

## PIECEWISE FUNCTIONS

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### INSTRUCTIONS FOR THE INSTRUCTOR

#### NCTM EXPECTATIONS THAT ARE MET BY THIS ACTIVITY – GRADES 9-12

After completing this activity, the students should be able to

- Interpret representations of functions of two variables;
- Use symbolic algebra to represent and explain mathematical relationships;
- Judge the meaning, utility and reasonableness of the results of symbol manipulations, including those carried out by technology;
- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships.

#### PUERTO RICO INDICATORS THAT ARE MET BY THIS ACTIVITY – 10<sup>th</sup> GRADE

After completing this activity, the students should be able to

**A.PR.10.8.1** Analyze a situation to determine and interpret the domain and range of a piecewise function;

**A.PR.10.8.2** Interpret, construct and apply the floor and ceiling functions, and other piecewise functions such as the absolute value function, to model and solve problems;

**A.PR.10.8.3** Transit between verbal, graphical, tabular and symbolic representations of the floor and ceiling functions and other piecewise functions.

#### CONCEPTS

Piecewise functions

Floor and ceiling functions

Range and domain of functions

Verbal, graphical, tabular and symbolic representations of piecewise functions

Modeling of situations using piecewise functions

#### MATERIALS

TI-Nspire with activity **piecewise\_functions.tns**

#### INTRODUCTION

This activity is aimed for high school students and university students enrolled in a pre-calculus course. The goals of this activity are:

- To establish the connection between a situation and its graphical representation;
- To establish the importance of the different representations of a situation;
- To model a situation mathematically.

#### INSTRUCTIONS

Students should work in pairs. Each student should have a printed version of the document **piecewise\_functions\_student.doc**. Each group should have a TI-Nspire with the archive **piecewise\_functions.tns**. They should read the instructions and answer every question with the help of the TI-Nspire. After each group has analyzed problems 1 and 2, the instructor will lead a group discussion to emphasize some of the important aspects.

The first problem has two parts: writing the stories and assigning the units of measurement to the variables and the scale. For each graph, the instructor should ask questions to verify that the concepts have been understood:

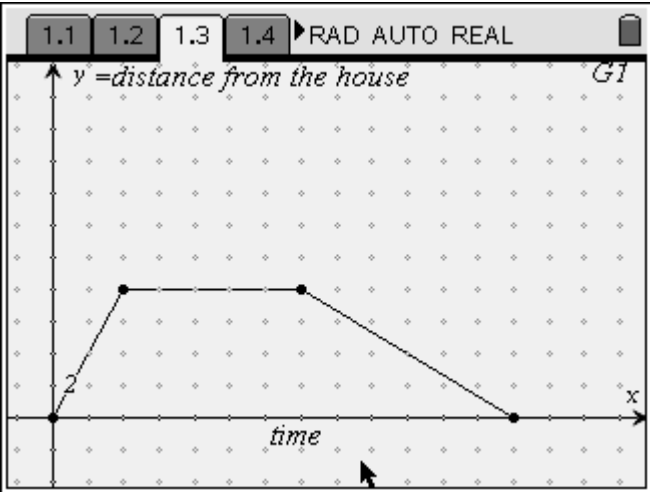
- What does the horizontal segment in the first graph mean?
- What does the change in the slope of the different segments of the first graph mean?
- What differences and similarities are there between the first and second graph? Between the second and third graph?

For the second part, the instructor should discuss the reasonability of the units of measurement and the scale that were used.

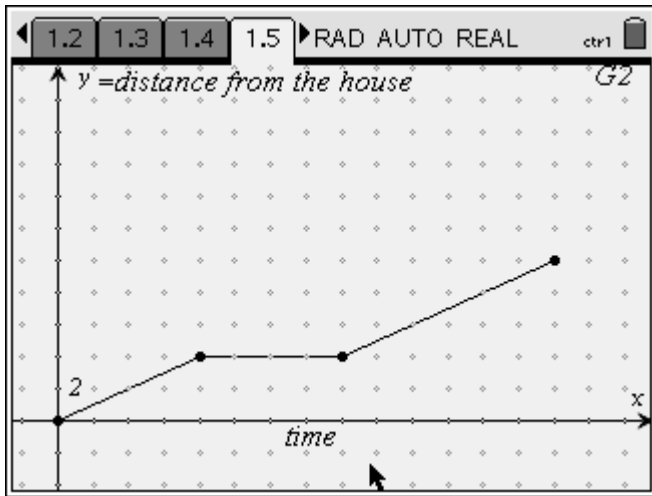
The second problem has three parts. In the first part, students should analyze whether the graph corresponds to the situation. After the students have completed the assignment, the instructor should lead a group discussion in which the students explain, for each graph, why they think it is a good representation of the situation or not. Then, the students should proceed to vote for the graph that best represents the situation. This permits the students to explore the situation from different points of view. Once the students have selected the graph that best models the situation, they should make a table of values, taking special care of the values of the function at integer values. Once the controversial points have been settled, the students should proceed to use the ceiling function to model the situation.

### Problem #1

1. Mrs. Smith lives in Brooklyn, New York. She goes out early each day in the morning. The graphs represent Mrs. Smith's displacement around the city throughout different days. The horizontal axis represents the time since she left the house. The vertical axis represents the distance between Mrs. Smith and her house. Write a story for each graph.

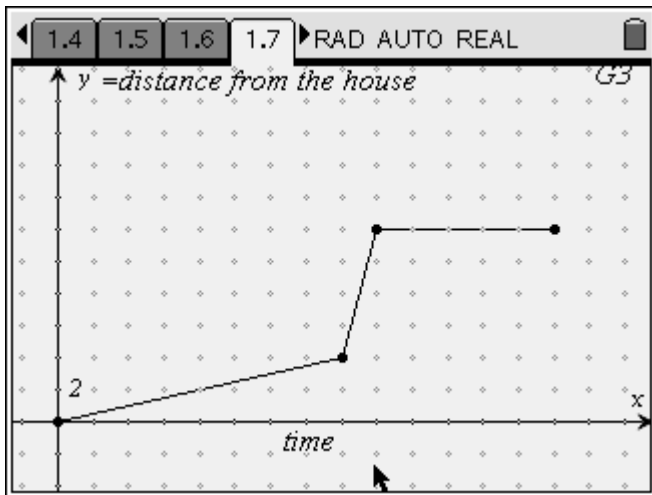
<p><b>G1</b></p> 	<p><b>STORY 1</b></p>
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G2



STORY 2

G3



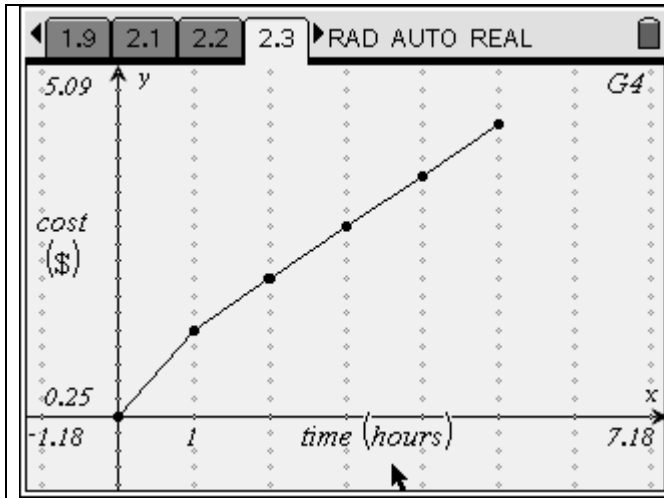
STORY 3

2. For each graph and its story, indicate the units of measurement of the variables, as well as the domain and range of the function.

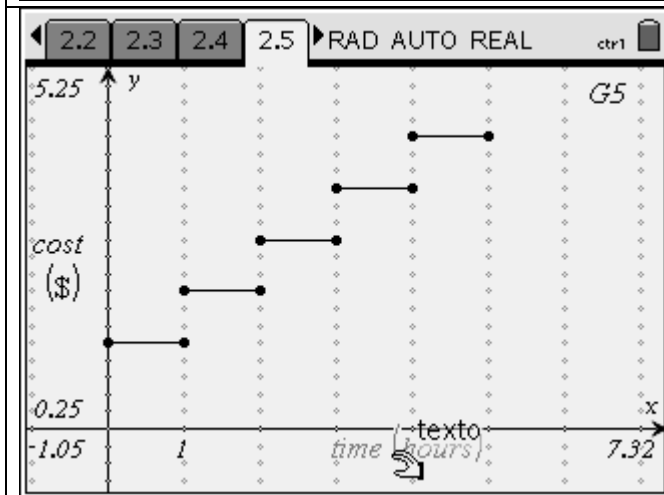
	Units of measurement	Scale	Domain	Range
Story #1				
Story #2				
Story #3				

**Problem #2**

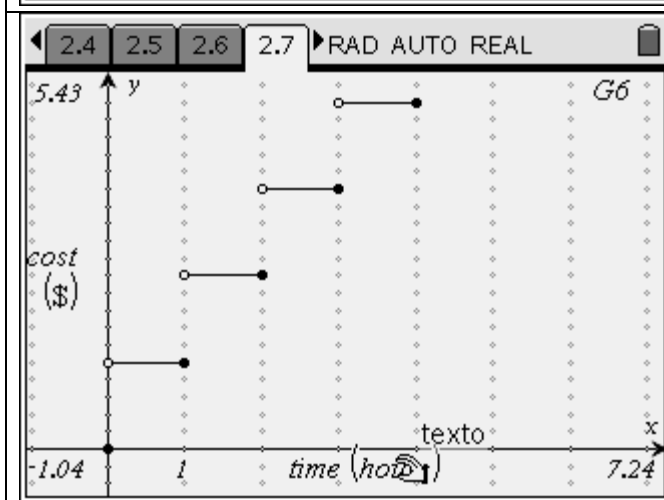
1. A parking lot in Fort Lauderdale charges \$1.25 for the first hour and \$0.75 for each additional hour or fraction of an hour. Determine whether the following graphs appropriately illustrate the situation.



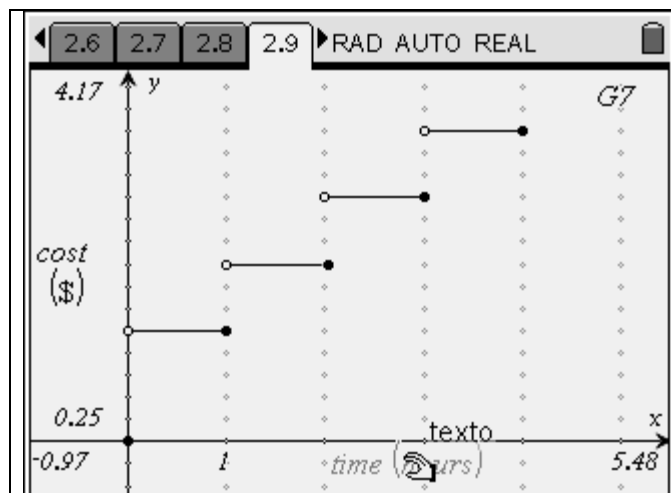
Yes/No. Why?



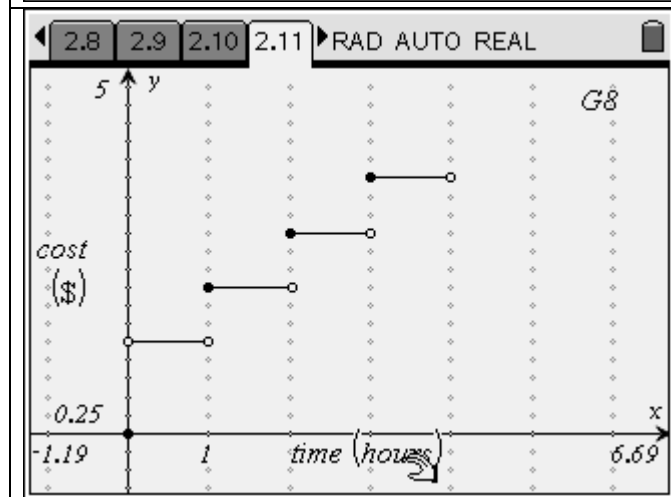
Yes/No. Why?



Yes/No. Why?



Yes/No. Why?



Yes/No. Why?

2. Fill out the following table.

<i>Time (in hours)</i>	<i>Cost (in dollars)</i>
0	
0.5	
1	
1.5	
2	
2.5	
3	

- Use the ceiling function to give an algebraic representation of the function.

### POSSIBLE EXTENSIONS

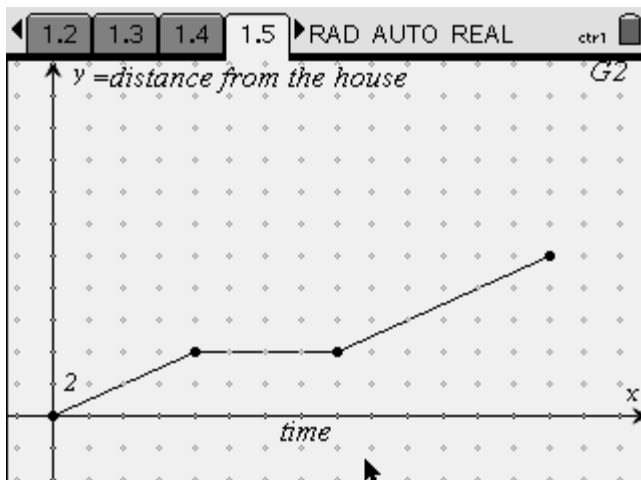
Problem #1 can be extended so that students are asked to find the tabular and algebraic representation for each of the stories they came up with.

Problem #2 can be extended so that students are asked to come up with everyday situations that are similar to it.

### SCREENS

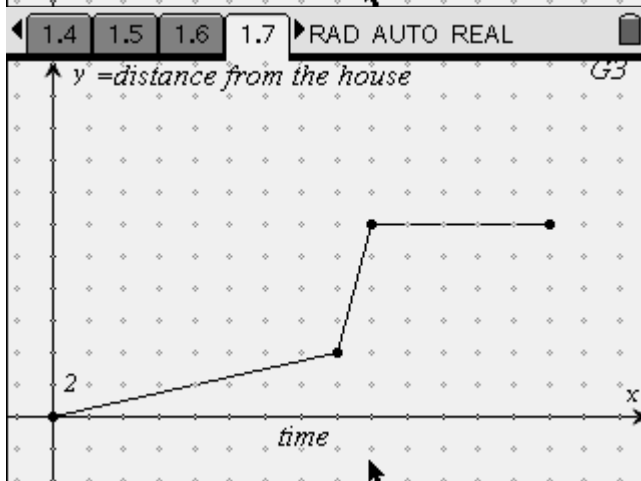
The screenshot shows a software interface with four panels, each with a tabbed menu (1.1, 1.2, 1.3, 1.4) and a 'RAD AUTO REAL' button.

- Top-Left Panel:** Title "Modeling Situations using Piecewise Functions" and author information: "Omar Hernández Rodríguez, MS, EdD, University of Puerto Rico".
- Top-Right Panel:** Text prompt: "Mrs. Smith lives in Brooklyn, New York. She goes out early every the morning. The graphs represent Mrs. Smith's displacement around the city throughout different days. The horizontal axis represents the time since she goes out of the house. The vertical axis represents the distance between Mrs. Smith and her house. Write a story for each graph."
- Bottom-Left Panel:** A coordinate plane with a grid. The vertical axis is labeled  $y = \text{distance from the house}$  and the horizontal axis is labeled  $x = \text{time}$ . A piecewise linear graph  $G1$  is plotted with three segments: a line from the origin to a point, a horizontal line segment, and a line segment ending on the x-axis. A mouse cursor is pointing at the x-axis.
- Bottom-Right Panel:** A text input area with the prompt: "Write a story for graph G1."



1.3 1.4 1.5 1.6 RAD AUTO REAL

Write a story for graph G2.



1.5 1.6 1.7 1.8 RAD AUTO REAL

Write a story for graph G3.

1.6 1.7 1.8 1.9 RAD AUTO REAL

For each graph and its story, indicate the units of measurement of the variables, as well as the domain and range of the function.

1.7 1.8 1.9 2.1 RAD AUTO REAL

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**Piecewise Functions**

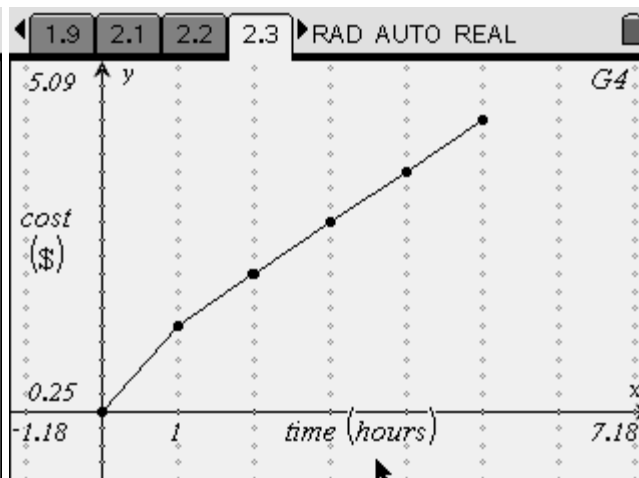
**Graph of a situation**

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1.8 1.9 2.1 2.2 RAD AUTO REAL

A parking lot in Fort Lauderdale charges \$1.25 for the first hour and \$0.75 for each additional hour or fraction of an hour.

Determine whether the following graphs appropriately illustrate the situation.

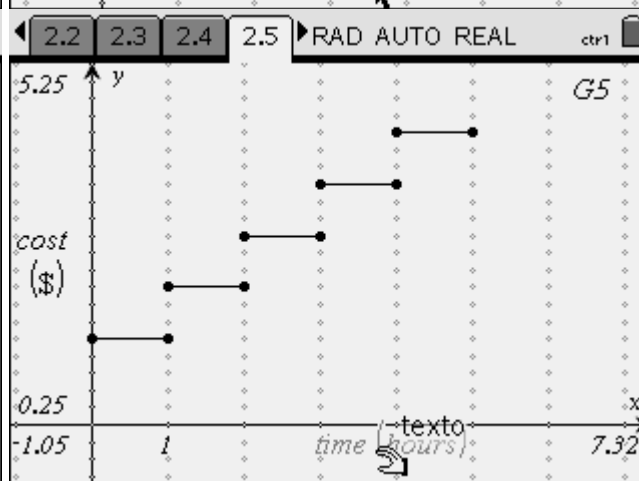


2.1 2.2 2.3 2.4 RAD AUTO REAL

**Pregunta**

Does the graph G4 correctly model the situation? Why or why not?

**Respuesta**

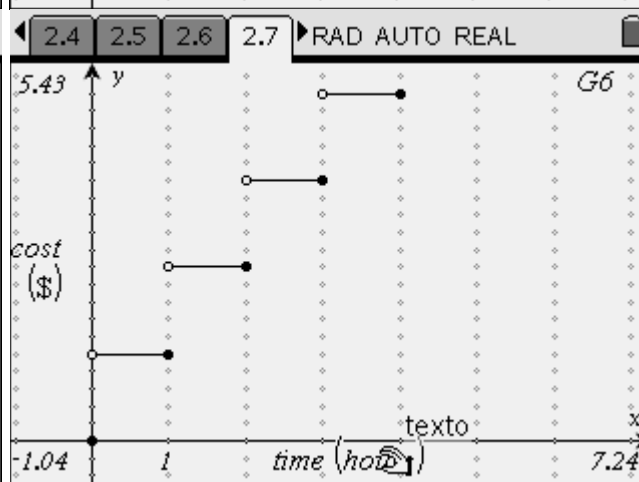


2.3 2.4 2.5 2.6 RAD AUTO REAL

**Pregunta**

Does the graph G5 correctly model the situation? Why or why not?


**Respuesta**

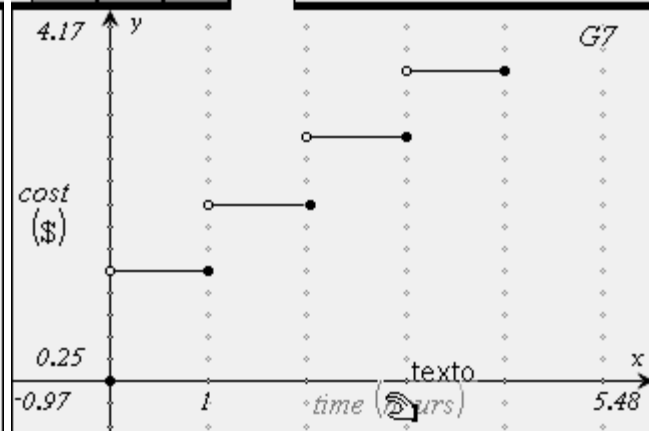




**Pregunta**

Does the graph G6 correctly model the situation? Why or why not?

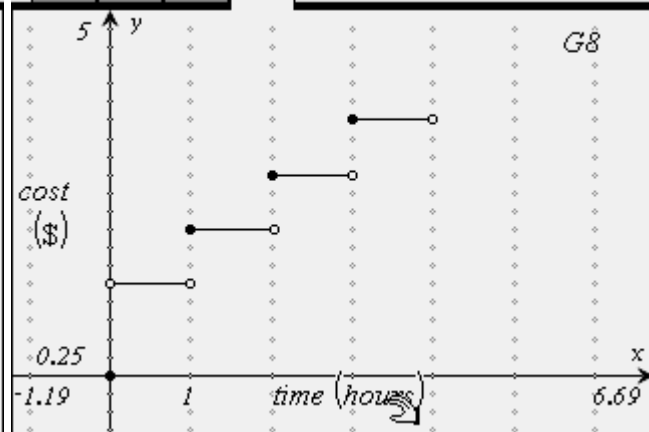
**Respuesta** 



**Pregunta**

Does the graph G7 correctly model the situation? Why or why not?


**Respuesta** 



2.9 2.10 2.11 2.12 RAD AUTO REAL

**Pregunta**

Does the graph G8 correctly model the situation? Why or why not?

**Respuesta** 

2.10 2.11 2.12 2.13 RAD AUTO REAL

On the next page, complete the table of values.

2.11 2.12 2.13 2.14 RAD AUTO REAL

	A time	B cost	C	D
1	0			
2	0.5			
3	1			
4	1.5			
5	2			
A7	0			

2.12 2.13 2.14 2.15 RAD AUTO REAL

Write the algebraic representation of the situation.