rotate ΔABC until the Rotation Circle shows 360^o.
Discuss in your groups what you observe.

<u>Sample Answer:</u> Each pre-image is shown as a ghosted image.

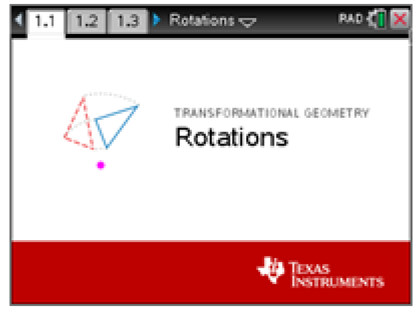
8. a. In a rotation ΔABC is typically called the **<u>pre-image</u>** triangle and $\Delta A'B'C'$ is typically called the **<u>image</u>** triangle.

b. How is $\Delta A'B'C'$ read? **<u>Triangle A prime, B prime, C prime</u>**

c. Explain what happens when a triangle is rotated about a point through an angle of 45^o.

<u>Sample Answer:</u> Each point on the triangle moves to the left and “around”.

Lesson 1: Rotation Point



In this lesson, you will investigate the point of rotation and discover how to rotate a triangle about a point by hand (paper and pencil, without technology).

Open the document: *Rotations.tns.*

It is important that the Rotations Tour be done before any Rotations lessons.

PLAY INVESTIGATE EXPLORE DISCOVER

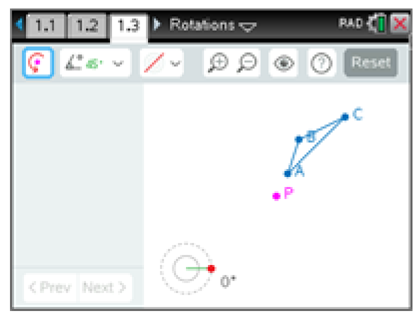
Move to page 1.3. (ctrl ▶ two times)

On the handheld, press ctrl ▶ and ctrl ◀ to navigate through the pages of the lesson. (On the iPad®, select the page thumbnail in the page sorter panel.)

1. Press menu to open the menu.

   (On the iPad®, tap on the wrench icon to open the menu.)

   Press 1 (1: Templates), 1 (1: Tour).



2. Rotate ΔABC 45° about point P (click on or press Q).

   Zoom in (+) or out (−) as needed.

   a. Think about how you might rotate a triangle about a point 45° by hand. Discuss in your groups.

   Sample Answer: Normally, most students will admit that they have no idea at this point. And that is OK for now.

   b. Rotate ΔABC about point P another 45° (click on or press Q).

   Rotate ΔABC about point P a third 45° (click on or press Q).

   How many degrees has the pre-image ΔABC been rotated about point P? **135°**

   Think about how you might rotate a triangle about a point 135° by hand. Discuss in your groups.

   Sample Answer: Students are still not sure how to do this yet.

c. To help visualize this better, click on the Multiple Icon or press M.
Press the space bar (␣) to select the first choice in the dropdown menu.
Look at what is displayed on the screen.
Rotate ΔABC about point P another 45° (click on or press Q).
Discuss in your groups what you notice.

<u>Sample Answer:</u> Arcs (parts of circles) connect vertices A', B', and C' with corresponding vertices on the previous ghosted pre-image.

d. Click on the Multiple Icon or press M. Press the down arrow (▼) once and press the space bar (␣) to select the second choice in the dropdown menu.
Look at what is displayed on the screen.
Rotate ΔABC about point P another 45° (click on or press Q).
Discuss in your groups what you notice.

<u>Sample Answer:</u> The vertices of A and A' are on one circle, B and B' are on a second circle, and C and C' are on a third circle.

e. Click on the Multiple Icon or press M. Press the down arrow (▼) once and press the space bar (␣) to select the third choice in the dropdown menu.
Look at what is displayed on the screen.
Rotate ΔABC about point P another 45° (click on or press Q).
Continue to rotate ΔABC about point P several more times.
Discuss in your groups what you notice.

<u>Sample Answer:</u> The arcs and vertices "travel along" the concentric circles with P as the center of each circle.

f. Discuss in your groups how you might rotate a triangle about a point 45° by hand?

<u>Sample Answer:</u> We will need a compass ("circle maker") and a protractor ("angle measure-er").

3. Reset the page. Press Reset (ctrl del).
Now explore how moving the point of rotation affects the result of the rotation.
a. Rotate ΔABC 45° about point P (click on or press Q).
Zoom in (+) or out (−) as needed.

b. Move point P to many places on the screen (press [P] and use the directional arrows(▲ ▼ ◀ ▶).
As the point of rotation, P, moves about the screen, look at what happens to pre-image ΔABC and image ΔA'B'C'.
Discuss in your groups what you observe.

<u>Sample Answer:</u> The blue pre-image triangle does not move. The red image triangle moves about the screen. The two triangles appear to be congruent.

4. Reset the page. Press Reset ([ctrl] [del]).
Let's now explore what happens when the point of rotation coincides with a vertex of the triangle.
   a. Change the angle of rotation to be 60°: Click on ∠° 45° ⌄ or press [E] .
   Use the directional arrows (▲ ▼ ◀ ▶) to move to 60°.
   Press the space bar ([␣]) to select that measure and to close the menu.

   Move point P to coincide with point A (press [P] and use the directional arrows (▲ ▼ ◀ ▶).

   Rotate ΔABC 60° about point P (click on or press [Q]).
   Observe what is on the screen.

   b. Continue to rotate ΔABC 60° about point P (click on or press [Q]) until the total number of degrees rotated is 360°. Observe the screen as you rotate.

   c. Reset the page. Press Reset ([ctrl] [del]).
   Press [menu] to open the menu.
   (On the iPad®, tap on the wrench icon to open the menu.
   Press [1] (1: Templates), [7] (7: Point P).
   Move point P to coincide with point A (press [P] and use the directional arrows (▲ ▼ ◀ ▶).
   Rotate ΔABC 45° about point P (click on or press [Q]).
   Continue to rotate ΔABC 45° about point P (click on or press [Q]) until the total number of degrees rotated is 360°. Observe the screen while you do so.
   Discuss what you observe in your groups.

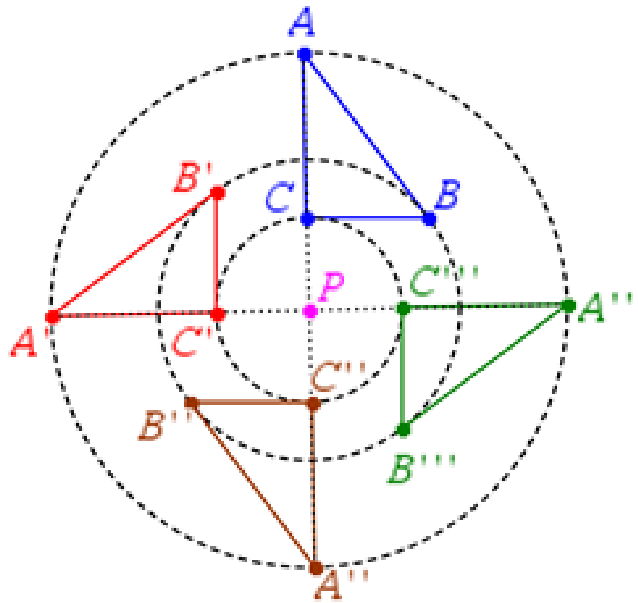
<u>Sample Answer:</u> All the previous images appear ghosted on the screen. It looks like a pinwheel.

5. Using only a straightedge or ruler, do the following:

   a. Rotate ΔABC about point P 90°.
      Label this image ΔA'B'C'.

   b. Rotate ΔABC about point P 180°.
      Label this image ΔA"B"C".

   c. Rotate ΔABC about point P 270°.
      Label this image ΔA'"B'"C'".



Notice that perpendicular lines were drawn with a straightedge through A and A'', A' and A'''. These help in the placement of the vertices of the images.

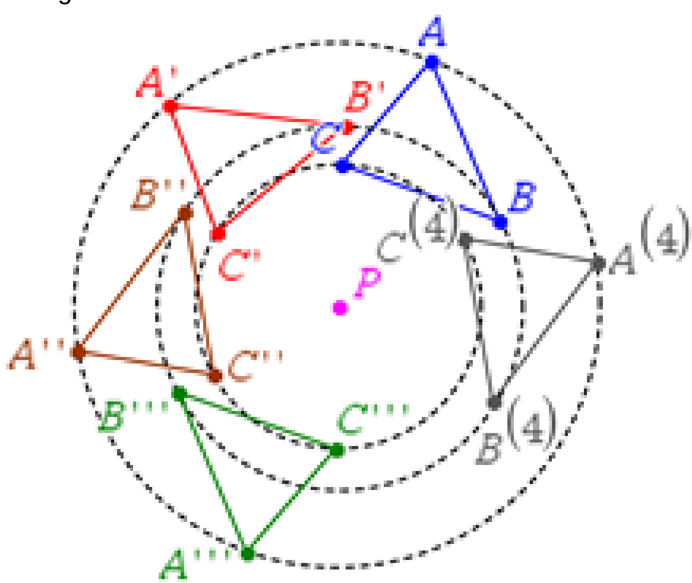
6. Use a compass and protractor to do the following:

   a. Rotate ΔABC about point P 60°.
      Label this image ΔA'B'C'.

   b. Rotate ΔABC about point P 120°.
      Label this image ΔA"B"C".

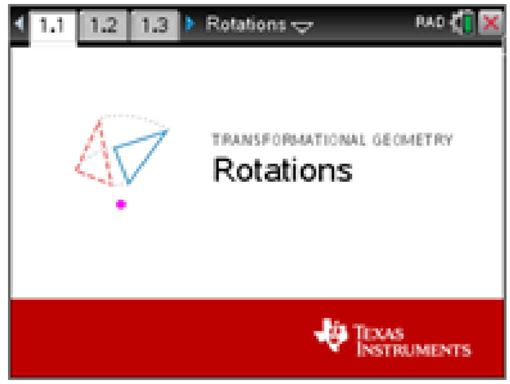
   c. Rotate ΔABC about point P 180°.
      Label this image ΔA'"B'"C'".

   d. Rotate ΔABC about point P 300°.
      Label this image $\Delta A^{(4)}B^{(4)}C^{(4)}$.



Encourage students to help each other with creating a procedure, let them "figure it out".

Lesson 2: Angles & Sides



In this lesson, you will investigate the measures of angles and lengths of sides of triangles that have been rotated in different ways. Open the document: *Rotations.tns.*

It is important that the Rotations Tour be done before any Rotations lessons.

PLAY INVESTIGATE EXPLORE DISCOVER

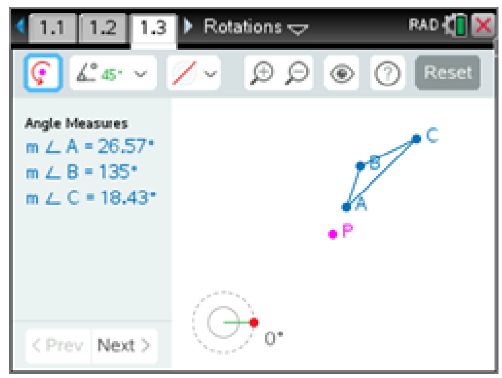
Move to page 1.3. (ctrl ▸ two times)

On the handheld, press ctrl ▸ and ctrl ◂ to navigate through the pages of the lesson.
(On the iPad®, select the page thumbnail in the page sorter panel.)

1. Press menu to open the menu.

   (On the iPad®, tap on the wrench icon to open the menu.)

   Press 1 (1: Templates), 2 (2: Angles & Sides).



2. Rotate ΔABC 45° about point P (click on or press Q).

   Zoom in (+) or out (−) as needed.

   a. Record the Original angle measures (first measures displayed) in the first row of the following table.

   b. Investigate and mentally make note of Angle Measures by grabbing and moving each of the three vertices of ΔABC (A, B, C) to create different shaped triangles.

   Record a set of data observed in row ”Figure 1” in the following table.

Sample Answer(s):

Rotate 45°	$m\angle A$	$m\angle B$	$m\angle C$	$m\angle A'$	$m\angle B'$	$m\angle C'$
Original	**26.57°**	**135°**	**18.43°**	**26.57°**	**135°**	**18.43°**
Figure 1	**45°**	**101.31°**	**33.69°**	**45°**	**101.31°**	**33.69°**

   c. Discuss observations in your group. Write a conjecture about the angle measures.

Sample Answer: Corresponding angles have the same measure. (Do not allow students to incorrectly say that “all angles have the same measure or are equal.”)

d. Click on Next or press) to see the lengths of the sides of the triangles.
Record the Original side lengths (first measures displayed) in the first row of the following table.

e. Investigate and mentally make note of Side Lengths by grabbing and moving each of the three vertices of ΔABC (A, B, C) to create different shaped triangles.
Record a set of data observed in row “Figure 1” in the following table.

Sample Answer(s):

Rotate 45°	$\overline{AB}$	$\overline{BC}$	$\overline{CA}$	$\overline{A'B'}$	$\overline{B'C'}$	$\overline{C'A'}$
Original	**4 u**	**5.1 u**	**7.07 u**	**4 u**	**5.1 u**	**7.07 u**
Figure 1	**5.1 u**	**5.39 u**	**5 u**	**5.1 u**	**5.39 u**	**5 u**

f. Discuss observations in your group. Write a conjecture about the lengths of the sides.
Sample Answer: Corresponding sides have the same length. (Do not allow students to incorrectly say that “all sides have the same length or are equal.”)

3. Reset the page. Press Reset (ctrl del).
Repeat what was done in exercise 2, but with each person in the group doing a different rotation.
Each person in the group should choose one from the following:
i) Rotate ΔABC 60° about point P.
ii) Rotate ΔABC 90° about point P.
iii) Rotate ΔABC – 60° about point P.
iv) Rotate ΔABC – 45° about point P.
(Note: to change the angle of rotation, click on 45° or press E to open the menu, and press the space bar (␣) to select that measure and to close the menu.)
Rotate ΔABC about point P (click on or press Q).

a. Record the Original angle measures (first measures displayed) in the first row of the table below.

b. Investigate and mentally make note of Angle Measures by grabbing and moving each of the three vertices of ΔABC (A, B, C) to create different shaped triangles.
Record a set of data observed in row ”Figure 1” in the following table.

Sample Answer(s):

Circle: i ii iii iv	$m\angle A$	$m\angle B$	$m\angle C$	$m\angle A'$	$m\angle B'$	$m\angle C'$
Original	**26.57°**	**135°**	**18.43°**	**26.57°**	**135°**	**18.43°**
Figure 1	**29.74°**	**116.57°**	**33.69°**	**29.74°**	**116.57°**	**33.69°**

c. Discuss observations in your group. Is your conjecture about the angle measures still true?
Sample Answer: Yes, the measures of the *corresponding* angles are equal.

d. Click on Next or press) to see the lengths of the sides of the triangles.

Record the Original side lengths (first measures displayed) in the first row of the table below.

e. Investigate and mentally make note of Side Lengths by grabbing and moving each of the three vertices of ΔABC (A, B, C) to create different shaped triangles.
Record a set of data observed in row "Figure 1" in the following table.

Sample Answer(s):

Circle: i ii iii iv	$\overline{AB}$	$\overline{BC}$	$\overline{CA}$	$\overline{A'B'}$	$\overline{B'C'}$	$\overline{C'A'}$
Original	**4.47 u**	**4 u**	**7.21 u**	**4.47 u**	**4 u**	**7.21 u**
Figure 1	**5.83 u**	**5 u**	**7.28 u**	**5.83 u**	**5 u**	**7.28 u**

f. Discuss observations in your group. Is your conjecture about the lengths of the sides still true?
Answer: Yes, the lengths of the *corresponding* sides are equal.

4. Many different triangles have been rotated in several directions.
Generalize explorations and investigations by responding to the following:
   a. If a triangle is rotated about a point through an angle, what appears to be true about the measures of the angles of the pre-image and image triangles?
   Answer: The measures of the *corresponding* angles are equal.

   b. If a triangle is rotated about a point through an angle, what appears to be true about the lengths of the sides of the pre-image and image triangles?
   Answer: The lengths of the *corresponding* sides are equal.

5. Because the corresponding angles and the corresponding sides of the pre-image and image triangles are congruent, the triangles are congruent.
Therefore, a rotation is a **rigid motion**, or an **isometry**.
We also say that a rotation is a **distance-preserving** and an **angle-preserving** transformation.

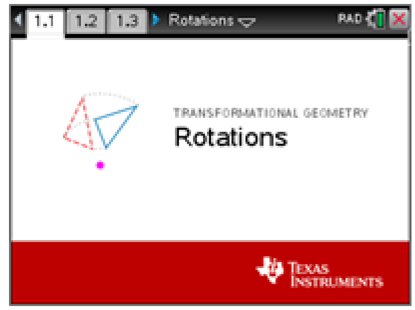
6. ΔDEF has been rotated 75^o about a point. Answer the following.
   a. If $m\angle D = 55^o$, $m\angle D' =$ **55^o.**

   b. If EF = 9 cm, E'F' = **9 cm.**

   c. If $m\angle E = 110^o$, which other angle has a measure of 110^o? $\angle E'$

   d. If DF = 4 in, which other segment has a length of 4 in? $\overline{D'F'}$

Lesson 3: Perimeters & Areas



In this lesson, you will investigate the perimeters and areas of triangles that have been rotated in different ways.

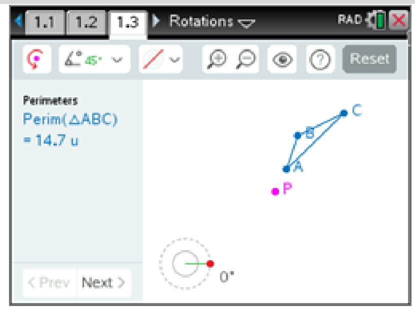
Open the document: *Rotations.tns.*

It is important that the Rotations Tour be done before any Rotations lessons.

PLAY INVESTIGATE EXPLORE DISCOVER

Move to page 1.3. (ctrl ▶ two times)

On the handheld, press ctrl ▶ and ctrl ◀ to navigate through the pages of the lesson. (On the iPad®, select the page thumbnail in the page sorter panel.)



1. Press menu to open the menu.

   (On the iPad®, tap on the wrench icon to open the menu.)

   Press 1 (1: Templates), 3 (3: Perimeters & Areas).

2. Rotate ΔABC 45° about point P (click on or press Q).

   a. Record the Original perimeters (first measures displayed) in the appropriate places of the **Rotate 45°** section in the table below.

   b. Investigate and mentally make note of the perimeters by grabbing and moving each of the three vertices of ΔABC (A, B, C) to create different shaped triangles. Record a set of data observed in row "Figure 1" in the following table.

Sample Answer(s):

Rotate 45°	Perimeter ΔABC	Perimeter $\Delta A'B'C'$		**Rotate 60°**	Perimeter ΔABC	Perimeter $\Delta A'B'C'$
Original	**14.7 u**	**14.7 u**		Original	**14.7 u**	**14.7 u**
Figure 1	**15.47 u**	**15.47 u**		Figure 1	**10.79 u**	**10.79 u**

   c. Reset the page. Press Reset (ctrl del).

   Change the angle of rotation to 60°: click on 45° or press E to open the menu, and press the space bar (␣) to select that measure and to close the menu.

   Click on or press Q to rotate ΔABC 60° about point P.

Record the Original perimeters in the appropriate places of the **Rotate 60°** section in the previous table.

d. Investigate and mentally make note of the perimeters by grabbing and moving each of the three vertices of ΔABC (A, B, C) to create different shaped triangles. Record a set of data observed in row "Figure 1" in the previous table.

e. Reset the page. Press Reset (ctrl del).
Repeat what was done in parts a – d, but with each person in the group choosing a different rotation. Each person in the group should choose one from the following:
i) Rotate ΔABC 30° about point P.
ii) Rotate ΔABC 90° about point P.
iii) Rotate ΔABC – 60° about point P.
iv) Rotate ΔABC – 45° about point P.

(Note: to change the angle of rotation, click on 45° or press E to open the menu, and press the space bar (␣) to select that measure and to close the menu.)

Click on or press Q to rotate ΔABC about point P.

Record the Original perimeters in the appropriate places in the following table.

Sample Answer(s):

Circle: i ii iii iv	Perimeter ΔABC	Perimeter $\Delta A'B'C'$
Original	**14.7 u**	**14.7 u**
Figure 1	**21.95 u**	**21.95 u**

f. Investigate and mentally make note of the perimeters by grabbing and moving each of the three vertices of ΔABC (A, B, C) to create different shaped triangles. Record a set of data observed in row" Figure 1" in the previous table.

g. Many different triangles were rotated in several different directions.
Make a conjecture about the perimeters of rotated triangles.
A ***conjecture*** is an opinion or conclusion based upon what is observed.

Sample Answer: If a triangle is rotated about a point through any angle measure, the perimeter of the pre-image triangle is equal to the perimeter of the image triangle.

h. Based on explorations of rotated triangles in previous lessons, explain why this conjecture is true.

<u>Sample Answer:</u> **Previously we had discovered that the lengths of corresponding sides are equal for rotated triangles. Therefore, their sums should be equal, also.**

3. Do a similar exploration about the areas of rotated triangles in several directions.

a. Reset the page. Press Reset (ctrl del).

Change the angle of rotation to 60^o: click on 45° or press E to open the menu, and press the space bar (␣) to select that measure and to close the menu.

Click on or press Q to rotate ΔABC 60^o about point P.

Click on Next or press) to explore the areas of the triangles.

Record the Original areas (first measures displayed) in the appropriate places of the **Rotate 60°** section in the table below.

<u>**Sample Answer(s):**</u>

Rotate 60°	Area ΔABC	Area $\Delta A'B'C'$		**Rotate – 45°**	Area ΔABC	Area $\Delta A'B'C'$
Original	**5 sq u**	**5 sq u**		Original	**5 sq u**	**5 sq u**
Figure 1	**9.5 sq u**	**9.5 sq u**		Figure 1	**7.5 sq u**	**7.5 sq u**

b. Investigate and mentally make note of the areas by grabbing and moving each of the three vertices of ΔABC (A, B, C) to create different shaped triangles. Record a set of data observed in row "Figure 1" in the previous table.

c. Reset the page. Press Reset (ctrl del).

Change the angle of rotation to -45^o: Click on 45° or press E to open the menu, and press the space bar (␣) to select that measure and to close the menu.

Click on or press Q to rotate ΔABC -45^o about point P.

Click on Next or press) to explore the areas of the triangles.

Record the Original areas in the appropriate places of the **Rotate – 45°** section in the previous table.

d. Investigate and mentally make note of the areas by grabbing and moving each of the three vertices of ΔABC (A, B, C) to create different shaped triangles.

Record a set of data observed in row "Figure 1" in the previous table.

e. Reset the page. Press Reset (ctrl del).
Repeat what was done in parts a – d, but each person in the group choosing a different rotation. Record the Original areas in the appropriate place in the following table. Each person in the group should choose one from the following:

i) Rotate Δ ABC 30^o about point P.
ii) Rotate Δ ABC 90^o about point P.
iii) Rotate Δ ABC -90^o about point P.
iv) Rotate Δ ABC -60^o about point P.

Click on or press Q to rotate Δ ABC about point P.
Click on Next or press) to explore the areas of the triangles.
Record the Original areas in the appropriate place in the following table.

Sample Answer(s):

Circle: i ii iii iv	Area ΔABC	Area $\Delta A'B'C'$
Original	**5 sq u**	**5 sq u**
Figure 1	**10 sq u**	**10 sq u**

f. Investigate and mentally make note of the areas by grabbing and moving each of the three vertices of Δ ABC (A, B, C) to create different shaped triangles. Record a set of data observed in row “Figure 1” in the previous table.

g. Many different triangles were rotated in several different directions.
Make a conjecture about the areas of rotated triangles.
A **conjecture** is an opinion or conclusion based upon what is observed.

Sample Answer: If a triangle is rotated about a point through any angle measure, the area of the pre-image triangle is equal to the area of the image triangle.

h. Based on explorations of rotated triangles in previous lessons, explain why this conjecture is true.

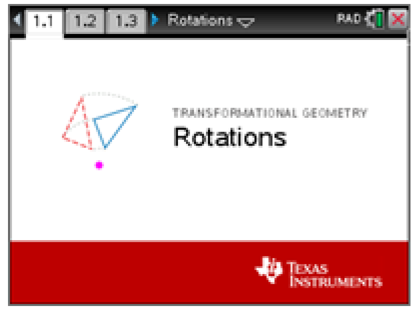
Sample Answer: Previously we had discovered that the lengths of corresponding sides are equal for rotated triangles, which means the lengths of their corresponding bases and heights would be equal, also. Therefore, their products times one-half, A = ½bh, should be equal, also.

4. ΔJKL is rotated 120^o about a point. The perimeter of ΔJKL is 40 cm and its area is 60 sq. cm.

a. What is the perimeter of $\Delta J'K'L'$? **40 cm**

b. What is the area of $\Delta J'K'L'$? **60 sq cm**

Lesson 4: Grid & Coordinates



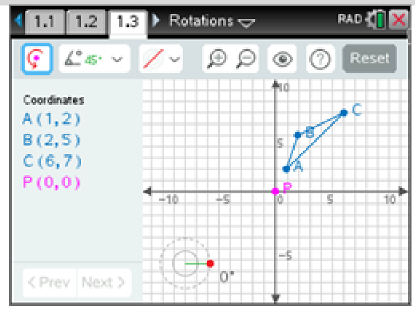
In this lesson, you will investigate the coordinates of vertices of rotated triangles and look for patterns. Open the document: *Rotations.tns.*

It is important that the Rotations Tour be done before any Rotations lessons.

PLAY   INVESTIGATE   EXPLORE   DISCOVER

Move to page 1.3. (ctrl ▸ two times)

On the handheld, press ctrl ▸ and ctrl ◂ to navigate through the pages of the lesson. (On the iPad®, select the page thumbnail in the page sorter panel.)



1. Press menu to open the menu.

   (On the iPad, tap the wrench icon  to open the menu.)
   Press 1 (1: Templates), 5 (5: Grid & Coordinates).

2. Rotate a triangle about the origin through angles of 90°, 180°, 270°, and 360° and look for patterns among the coordinates.

   Change the angle of rotation to 90°. Click on  or press E to open the menu, and press the space bar (␣) to select 90° and to close the menu.

   Click on  or press Q to rotate ΔABC 90° about the origin.

   a. Record the Original coordinates (first coordinates displayed) in the first row of the table on the next page. Look for patterns.

   b. Investigate and mentally make note of the coordinates by grabbing and moving each of the three vertices of ΔABC (A, B, C) to create different shaped triangles.
   Record a set of data observed in row "Figure 1" in the following table.
   Repeat and move each of the three vertices and record a set of data in row "Figure 2" in the following table.
   Suggestion: Move point A to Quadrant II and move point C to Quadrant IV.
   Look for patterns among the coordinates of corresponding vertices.
   Which coordinates remain the same? Which coordinates change? How? Discuss.

Answer: The x-coordinate of the image is the opposite of the y-coordinate of the pre-image, while the y-coordinate of the image is the x-coordinate of the pre-image.

Rotate 90°	A		B	C	A'	B'	C'
Original	**(1, 2)**		**(2, 5)**	**(6, 7)**	**(– 2, 1)**	**(– 5, 2)**	**(– 7, 6)**
Figure 1	**(3, 2)**		**(1, 6)**	**(8, 7)**	**(– 2, 3)**	**(– 6, 1)**	**(– 7, 8)**
Figure 2	**(– 1, 2)**		**(2, 5)**	**(6, – 1)**	**(– 2, –1)**	**(– 5, 2)**	**(1, 6)**

c. Using the pattern observed in the previous table, if a point on the pre-image triangle has coordinates (5, 8), what are the coordinates of the corresponding point on the image triangle?
That is (5, 8) → **(– 8, 5).** '→' means "maps to"
Similarly, the point (– 3, 7) would map to what point? That is (– 3, 7) → **(– 7, –3).**

d. In general, if a point on the pre-image triangle has coordinates (x, y), what are the coordinates of the corresponding point on the image triangle?
That is (x, y) → **(– y, x)** '→' means "maps to"

Encourage students to read as "the opposite of y comma x."

3. Reset the page. Press Reset (ctrl del).

Change the angle of rotation to 180°. Click on 45° or press E to open the menu, and press the space bar (␣) to select 180° and to close the menu.

Click on or press Q to rotate ∆ABC 180° about the origin.

a. Record the Original coordinates (first coordinates displayed) in the first row of the following table. Look for patterns.

b. Investigate and mentally make note of the coordinates by grabbing and moving each of the three vertices of ∆ABC (A, B, C) to create different shaped triangles.
Record a set of data observed in row "Figure 1" in the following table.
Repeat and move each of the three vertices and record a set of data in row "Figure 2" below.
Suggestion: Move point A to Quadrant IV and move point C to Quadrant II.
Look for patterns among the coordinates of corresponding vertices.
Which coordinates remain the same? Which coordinates change? How? Discuss.

Sample Answer(s): The x-coordinate of the image is the opposite of the x-coordinate of its pre-image. The y-coordinate of the image is the opposite of the y-coordinate of its pre-image.

Rotate 180°	A	B	C	A'	B'	C'
Original	**(1, 2)**	**(2, 5)**	**(6, 7)**	**(– 1, –2)**	**(– 2, –5)**	**(– 6, –7)**
Figure 1	**(0, 2)**	**(3, 5)**	**(6, 1)**	**(0, –2)**	**(– 3, –5)**	**(– 6, –1)**
Figure 2	**(3, – 1)**	**(2, 5)**	**(– 3, 7)**	**(– 3, 1)**	**(– 2, –5)**	**(3, –7)**

c. Using the pattern observed in the previous table, if a point on the pre-image triangle has coordinates (5, 8), what are the coordinates of its corresponding point on the image triangle?
That is (5, 8) $\rightarrow$ **(– 5, – 8)** '$\rightarrow$' means "maps to"

Similarly, the point (– 3, 7) would map to what point? That is (– 3, 7) $\rightarrow$ **(3, – 7).**

d. In general, if a point on the pre-image triangle has coordinates (x, y), what are the coordinates of its corresponding point on the image triangle?

That is (x, y) $\rightarrow$ **(– x, – y)** '$\rightarrow$' means "maps to"

Encourage students to read as "the opposite of x comma the opposite of y."

4. Reset the page. Press Reset (ctrl del).
Change the angle of rotation to 270°. Click on 45° or press E to open the menu, and press the space bar (␣) to select 270° and to close the menu.
Click on or press Q to rotate ΔABC 270° about the origin.

a. Record the Original coordinates (first coordinates displayed) in the first row of the following table. Look for patterns.

b. Investigate and mentally make note of the coordinates by grabbing and moving each of the three vertices of ΔABC (A, B, C) to create different shaped triangles.
Record a set of data observed in row "Figure 1" in the following table.
Repeat and move each of the three vertices and record a set of data in row "Figure 2" below.
Suggestion: Move point A to Quadrant II and move point C to Quadrant IV.
Look for patterns among the coordinates of corresponding vertices.

Sample Answer(s):

Rotate 270°	A	B	C	A'	B'	C'
Original	**(1, 2)**	**(2, 5)**	**(6, 7)**	**(2, –1)**	**(5, –2)**	**(7, –6)**
Figure 1	**(0, 2)**	**(2, 7)**	**(6, 5)**	**(2, 0)**	**(7, –2)**	**(5, –6)**
Figure 2	**(– 1, 2)**	**(3, 7)**	**(6, – 3)**	**(2, 1)**	**(7, –3)**	**(– 3, –6)**

c. Using the pattern observed in the previous table, if a point on the pre-image triangle has coordinates (5, 8), what are the coordinates of its corresponding point on the image triangle?
That is (5, 8) $\rightarrow$ **(8, – 5)** '$\rightarrow$' means "maps to"

Similarly, the point (– 3, 7) would map to what point? That is (– 3, 7) $\rightarrow$ **<u>(7, 3).</u>**

d. In general, if a point on the pre-image triangle has coordinates (x, y), what are the coordinates of the corresponding point on the image triangle?

That is (x, y) $\rightarrow$ **<u>(y, – x)</u>** '$\rightarrow$' means "maps to"

Encourage students to read as "y comma the opposite of x."

5. Reset the page. Press Reset (ctrl del).

Change the angle of rotation to 360°. Click on 45° or press E to open the menu, and press the space bar (␣) to select 360° and to close the menu.

Click on or press Q to rotate ΔABC 360° about the origin.

a. Discuss in your groups what you see on the screen.
Investigate and mentally make note of the coordinates by grabbing and moving each of the three vertices of ΔABC (A, B, C) to create different shaped triangles.

b. Based upon your observations, to what point would (5, 8) map? That is (5, 8) $\rightarrow$ **<u>(5, 8)</u>**

c. In general, the point (x, y) would map to what point? That is (x, y) $\rightarrow$ **<u>(x, y).</u>**

6. Summarize the results of these investigations below:
ΔABC is rotated about the origin. The number of degrees of rotation is given below along with the coordinates of a point, (p, q), on the pre-image ΔABC.
Write the coordinates of the corresponding point on its image.

a. ΔABC is rotated 90° about the origin. (p, q) $\rightarrow$ **<u>(– q, p)</u>**

b. ΔABC is rotated 180° about the origin. (p, q) $\rightarrow$ **<u>(– p, – q)</u>**

c. ΔABC is rotated 270° about the origin. (p, q) $\rightarrow$ **<u>(q, – p)</u>**

d. ΔABC is rotated 360° about the origin. (p, q) $\rightarrow$ **<u>(p, q)</u>**

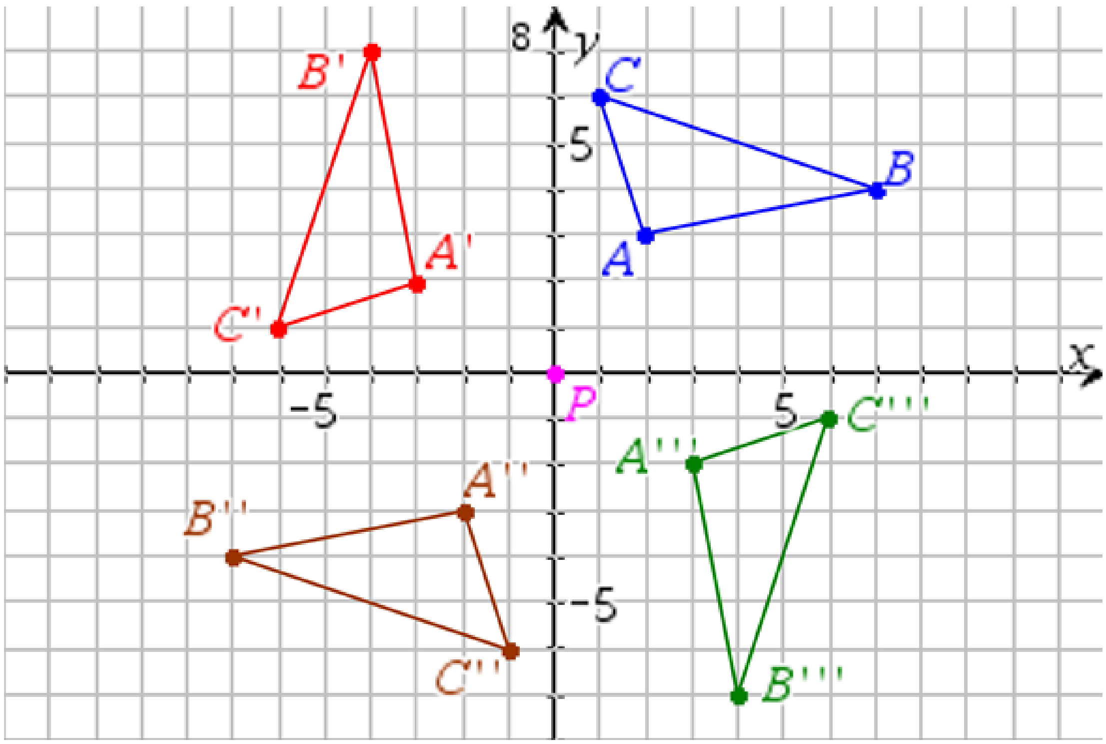
Suggested discussion: notice that the answer for part b is found by rotating part a's answer 90°, and the answer for part c is found by rotating part b's answer another 90°, etc.

7. Using the results from exercise 6 a – d, perform the following rotations on the grid supplied.
   a. Rotate ΔABC 90^o about the origin, P. Label the vertices A', B', and C'.
   b. Rotate ΔABC 180^o about the origin, P. Label the vertices A'', B'', and C''.
   c. Rotate ΔABC 270^o about the origin, P. Label the vertices A''', B''', and C'''.

   List the coordinates of the points below:

A: **(2, 3)** B: **(7, 4)** C: **(1, 6)** A': **(– 3, 2)** B': **(– 4, 7)** C': **(– 6, 1)**

A'': **(– 2, – 3)** B'': **(– 7, – 4)** C'': **(– 1, – 6)** A''': **(3, – 2)** B''': **(4, – 7)** C''': **(6, – 1)**



d. What is true about these four triangles? **Answer: They are congruent to each other.**

e. Draw $\angle APA'$. What is the measure of $\angle APA'$? **Answer: 90°**

f. Draw $\angle CPC''$. What is the measure of $\angle CPC''$? **Answer: 180°**

g. Draw $\angle BPB'''$. What is the measure of $\angle BPB'''$? **Answer: 270° or 90° or – 90°**

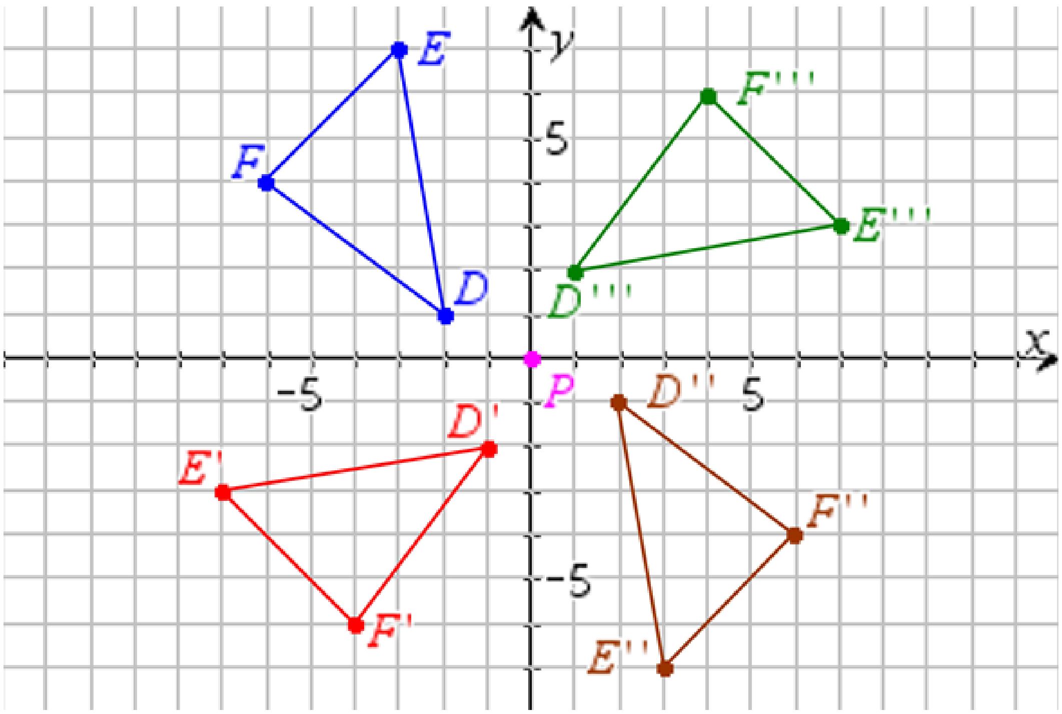
h. What is the image of ΔABC when it is rotated 360^o about the origin **Answer:** **Δ ABC (itself)**

8. Perform the following rotations on the grid supplied.
   a. Rotate ΔDEF 90^o about the origin, P. Label the vertices D’, E’, and F’.
   b. Rotate ΔDEF 180^o about the origin, P. Label the vertices D’’, E’’, and F’’.
   c. Rotate ΔDEF 270^o about the origin, P. Label the vertices D’’’, E’’’, and F’’’.

   List the coordinates of the points below:

D: **(– 2, 1)** E: **(– 3, 7)** F: **(– 6, 4)** D’: **(– 1, – 2)** E’: **(– 7, – 3)** F’: **(– 4, – 6)**

D”: **(2, – 1)** E”: **(3, – 7)** F”: **(6, – 4)** D’”: **(1, 2)** E’”: **(7, 3)** F’”: **(4, 6)**



   d. What is true about these four triangles? **Answer: They are congruent to each other.**

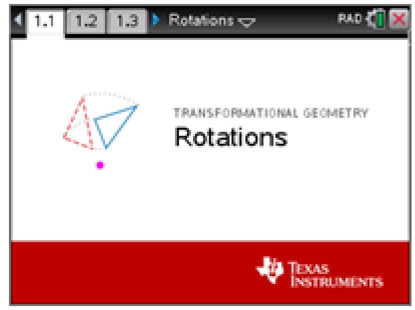
   e. Draw $\angle DPD'$. What is the measure of $\angle DPD'$? **Answer: 90^o**

   f. Draw $\angle D''PD'''$. What is the measure of $\angle D''PD'''$? **Answer: 90^o or -90^o**

   g. Draw $\angle E'PE'''$. What is the measure of $\angle E'PE'''$? **Answer: 180^o or -180^o**

   h. What is the image of ΔDEF when it is rotated 360^o about the origin? **Answer: ΔDEF (itself)**

Lesson 5: Grid and Coordinates 2

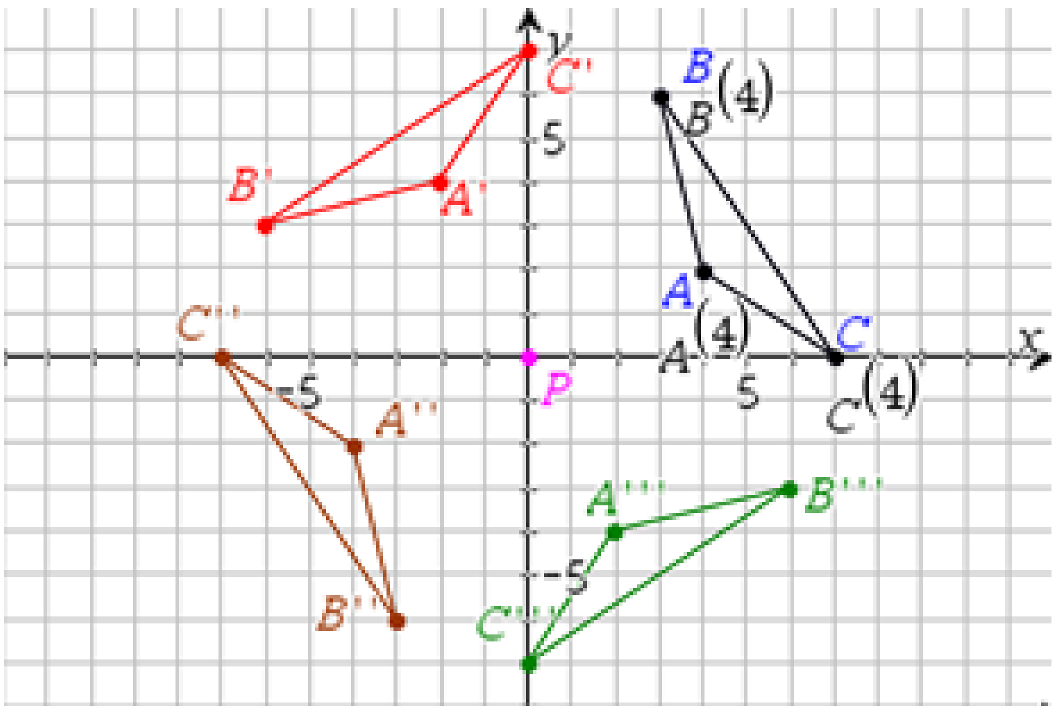


In this lesson, you will continue to investigate the coordinates of vertices of rotated triangles and look for patterns.
Open the document: *Rotations.tns.*

It is important that Rotations Lesson 4 be completed before doing this Lesson.

PLAY INVESTIGATE EXPLORE DISCOVER

1.



Recall from Lesson 4:

Δ ABC is rotated n^o about the origin. Use the grid above to help write the coordinates of the vertices of the image triangles in the table below.

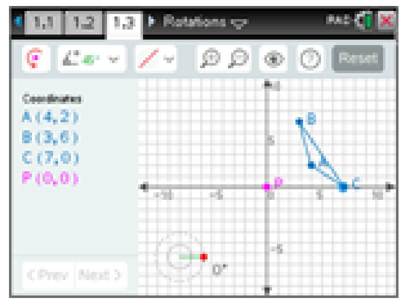
Answer(s):

n^o			
0^o	A: (4, 2)	B: (3, 6)	C: (7, 0)
90^o	**A’: (– 2, 4)**	**B’: (– 6, 3)**	**C’: (0, 7)**
180^o	**A’’: (– 4, – 2)**	**B’’: (– 3, – 6)**	**C’’: (– 7, –0)**
270^o	**A’’’: (2, – 4)**	**B’’’: (6, – 3)**	**C’’’: (0, – 7)**
360^o	**$A^{(4)}$: (4, 2)**	**$B^{(4)}$: (3, 6)**	**$C^{(4)}$: (7, 0)**

Check your answers in the previous table by using the *Rotations.tns* file in exercise 2 below.

Move to page 1.3. (ctrl ▸ two times)

On the handheld, press ctrl ▸ and ctrl ◂ to navigate through the pages of the lesson.
(On the iPad®, select the page thumbnail in the page sorter panel.)



2. Press menu to open the menu.

   (On the iPad, tap the wrench icon to open the menu.)
   Press 1 (1: Templates), 5 (5: Grid & Coordinates).

   Grab and move each of the three vertices of ΔABC
   (A, B, C) so that: A: (4, 2) B: (3, 6) C: (7, 0)

3. To check your answers, change the angle of rotation to 90°. Click on 45° or press E to open the menu, and press the space bar (␣) to select 90° and close the menu.
   Make corrections as needed.

   a. Click on or press Q to rotate ΔABC 90° about the origin.
      Compare the ordered pairs listed on the screen to the ones in the table on the previous page.

   b. Click on or press Q to rotate ΔABC an additional 90° about the origin, a total of 180°.
      Compare the ordered pairs listed on the screen to the ones in the table on the previous page.

   c. Click on or press Q to rotate ΔABC an additional 90° about the origin, a total of 270°.
      Compare the ordered pairs listed on the screen to the ones in the table on the previous page.

   d. Click on or press Q to rotate ΔABC an additional 90° about the origin, a total of 360°.
      Compare the ordered pairs listed on the screen to the ones in the table on the previous page.

4. Reset the page. Press Reset (ctrl del).
   Change the angle of rotation to 90°. Click on 45° or press E to open the menu, and press the space bar (␣) to select 90° and to close the menu.
   Click on or press Q to rotate ΔABC 90° about the origin.

   a. Look at the coordinates of corresponding vertices. Does each point (x, y) on ΔABC map to (– y, x) on ΔA’B’C’? **Yes.**

b. Grab and move point P (click on point P or press [P] and use the directional arrows) and look at the coordinates of corresponding vertices. Move point P to several places on the grid. Does each point (x, y) on ΔABC always map to (– y, x) on ΔA'B'C'? Explain.

Sample Answer: No. Only when the point of rotation, P, is at the origin.

c. Click on or press [Q] to rotate ΔABC an additional 90° about point P, a total of 180°. Move point P to several places on the grid. (click on point P or press [P] and use the directional arrows)
Does each point (x, y) on ΔABC always map to (– x, – y) on ΔA'B'C'?
Explain.

Sample Answer: No. Only when the point of rotation, P, is at the origin.

d. Move point P back to the origin (click on point P or press [P] and use the directional arrows).
Does each point (x, y) on ΔABC now map to (– x, – y) on ΔA'B'C'?
Explain.

Sample Answer: Yes. As long as the point of rotation, P, is at the origin.

e. Click on or press [Q] to rotate ΔABC an additional 90° about point P, a total of 270°.
Move point P to several places on the grid.
Does each point (x, y) on ΔABC always map to (y, – x) on ΔA'B'C'?
Explain.

Sample Answer: No. Only when the point of rotation, P, is at the origin.

f. Move point P back to the origin.
Does each point (x, y) on ΔABC now map to (y, – x) on ΔA'B'C'?
Explain.

Sample Answer: Yes. As long as the point of rotation, P, is at the origin.

g. Discuss in your groups and make a generalization.

Sample Answer: For the patterns that we noticed, the point of rotation, P, must be at the origin.

5. Reset the page. Press Reset ([ctrl] [del]).
Change the angle of rotation to – 90°. Click on 45° or press [E] to open the menu, and press the space bar ([␣]) to select – 90° and to close the menu.
Click on or press [Q] to rotate ΔABC – 90° about the origin.

a. Record the Original coordinates (first coordinates displayed) in the first row of the following table. Look for patterns.

b. Investigate and mentally make note of the coordinates by grabbing and moving each of the three vertices of ΔABC ($\boxed{A}$, $\boxed{B}$, $\boxed{C}$) to create different shaped triangles.
Record a set of data observed in row "Figure 1" in the following table.
Repeat and move each of the three vertices and record a set of data in row "Figure 2" below.
Look for patterns among the coordinates of corresponding vertices.
Which coordinates remain the same? Which coordinates change? How? Discuss.

The x-coordinate of the image is the same as the y-coordinate of the pre-image.
The y-coordinate of the image is the opposite of the x-coordinate of the pre-image.

Sample Answer(s):

Rotate – 90°	A	B	C	A'	B'	C'
Original	**(1, 2)**	**(2, 5)**	**(6, 7)**	**(2, – 1)**	**(5, – 2)**	**(7, – 6)**
Figure 1	**(1, 3)**	**(2, 6)**	**(7, 5)**	**(3, – 1)**	**(6, – 2)**	**(5, – 7)**
Figure 2	**(– 1, 3)**	**(2, 7)**	**(8, 4)**	**(3, 1)**	**(7, – 2)**	**(4, – 8)**

c. Using the pattern observed in the previous table, if a point on the pre-image triangle has coordinates (5, 8), what are the coordinates of its corresponding point on the image triangle?
That is (5, 8) $\rightarrow$ **(8, – 5)** '$\rightarrow$' means "maps to"
Similarly, the point (– 3, 7) would map to what point? That is (– 3, 7) $\rightarrow$ **(7, 3).**

d. In general, if a point on the pre-image triangle has coordinates (x, y), what are the coordinates of its corresponding point on the image triangle?

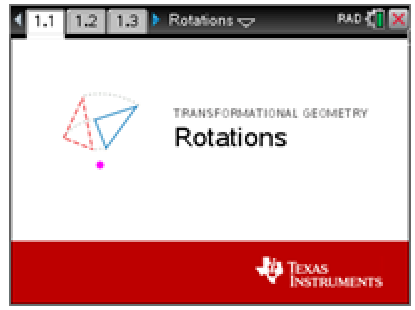
That is (x, y) $\rightarrow$ **(y, – x)** '$\rightarrow$' means "maps to"

e. Rotating a triangle – 90° about the origin is equivalent to a different rotation. Explain.
Answer: It is equivalent to rotating 270° about the origin.

f. What rotation is equivalent to rotating a triangle – 180° about the origin?
Answer: It is equivalent to rotating 180° about the origin.

g. What rotation is equivalent to rotating a triangle – 270° about the origin?
Answer: It is equivalent to rotating 90° about the origin.

Lesson 6: Distance to Vertices



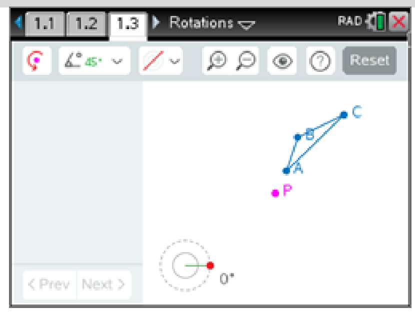
In this lesson, you will investigate the distances from the point of rotation to each of the vertices of rotated triangles and look for patterns. Open the document: *Rotations.tns.*

<u>It is important that the Rotations Tour be done before any Rotations lessons.</u>

PLAY INVESTIGATE EXPLORE DISCOVER

Move to page 1.3. (ctrl ▸ two times)

On the handheld, press ctrl ▸ and ctrl ◂ to navigate through the pages of the lesson. (On the iPad®, select the page thumbnail in the page sorter panel.)



1. Click on or press Q to rotate Δ ABC 45° about point P.

   Look at segments: $\overline{PA}$ and $\overline{PA'}$.

   What seems to be true about the lengths of $\overline{PA}$ and $\overline{PA'}$?

   Discuss in your groups. **<u>The lengths appear to be equal.</u>**

   Grab point A (A) and move it about the screen.

   Click on or press Q to rotate Δ ABC an additional 45°.

   Grab point A (A) and move it about the screen.

   Make a conjecture about the lengths of $\overline{PA}$ and $\overline{PA'}$

   <u>Sample Answer:</u> When a triangle is rotated about a point, the distance from the point of rotation to a vertex on the pre-image is equal to the distance from the point of rotation to the corresponding vertex on the image.

2. Reset the page. Press Reset (ctrl del).

   a. Each person in the group select one of the pairs of segments to observe:

      i) the lengths of $\overline{PB}$ and $\overline{PB'}$ ii) the lengths of $\overline{PC}$ and $\overline{PC'}$

   Click on or press Q to rotate Δ ABC 45° about point P.

   Look at the lengths of segments: $i)$ $\overline{PB}$ and $\overline{PB'}$ or $ii)$ $\overline{PC}$ and $\overline{PC'}$.

   What seems to be true about the lengths of: $i)$ $\overline{PB}$ and $\overline{PB'}$ or $ii)$ $\overline{PC}$ and $\overline{PC'}$?

   Discuss in your groups.

   Grab either point B (B) or point C (C) and move it about the screen.

b. Click on or press Q to rotate ΔABC an additional 45°.
Grab either point B (B) or point C (C) and move it about the screen.

c. Make a conjecture about the lengths of: $i)\ \overline{PB}\ and\ \overline{PB'}\ or\ ii)\overline{PC}\ and\ \overline{PC'}$.

Sample Answer: When a triangle is rotated about a point, the distance from the point of rotation to a vertex on the pre-image is equal to the distance from the point of rotation to the corresponding vertex on the image.

You may want to extend the discussion: the distance from P to any point on the pre-image triangle is equal to the distance from P to its corresponding point on the image triangle.

3. Reset the page. Press Reset (ctrl del).

a. Click on or press Q to rotate ΔABC 45° about point P.

To assist in validating your conjectures, do the following:

Click on the Multiple Icon or press M . Press the down arrow (▼) once and press the space bar (␣) to select the second choice in the dropdown menu.
Discuss in your groups what is displayed on the screen.

b. Three dashed circles appeared on the screen. The circles all have the same center, P, but have different radii. They are called **concentric circles**.

c. Continue to rotate ΔABC about point P until it shows 360° on the screen.
Look at $\overline{PA}\ and\ \overline{PA'},\ \overline{PB}\ and\ \overline{PB'},\ and\ \overline{PC}\ and\ \overline{PC'}$ as you rotate ΔABC.

d. To see all previous images, open the Options menu (press or O).
Use the directional arrows (▲ ▼ ◀ ▶) to move to the box next to "Historical Images".
Press the space bar key (␣) to put a check mark in the box. Press enter or esc .
Observe the screen.

e. Click on or press Q to rotate ΔABC 45° about point P.
Continue to rotate ΔABC about point P until it shows 360° on the screen.
Look at $\overline{PA}\ and\ \overline{PA'},\ \overline{PB}\ and\ \overline{PB'},\ and\ \overline{PC}\ and\ \overline{PC'}$ as you rotate ΔABC.

f. Discuss in your groups how the concentric circles can help convince you why your conjecture is true.

Sample Answer: $\overline{PA}\ and\ \overline{PA'}$ **are radii of the same circle and are congruent. Similarly,** $\overline{PB}\ and\ \overline{PB'}$ **and** $\overline{PC}\ and\ \overline{PC'}$ **are radii of the same circle and are congruent.**

4. Press [menu] to open the menu.

   (On the iPad, tap the wrench icon to open the menu.)
   Press [1] (1: Templates), [4] (4: Dist P to Vertices).
   Click on or press [Q] to rotate ΔABC 45° about point P.

   a. Record the Original lengths (first lengths displayed) in the first row of the table below.
      Look for patterns.

   b. Investigate and mentally make note of the lengths by grabbing and moving each of the three vertices of ΔABC ([A], [B], [C]) to create different shaped triangles.
      Record a set of data observed in row ”Figure 1” in the following table.
      Repeat and move each of the three vertices and record a set of data in row “Figure 2” below.
      Look for patterns among the lengths of corresponding sides.

Sample Answer(s):

Rotate 45°	PA	PB	PC	PA’	PB’	PC’
Original	**2.24 u**	**5.39 u**	**9.22 u**	**2.24 u**	**5.39 u**	**9.22 u**
Figure 1	**2 u**	**6.32 u**	**9.9 u**	**2 u**	**6.32 u**	**9.9 u**
Figure 2	**3 u**	**7.28 u**	**8.49 u**	**3 u**	**7.28 u**	**8.49 u**

   c. Based upon the data in the table above, make a conjecture.

   Sample Answer: When a triangle is rotated about a point, the distance from the point of rotation to a vertex on the pre-image is equal to the distance from the point of rotation to the corresponding vertex on the image.

5. Reset the page. Press Reset ([ctrl] [del]).

   a. Each person in the group will select a different angle for the step size (45° or press [E]).
      i) 30° ii) 60° iii) – 60° iv) – 45°
      Press the space bar ([␣]) to select that measure and to close the menu.

   b. Click on or press [Q] to rotate ΔABC about point P through the angle you chose.
      Record the Original lengths (first lengths displayed) in the first row of the following table.
      Look for patterns.

   c. To see all previous images, open the Options menu (press or [O]).
      Use the directional arrows (▲ ▼ ◀ ▶) to move to the box next to “Historical Images”.
      Press the space bar key ([␣]) to put a check mark in the box. Press [enter] or [esc] .

Click on the Multiple Icon or press [M] . Press the down arrow (▼) once and press the space bar ([␣]) to select the second choice in the dropdown menu.

d. Investigate and mentally make note of the lengths by grabbing and moving each of the three vertices of Δ ABC ([A], [B], [C]) to create different shaped triangles.
Record a set of data observed in row "Figure 1" in the following table.
Repeat and move each of the three vertices and record a set of data in row "Figure 2" below.
Look for patterns among the lengths of corresponding sides.

Sample Answer(s):

Circle: **i ii iii iv**	PA	PB	PC	PA'	PB'	PC'
Original	**2.24 u**	**5.39 u**	**9.22 u**	**2.24 u**	**5.39 u**	**9.22 u**
Figure 1	**1.41 u**	**6.08 u**	**8.49 u**	**1.41 u**	**6.08 u**	**8.49 u**
Figure 2	**1 u**	**7.07 u**	**10 u**	**1 u**	**7.07 u**	**10 u**

e. Continue to rotate Δ ABC about point P until it shows 360^{o} on the screen.
Look at $\overline{PA}$ *and* $\overline{PA'}$, $\overline{PB}$ *and* $\overline{PB'}$, *and* $\overline{PC}$ *and* $\overline{PC'}$ as you rotate Δ ABC.

f. Based upon the data in the table above, is your conjecture still true?
Answer: Yes

6. Δ DEF has been rotated 65^{o} about point Z. Answer the following questions.

a. List 3 pairs of segments that have point Z as one of the endpoints that are congruent.

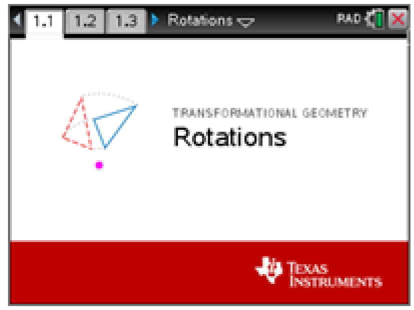
Sample Answer: $\overline{ZD} \cong \overline{ZD'}, \ \overline{ZE} \cong \overline{ZE'}, \ \overline{ZF} \cong \overline{ZF'}$

b. If ZD = 5 cm, then **ZD'** = 5 cm.

c. If ZE' = 4 in, then **ZE** = 4 in.

7. Define concentric circles.
Sample Answer: **Concentric circles are circles that have the same center, but different radii (and are in the same plane).**

Lesson 7: Corresponding Sides



In this lesson, you will investigate the corresponding sides (not their lengths) of rotated triangles and look for patterns.

Open the document: *Rotations.tns.*

It is important that the Rotations Tour be done before any Rotations lessons.

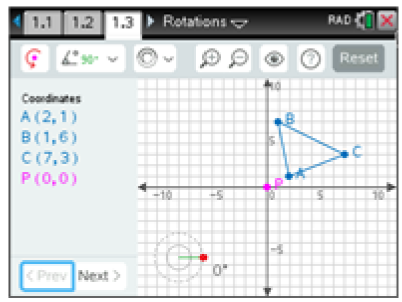
PLAY INVESTIGATE EXPLORE DISCOVER

Move to page 1.3. (ctrl ▶ two times)

On the handheld, press ctrl ▶ and ctrl ◀ to navigate through the pages of the lesson.
(On the iPad®, select the page thumbnail in the page sorter panel.)

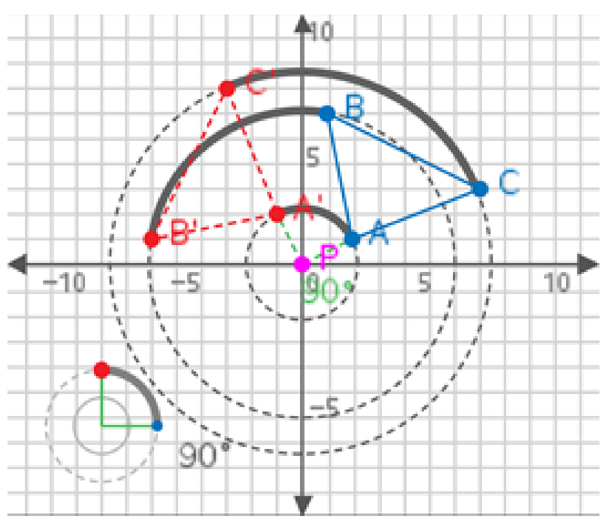
1. Press menu to open the menu.

   (On the iPad, tap the wrench icon to open the menu.)
   Press 1 (1: Templates), 6 (6: Slopes).



2. Click on or press Q to rotate Δ ABC 90° about the origin, point P.
   Each person in the group will pick a different pair of corresponding sides. Circle your choice.
   i) $\overline{AB}$ *and* $\overline{A'B'}$ ii) $\overline{BC}$ *and* $\overline{B'C'}$ iii) $\overline{CA}$ *and* $\overline{C'A'}$

   Calculate the slopes of corresponding sides by hand – either graphically or by slope formula. Show your work in the space provided below. Write your answers as fractions in simplest form. Record the slopes in the first row (Original) of the table on the next page. Look for patterns.



$$m\left(\overline{AB}\right) = -\frac{5}{1} \qquad m\left(\overline{A'B'}\right) = \frac{1}{5}$$

$$m\left(\overline{BC}\right) = -\frac{1}{2} \qquad m\left(\overline{B'C'}\right) = \frac{2}{1}$$

$$m\left(\overline{CA}\right) = \frac{2}{5} \qquad m\left(\overline{C'A'}\right) = -\frac{5}{2}$$

a. Collaborate and summarize the answers in the table below:

Answer(s):

Rotate 90°	$m(\overline{AB})$	$m(\overline{A'B'})$	$m(\overline{BC})$	$m(\overline{B'C'})$	$m(\overline{CA})$	$m(\overline{C'A'})$
Original	$-\frac{5}{1}$	$\frac{1}{5}$	$-\frac{1}{2}$	$\frac{2}{1}$	$\frac{2}{5}$	$-\frac{5}{2}$
Figure i	$\frac{2}{1}$	$-\frac{1}{2}$	$-\frac{4}{5}$	$\frac{5}{4}$	$\frac{1}{4}$	$-\frac{4}{1}$
Figure ii	$\frac{2}{1}$	$-\frac{1}{2}$	$-\frac{4}{1}$	$\frac{1}{4}$	**0**	**undef**
Figure iii	$-\frac{1}{1}$	$\frac{1}{1}$	$-\frac{1}{4}$	$\frac{4}{1}$	$\frac{1}{2}$	$-\frac{2}{1}$
Figure iv	**undef**	**0**	$-\frac{1}{2}$	$\frac{2}{1}$	**0**	**undef**

b. Check your answers. To see the slopes, click on Next > or press $\boxed{)}$.
The slopes are listed as decimals on the screen. Rewrite them as fractions in simplest form and compare these fractions to the answers in the 'Original' row above.
Make corrections as needed.

c. Look at the slopes of corresponding sides. Discuss in your groups what pattern you notice about these numbers.

Answer: **Each pair of numbers are opposites and reciprocals of each other, except when the lines are horizontal and vertical.**

d. Continue to investigate for several more triangles and look for patterns.
Each person in the group will pick a different Figure i, ii, iii, or iv (from the following)

To see the coordinates of the vertices on the screen, click on < Prev or press $\boxed{(}$.

i) Grab and move the vertices so that A: (1, 2) B: (4, 8) C: (9, 4)
ii) Grab and move the vertices so that A: (1, 3) B: (3, 7) C: (4, 3)
iii) Grab and move the vertices so that A: (4, 3) B: (0, 7) C: (8, 5)
iv) Grab and move the vertices so that A: (2, 1) B: (2, 5) C: (10, 1)

To see the slopes, click on Next > or press $\boxed{)}$.
Write the slopes of $\overline{AB}$, $\overline{A'B'}$, $\overline{BC}$, $\overline{B'C'}$, $\overline{CA}$, $\overline{C'A'}$ as fractions in simplest form.
Show your work on this paper.
When all the students in your group are finished, record all the slopes as fractions in the appropriate places (Figure i, ii, iii, or iv) in the previous table.

e. Look at the slopes of each pair of corresponding sides $\overline{AB}$ *and* $\overline{A'B'}$ listed in the table.

What is true about the slopes of these two segments?

Answer: The slopes are opposites and reciprocals of each other, except when they are '0' and 'undef' (horizontal and vertical lines).

f. Look at the slopes of each pair of corresponding sides $\overline{BC}$ *and* $\overline{B'C'}$ listed in the table.

What is true about the slopes of these two segments?

Answer: The slopes are opposites and reciprocals of each other, except when they are '0' and 'undef' (horizontal and vertical lines).

g. Look at the slopes of each pair of corresponding sides $\overline{CA}$ *and* $\overline{C'A'}$ listed in the table.

What is true about the slopes of these two segments?

Answer: The slopes are opposites and reciprocals of each other, except when they are '0' and 'undef' (horizontal and vertical lines).

h. If segments (lines) are to be parallel, what must be true about their slopes?

Answer: Slopes must be equal (the same).

i. If segments (lines) are to be perpendicular, what must be true about their slopes?

Answer: Slopes must be opposites and reciprocals of each other, except when they are '0' and 'undef' (horizontal and vertical lines).

j. Based upon your observations, complete the following:

If a triangle is rotated about the origin 90^o, the slopes of corresponding sides are **opposites and reciprocals of each other**. The lines that contain these corresponding sides will be **perpendicular** to each other.

3. Reset the page. Press Reset (ctrl del).

a. Rotate Δ ABC 180^o about the origin by clicking on [rotate icon] twice or by pressing on (Q) twice. To see the slopes, click on Next > or press). Record the slopes as decimals in the first row (Original) of the table below. Look for patterns.

Sample Answer(s):

Rotate 180^o	$m(\overline{AB})$	$m(\overline{A'B'})$	$m(\overline{BC})$	$m(\overline{B'C'})$	$m(\overline{CA})$	$m(\overline{C'A'})$
Original	**– 5**	**– 5**	**– 0.5**	**– 0.5**	**0.4**	**0.4**
Figure 1	**undef**	**undef**	**– 0.8**	**– 0.8**	**0**	**0**
Figure 2	**1**	**1**	**– 2**	**– 2**	**– 0.29**	**– 0.29**

b. Continue to investigate.
Grab and move each of the three vertices of ΔABC (A, B, C).
Record the slopes observed in row "Figure 1" in the previous table.

c. Grab and move each of the three vertices of ΔABC (A, B, C).
Record the slopes observed in row "Figure 2" in the previous table.

d. Based upon your observations, complete the following:
If a triangle is rotated about the origin180^o, the slopes of corresponding sides

are **<u>equal (the same).</u>** The lines that contain these corresponding sides will be

<u>parallel (or overlap)</u> to each other.

4. Reset the page. Press Reset (ctrl del).

a. Rotate ΔABC 270^o about the origin by clicking on three times or by pressing on (Q) three times.
To see the slopes, click on Next or press). Record the slopes as fractions in simplest form in the first row (Original) of the table below. Look for patterns.

<u>Sample Answer(s):</u>

Rotate 270^o	$m(\overline{AB})$	$m(\overline{A'B'})$	$m(\overline{BC})$	$m(\overline{B'C'})$	$m(\overline{CA})$	$m(\overline{C'A'})$
Original	$-\frac{5}{1}$	$\frac{1}{5}$	$-\frac{1}{2}$	$\frac{2}{1}$	$\frac{2}{5}$	$-\frac{5}{2}$
Figure 1	$-\frac{1}{1}$	$\frac{1}{1}$	$-\frac{1}{4}$	$\frac{4}{1}$	$\frac{1}{2}$	$-\frac{2}{1}$

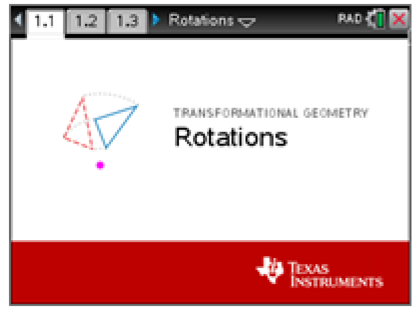
b. Click on Prev or press (. Grab and move the vertices to the following points:
A: (4, 3) B: (0, 7) C: (8, 5) To view the slopes, click on Next or press).
Record the slopes as fractions in simplest form in row "Figure 1" in the previous table.

c. Based upon your observations, complete the following:
If a triangle is rotated about the origin 270^o, the slopes of corresponding sides

are **<u>opposites and reciprocals of each other</u>**. The lines that contain

these corresponding sides will be **<u>perpendicular</u>** to each other.

Lesson 8: Self-Assessment

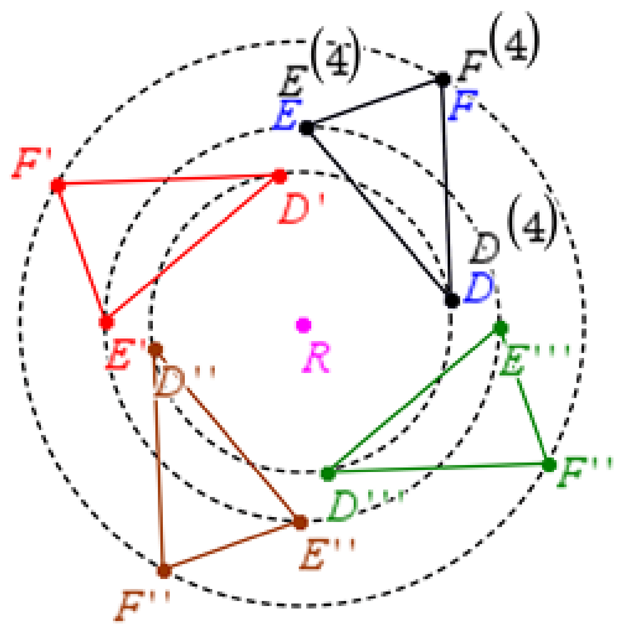


In this lesson, you will be given the opportunity to summarize, review, explore and extend ideas about Rotations.

<u>It is important that the Rotations Tour be done before any Rotations lessons.</u>

Use a compass and straightedge when needed.

1. Label the vertices of the images appropriately.



   a. Rotate ΔDEF 90° about point R. ($\Delta D'E'F'$)

   b. Rotate ΔDEF 180° about point R. ($\Delta D''E''F''$)

   c. Rotate ΔDEF 270° about point R. ($\Delta D'''E'''F'''$)

   d. Rotate ΔDEF 360° about point R. ($\Delta D^{(4)}E^{(4)}F^{(4)}$)

   e. If $m\angle D = 35^o$, then $m\angle D' =$ **<u>35°.</u>**

   f. If EF = 4.5 in, then E''F'' = **<u>4.5 in.</u>**

   g. If the slope of $\overline{ED}$ = – 2, then the slope of $\overline{E'D'}$ = $\underline{\frac{1}{2}}$.

   h. If the slope of $\overline{EF}$ = $\frac{2}{3}$, then the slope of $\overline{E''F''}$ = $\underline{\frac{2}{3}}$.

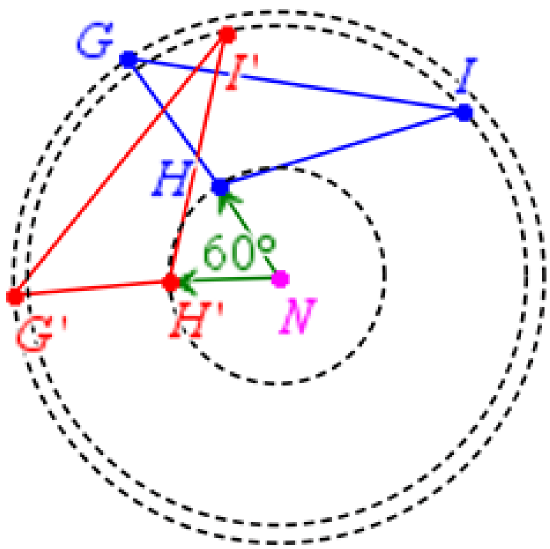
   i. If the perimeter of ΔDEF is 8 in, then the perimeter of $\Delta D''E''F''$ is **<u>8 in.</u>**

   j. If the coordinates of point D are (3, 2), what are the coordinates of:

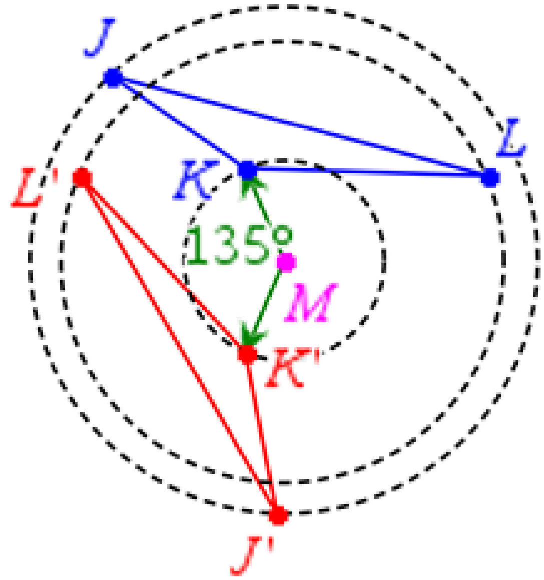
   D': **<u>(2, – 3)</u>** D'': **<u>(– 3, – 2)</u>** D''': **<u>(– 2, 3)</u>** $D^{(4)}$: **<u>(3, 2)</u>**

Use a compass and straightedge as needed.

2. Rotate ΔGHI 60^{o} about point N.



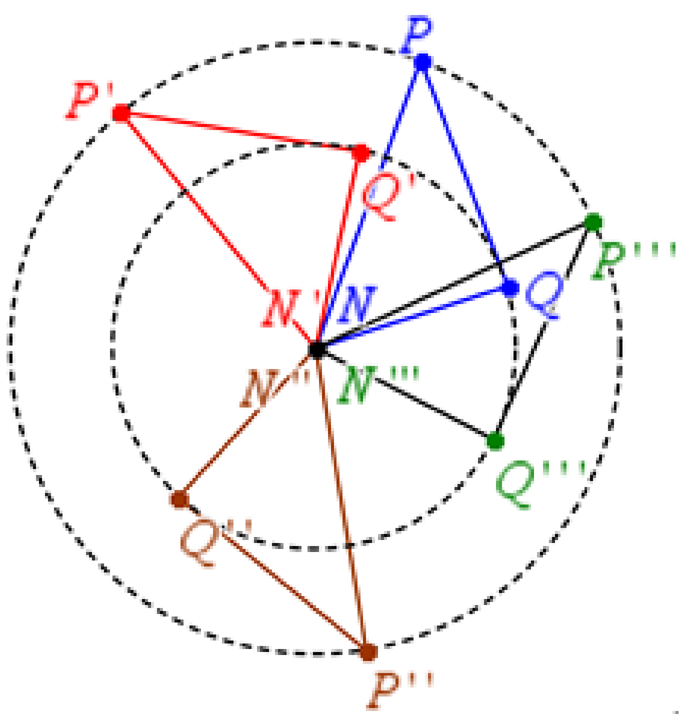
3. Rotate ΔJKL 135^{o} about point M.



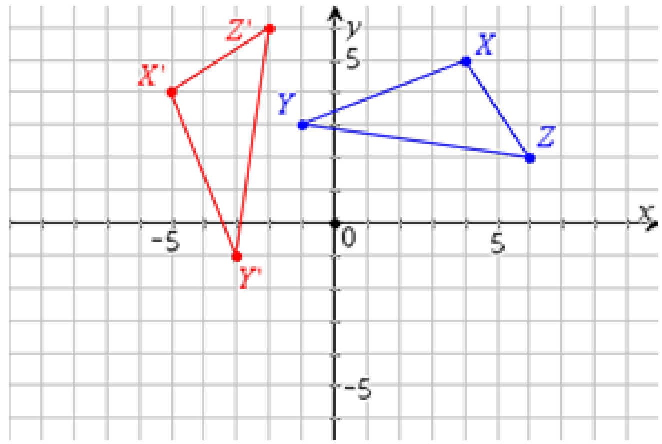
4. a. Rotate ΔNPQ 60^{o} about point N. Label the image $\Delta N'P'Q'$.

   b. Rotate ΔNPQ 210^{o} about point N. Label the image $\Delta N''P''Q''$.

   c. Rotate ΔNPQ -45^{o} about point N. Label the image $\Delta N'''P'''Q'''$.



5. Label the vertices of the images appropriately.



a. Rotate ΔXYZ 90^{o} about the origin.

$m(\overline{XY}) = \frac{2}{5}$ $m(\overline{X'Y'}) = -\frac{5}{2}$

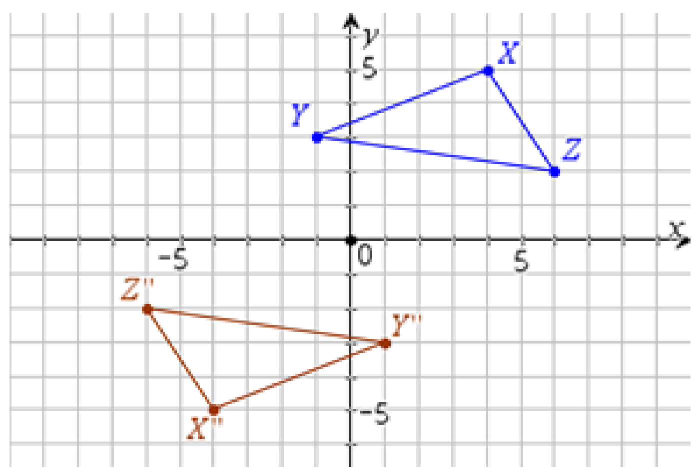
$m(\overline{YZ}) = -\frac{1}{7}$ $m(\overline{Y'Z'}) = \frac{7}{1}$

$m(\overline{XZ}) = -\frac{3}{2}$ $m(\overline{X'Z'}) = \frac{2}{3}$

Fill in the blanks with either $\parallel$ ('is parallel to') or $\perp$ (' is perpendicular to'):

$\overleftrightarrow{XY} \perp \overleftrightarrow{X'Y'}$ $\overleftrightarrow{YZ} \perp \overleftrightarrow{Y'Z'}$ $\overleftrightarrow{XZ} \perp \overleftrightarrow{X'Z'}$

Label the vertices of the images appropriately.



b. Rotate ΔXYZ 180^{o} about the origin.

$m(\overline{XY}) = \frac{2}{5}$ $m(\overline{X''Y''}) = \frac{2}{5}$

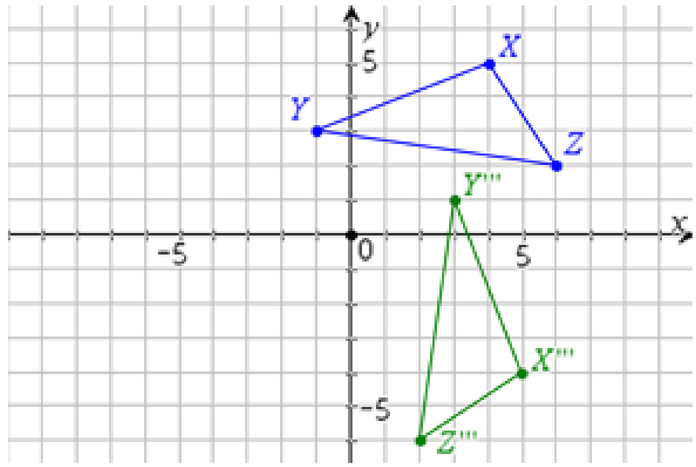
$m(\overline{YZ}) = -\frac{1}{7}$ $m(\overline{Y''Z''}) = -\frac{1}{7}$

$m(\overline{XZ}) = -\frac{3}{2}$ $m(\overline{X''Z''}) = -\frac{3}{2}$

Fill in the blanks with either $\parallel$ ('is parallel to') or $\perp$ (' is perpendicular to'):

$\overleftrightarrow{XY} \parallel \overleftrightarrow{X''Y''}$ $\overleftrightarrow{YZ} \parallel \overleftrightarrow{Y''Z''}$ $\overleftrightarrow{XZ} \parallel \overleftrightarrow{X''Z''}$

Label the vertices of the images appropriately.



c. Rotate ΔXYZ 270° about the origin.

$m(\overline{XY}) = \underline{\underline{\frac{2}{5}}}$ $m(\overline{X'''Y'''}) = \underline{\underline{-\frac{5}{2}}}$

$m(\overline{YZ}) = \underline{\underline{-\frac{1}{7}}}$ $m(\overline{Y'''Z'''}) = \underline{\underline{\frac{7}{1}}}$

$m(\overline{XZ}) = \underline{\underline{-\frac{3}{2}}}$ $m(\overline{X'''Z'''}) = \underline{\underline{\frac{2}{3}}}$

Fill in the blanks with either ∥ ('is parallel to') or ⊥ (' is perpendicular to'):

$\overleftrightarrow{XY}$ $\underline{\underline{\perp}}$ $\overleftrightarrow{X'''Y'''}$ $\overleftrightarrow{YZ}$ $\underline{\underline{\perp}}$ $\overleftrightarrow{Y'''Z'''}$ $\overleftrightarrow{XZ}$ $\underline{\underline{\perp}}$ $\overleftrightarrow{X'''Z'''}$

6. a. The corresponding sides of rotated triangles are **equal in length, congruent.**

   b. The corresponding angles of rotated triangles are **equal in measure, congruent.**

   c. If a triangle is rotated about a point through a given angle measure, then the pre-image triangle and the image triangle are **congruent** to each other.

7. If a triangle is rotated about a point through x°, the corresponding angles and the corresponding sides of the pre-image and image triangles are congruent and the triangles are **congruent.**

   Therefore, a rotation is a **rigid motion**, or an **isometry**.

   We also say that a rotation is a **distance-preserving**

   and an **angle-preserving** transformation.