



#### Math Objectives

- Students will interpret a confidence level as the average success rate of the process used to produce an interval intended to contain the true mean of the population.
- Students will recognize that as the confidence level increases, on average, the confidence interval increases in width.

#### Vocabulary

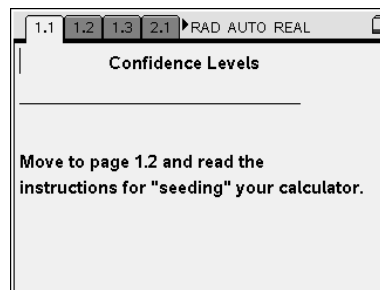
- confidence interval
- confidence level
- population parameter

#### About the Lesson

- This lesson involves interpreting the meaning of a confidence level that is associated with a confidence interval. It assumes that students know what confidence intervals are and have experience in calculating them.
- As a result, students will:
  - Generate confidence intervals for the mean of a population using random samples of size  $n$  and investigate how often these intervals actually contain the true population mean.
  - Use the actual population mean to check their estimates and to develop an understanding of the meaning of confidence level.
  - Experiment with different confidence levels and finally investigate the relationship between confidence level and interval size.

#### TI-Nspire™ Navigator™ System

- Send the .tns file to students.
- Use *Screen Capture* to display multiple sets of confidence intervals.
- Use *Quick Poll* to check student understanding and prior knowledge.



#### TI-Nspire™ Technology Skills:

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

#### Tech Tips:

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- In Graphs & Geometry, you can hide the function entry line by pressing **(ctrl) G**.

#### Lesson Materials:

##### *Student Activity*

- Confidence Levels.pdf
- Confidence Levels.doc

##### *TI-Nspire document*

- Confidence Levels.tns

Visit [www.mathnspired.com](http://www.mathnspired.com) for lesson updates and tech tip videos.



### Discussion Points and Possible Answers

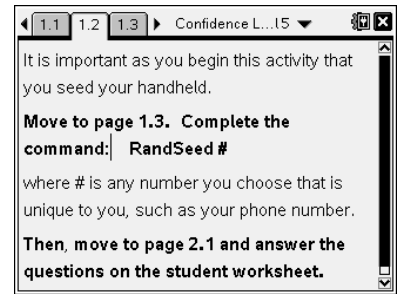
**Teacher Tip:** The variability in the lengths and centers of the intervals makes it reasonable to conjecture that not all of the intervals will contain the population mean; visualizing 50 possible intervals stacked vertically makes it possible to quantify that relative frequency.

**Student Prerequisites:**

- Calculation of a confidence interval
- Sampling distribution of a sample statistic

#### Move to page 1.2.

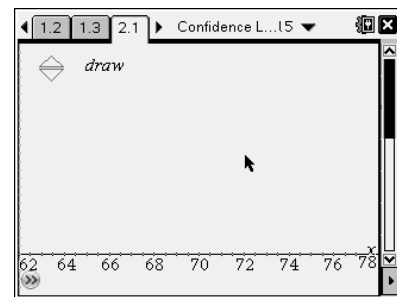
A confidence interval is an interval of plausible values for an unknown population parameter, (i.e., a 95% confidence level indicates that for every 100 times the process is repeated, approximately 95 of the intervals produced would contain the actual population mean). This lesson assumes that students have already learned to calculate a confidence interval.



**Teacher Tip:** Page 1.2 gives instructions about how to seed the random number generator of the TI-Nspire. Page 1.3 is a calculator page for the seeding process. Ensuring that students carry out this step will prevent them from generating data that are identical to others in the class. (Syntax: `RandSeed #`, where # should be a number unique to each student.)

#### Move to page 2.1.

1. Use the up arrow (▲) to draw a sample. The segment represents the calculated confidence interval based on that sample. Note that the intervals displayed are t-intervals for a population mean, calculated according to the formula  $\bar{x} \pm t^* \left( \frac{s}{\sqrt{10}} \right)$  using the same value of  $t^*$ .
  - a. What do the points on the horizontal axis represent?



**Sample Answers:** The points represent the elements from a sample of size 10 from the population under consideration.



**Teacher Tip:** The points might not be distinct due to the scaling. Several of them might lie very close to each other and appear as one point, so the total number of elements in the sample might not appear to be 10.

- b. Use the confidence interval to estimate the population mean. Are you confident that your estimate really captures the true mean of the population? Explain why or why not.

**Sample Answers:** The population mean could be anywhere in the confidence interval, for example from about 68 to 71. Students might give the center of the interval as their answer (i.e., 69.5). Students might say they are confident because they believe the confidence interval does capture the population mean, or they might be unsure that their sample is representative of the population.

**Teacher Tip:** Students' estimates should reflect the span of the interval—not just the center. They should remember this from their work finding confidence intervals, but it's possible they did not really understand why an interval estimate is used rather than a point estimate. If students do give only the center of the interval, pursue the "are you confident" question. Comparing the confidence of a student who guessed only one point to that of someone who estimated using an interval should help them see the value of interval estimates. The purpose of the question is to get students to start thinking about how confident they are that the process will produce an interval that contains the population mean.

**TI-Nspire Navigator Opportunity: *Quick Poll***  
**See Note 1 at the end of this lesson.**

- c. Although in real situations one sample is usually all you have, suppose you could draw more than one sample from the same population to find more than one confidence interval for the mean of that population. Draw another sample, and estimate the population mean from this new confidence interval. Repeat this process four more times, and write down your interval estimate for the population mean each time.

**Sample Answers:** Using the length of the interval, they might respond: 69 to 71.5; 70 to 73.2; 68.5 to 71.5; 69.5 to 72; 69 to 72.8



**Teacher Tip:** The intervals will be of unequal length because the sampling is done from a t-distribution and because the sample error has replaced the population standard deviation. This replacement will vary from sample to sample, which affects the amount that is added or subtracted from the sample mean. Students should not focus on this but rather on the questions asked about how to interpret a confidence level.

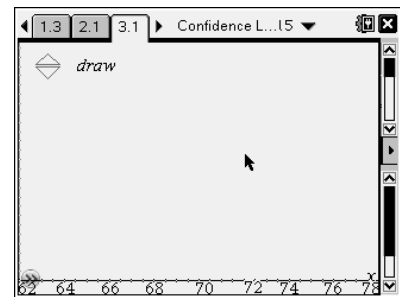
**TI-Nspire Navigator Opportunity: *Screen Capture***

**See Note 2 at the end of this lesson.**

**Move to page 3.1.**

2. a. Click ▲ to generate five confidence intervals. Using those intervals, estimate what you think the population mean is likely to be. Explain your reasoning.

**Sample Answers:** Students might give an interval estimate such as (69.5 to 70.5 or 69.5 to 72) for an interval that is the overlap among the five sample confidence intervals; they might give a "center" (70 or 70.75) for the intervals that seems to be in common with all of the sample confidence intervals; or they might give the overlapping whole numbers in the intervals (70; 70, 71 or 72). Their reasoning might go: "the number 71 was in all five of the confidence intervals so it seems likely 71 is the actual population mean." Some might recognize that it is difficult to give one number because the lengths of the intervals keep changing (one was four units long from 69 to 73 and another was three units long from 66 to 69) and offer a small interval that seems common to the five they have generated.



**Teacher Tip:** Students should share their solutions to 2a, so they can see the variability in their results and how their specific set of five confidence intervals would affect their estimates.

**TI-Nspire Navigator Opportunity: *Screen Capture***

**See Note 3 at the end of this lesson.**



- b. Click ▲ to generate five more confidence intervals. Make another estimate of the population mean using all ten of the intervals you generated. Explain your reasoning.

**Sample Answers:** Students might again answer using the same kind of reasoning as they did in 2a. They might begin to notice that 70 seems to be common to most of the intervals.

- c. Do you think all of the intervals you generated contain the population mean? Why or why not?

**Sample Answers:** If their intervals are fairly alike, students might say yes. If one or more of the intervals is quite different from the rest (i.e., one of the ten intervals only goes from 67 to 69 and the others all seem to contain 70), they might suggest that it looks like some intervals are so different they might not contain the population mean.

- d. Generate ten more confidence intervals and indicate the proportion of the 20 intervals you think fail to contain the population mean. Explain your thinking.

**Sample Answers:** Students should begin to suspect that one or two of the 20 intervals do not contain the population mean because they are so far to the right or to the left of the other intervals. They might answer something like the proportion  $1/20$  (5%) or  $2/20$  (10%).

**Teacher Tip:** It is easier to count the intervals that seem unlikely to contain the population mean because there are fewer of them. But confidence levels are given in terms of those that are likely to contain the population mean; be sure students see the switch as they move from 2d and 3 to problem 4a.

**TI-Nspire Navigator Opportunity: Screen Capture**  
**See Note 4 at the end of this lesson.**

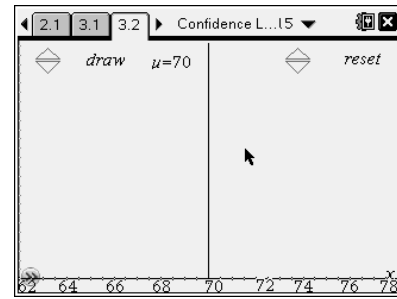
3. Generate confidence intervals until you have 50. Identify the intervals that seem to be quite different from the others and so might not contain the population mean. How many are there, and why do these seem different?

**Sample Answers:** Students might identify a specific number such as three that seem very different and so unlikely to contain the true population mean; for example, 3 of the 50 of the intervals seem unlikely to actually contain the population mean because they are way to the right or the left of the other intervals.



Move to page 3.2.

4. Page 3.2 shows the set of 50 confidence intervals you just generated and a vertical line that represents the true population mean, 70.
  - a. What proportion of the 50 confidence intervals you had generated in question 3 actually captured the true mean?



**Sample Answers:** If a student found 47 of the intervals contained 70, the proportion would be 47 out of 50 or 94%.

- b. Make a conjecture about the number of intervals in any set of 50 intervals generated from the same population with the same sample size that you think might actually contain the population mean.

**Sample Answers:** Students might respond in general terms (lots) or be specific and say all but four or five, so it would be 46/50 (92%) or 45/50 (90%).

- c. Click the reset up arrow (▲) to clear the 50 confidence intervals you generated in 3. Then generate 50 new confidence intervals. How well do your results support your conjecture?

**Sample Answers:** Students might respond, for example, that four intervals or 8% of them did not contain 70, so 92% included 70. They might indicate that they were unsure how to count the two that seemed to touch 70, raising the question of what to do about the end points of an interval.

**Teacher Tip:** Given the rough scale, it seems difficult to see exactly whether some intervals actually contain 70; remind students that each segment is just an approximation, and if it looks as if the "square" at the end of an interval is mostly next to 70, they might consider it as not including 70. The primary goal here is to show that for some relatively predictable proportion of times, the process of creating a confidence interval will produce intervals that do not contain the actual population mean. Do not get hung up on exact counts.

**TI-Nspire Navigator Opportunity: Screen Capture**  
See Note 5 at the end of this lesson.



**Teacher Tip:** You might want students to undo the confidence intervals they have on the screen and generate 50 more intervals. Ask them whether these intervals support their conjecture. Why or why not? If necessary, repeat this until they seem to have enough evidence to help them think about their conjecture(s).

5. Amie announced that in every 50 confidence intervals generated for this population, about 90% of them contain 70, the actual population mean.
  - a. Do you agree or disagree with Amie? Explain why or why not.

**Sample Answers:** This should be a reasonable estimate because in question 4, about 45 out of 50 or 90% contained the population mean.

**Teacher Tip:** This question is designed to move students from counting the number and reporting either that number (46) or a fraction (46/50) to using a percent.

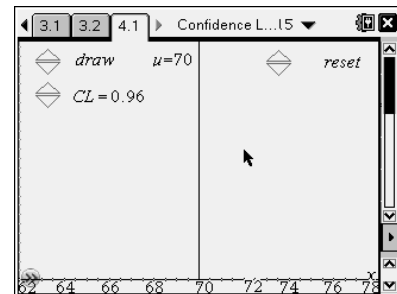
- b. The confidence *level* used to generate the confidence intervals on page 3.2 was 90%. Based on your work in questions 4 and 5, does 90% seem like a reasonable level? Why or why not?

**Sample Answers:** Students should recognize that 45 or 46 out of 50 would be 90% or 92%, which supports the claim that the confidence level for the work was 90%.

**Move to page 4.1.**

The CL up and down arrows (▲) on page 4.1 control the confidence level. (Note that the sample size stays constant.)

6. Set the confidence level to 80%. Generate 50 samples. Describe how well your confidence intervals support a confidence level of 80%.



**Sample Answers:** Student data should show that about 40 of the intervals contain the population mean, but some students might have fewer or more captures.



7. a. If the confidence level is 95%, about what proportion of 50 confidence intervals generated from the same population for a given sample size do you think will contain the true population mean?

**Sample Answers:** Students should begin to see that about 95% of the 50, i.e. 47 or 48, will contain the true mean. Some might still be unsure.

- b. Use the reset ▲ to undo the confidence intervals you generated in question 6. Set the confidence level to 95. Generate 50 confidence intervals. How do the results match your estimate in question 7a?

**Sample Answers:** Most students will find from 46 to 49 of the intervals actually contain the population mean of 70.

**Teacher Tip:** Counting those that "touch" the vertical line might come up again. The focus should continue to be on the notion that for a given confidence level, you can find a regularity in how many confidence intervals will include the actual population mean. If students need a "rule," indicate that if it looks clearly as though it touches, count it in the interval.

- c. Repeat questions 7a and b for another confidence level of your choice.

**Sample Answers:** Responses will vary depending on the confidence level they choose.

8. Click ▲ to return to a screen with no confidence intervals.
- a. Set the confidence level to 70%, and generate 50 confidence intervals. Then use the CL arrow to increase the confidence level to 99%. What effect does changing the confidence level from a low confidence level to a high confidence level seem to have on a confidence interval? Explain why your answer seems reasonable.

**Sample Answers:** As the confidence level increases, the width of the confidence intervals increases, which means that more of the intervals are likely to contain the true population mean. For example, in one set of 50 samples at the 70% confidence level, 38 of the intervals or 76% of them contained the true mean. However, when the confidence level was 95%, 48 of the intervals or 96% contained the true mean. When the confidence level is high, the intervals are longer, and thus it is more likely that more of them will contain the actual population mean.





**Teacher Tip:** In general, as the confidence level increases, the width of the corresponding confidence interval increases, all other factors being unchanged. One factor that also affects the width of confidence intervals is the standard deviation of the sampling distribution of the underlying statistic. In estimating the population mean when the population standard deviation is unknown, interval calculations are based on the standard deviation of the sample. Thus two different samples of equal sample size (but different sample standard deviations) from a single population can give rise to two intervals at the same confidence level that have very different lengths.

- b. Think about what a confidence level means. How does this help justify your conclusion in 8a?

**Sample Answers:** In order to be more confident that the interval contains the population mean, a longer interval will give more of a chance that it will actually contain the mean.

**Teacher Tip:** Students might compare their graphs and answers with a classmate. Understanding this idea is important to understanding the whole notion of confidence level as a probability that the confidence interval produced by the process will contain the true mean.

9. Consider the following: a 95% confidence interval for the mean number of hours per weekend day that teenagers play video games is between 5.35 hours and 7.25 hours. Comment on each statement.
- a. 95% of the time, the number of hours teenagers play video games on a weekend day is between 5.35 hours and 7.25 hours.

**Sample Answers:** This statement is incorrect; 95% confidence intervals do not contain 95% of all of the measurements.

- b. We are 95% confident that the mean number of hours that teenagers play video games on a weekend day is between 5.35 hours and 7.25 hours.

**Sample Answers:** This statement is correct.

- c. We are 95% confident that the mean number of hours that teenagers play video games on a weekend day is  $6.3 \pm 0.95$ .

**Sample Answers:** This statement is correct, just rephrasing the interval using the center and



distance to the ends of the interval.

- d. 95% of the confidence intervals we construct for the mean number of hours teenagers play video games on a weekend day will give us the interval (5.35, 7.25).

**Sample Answers:** This statement is incorrect; confidence intervals typically give different results for each sample that is taken.

- e. There is a 95% probability the mean number of hours that teenagers play video games on a weekend day is between 5.35 and 7.25 hours.

**Sample Answers:** This is wrong; probability does not apply to an event that either has occurred or has not occurred. There is no chance involved in the mean number of hours that the population of teenagers play video games on a weekend day; the chance is in the process of estimating that quantity from a sample.

10. Describe the difference between a confidence level and a confidence interval.

**Sample Answers:** A confidence level is a statement about the degree of certainty in the process of finding a confidence interval for an unknown parameter for a population. A confidence interval is the result of that process, is calculated based on a random sample from that population, and provides an interval of plausible estimates for the value of that parameter.

**TI-Nspire Navigator Opportunity: Screen Capture**  
**See Note 6 at the end of this lesson.**

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### Wrap Up

Upon completion of the discussion, the teacher should ensure that students are able to understand:

- That a confidence level applies to the *process* of generating a confidence interval.
- The process of generating a confidence interval for a specific confidence level will produce intervals that contain the true population parameter with relative frequency equal to that level of confidence.
- Changing the confidence level changes the width of the confidence interval; as the confidence level increases, the width of the confidence interval increases.



### TI-Nspire Navigator

#### Note 1

##### Question 1 part b, *Quick Poll*

Use *Quick Poll* to collect student responses for an estimate of the mean in question 1b. This should be a review of what they learned when they calculated confidence intervals and will allow you to see whether they have grasped the fact that an interval rather than point estimate is the goal.

#### Note 2

##### Question 1c, *Screen Capture*

Use *Screen Capture* to look at the different samples students are generating and discuss what they think the value of the true population mean is.

#### Note 3

##### Question 2a, *Screen Capture*

*Screen Capture* will allow you to display the results to the whole class and to ask them what they might estimate for the population mean, given that some of intervals are “interesting” (i.e., they lie primarily to one side or the other of 70, or some are very short intervals and some are very long intervals).

#### Note 4

##### Question 2d, *Screen Capture*

*Screen Capture* of each students' page 3.1 with 20 confidence intervals will show not only the variability possible but also the fact that in each case one or two of the intervals seem much different than the others. You might have them compare the proportions for the unlikely intervals and note that it seems to be 5% or 10% for each student's set of confidence intervals.

#### Note 5

##### Question 4c, *Screen Capture*

*Screen Capture* will show how different sets of 50 confidence intervals seem to have a relatively constant proportion of those intervals that do contain the population mean of 70.

#### Note 6

##### Question 9, *Quick Poll*

You might use Question 9 as a true/false *Quick Poll*, and then discuss student responses, focusing on "Why might some of you think d is false? Does anyone want to explain why they thought it would be true?"



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