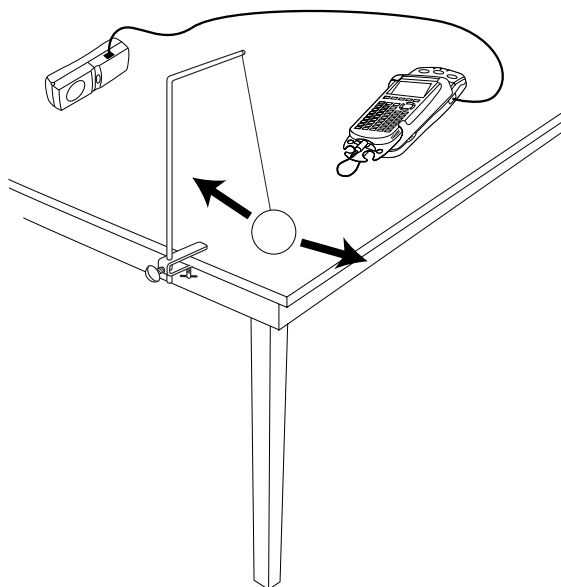


# Tic, Toc: Pendulum Motion

Pendulum motion has long fascinated people. Galileo studied pendulum motion by watching a swinging chandelier and timing it with his pulse. In 1851 Jean Foucault demonstrated that the earth rotates by using a long pendulum which swung in the same plane while the earth rotated beneath it.

As long as the swing is not too wide, the pendulum approximates simple harmonic motion and produces a sinusoidal pattern. In this activity, you will use a Motion Detector to plot the position versus time graph for a simple pendulum. You will time the motion to calculate the period, and use a ruler to measure the maximum displacement. You will then use the data to model the motion with the cosine function  $y = A \cos(B(x - C)) + D$  to mimic the position versus time graph.



## OBJECTIVES

- Record the horizontal position versus time for a swinging pendulum.
- Determine the period of the pendulum motion.
- Model the position data using a cosine function.

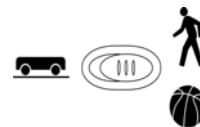
## MATERIALS

TI-83 Plus or TI-84 Plus graphing calculator  
EasyData application  
CBR 2 or Go!Motion and direct calculator cable  
or Motion Detector and data-collection interface

pendulum bob  
string  
meter stick  
stopwatch

## PROCEDURE

1. Set up the Motion Detector and calculator.
  - a. Open the pivoting head of the Motion Detector. If your Motion Detector has a sensitivity switch, set it to Normal as shown.
  - b. Turn on the calculator and connect it to the Motion Detector. (This may require the use of a data-collection interface.)
2. Set up EasyData for data collection.
  - a. Start the EasyData application, if it is not already running.
  - b. Select **(File)** from the Main screen, and then select **New** to reset the application.
3. Hang the pendulum bob from about 80 cm of string from a rigid support. Arrange the support so the bottom of the pendulum bob clears the table by several centimeters. Position the Motion Detector about 1 m away from the bob, pointing at the bob. Elevate the detector slightly so that it does not respond to the table.
4. Measure the distance from the bob to the Motion Detector; this distance must be at least 75 cm. Record this distance  $D$ , in meters, in the Data Table on the *Data Collection and Analysis* sheet.
5. Place the meter stick under the bob, along the line between the detector and the bob. Arrange the stick so that the zero point is under the bob when it is hanging still. Determine how far you will pull back the bob before releasing it. This distance should be less than 20 cm. Record this value, in meters, as the amplitude  $A$  in the Data Table.
6. Use the stopwatch to measure the period of the pendulum. The period is the time taken by the pendulum to complete one back and forth cycle. Use the amplitude of motion you determined in the previous step. Measure the time for *ten* complete cycles, and record this time in the Data Table.



Take care to count carefully: One way to do this is to start the stopwatch when the bob is farthest from the Motion Detector, and count one cycle when it returns to that spot. Keep the stopwatch running until ten cycles are completed.

7. Practice swinging the ball by pulling it back the distance you recorded above, and then releasing it so that the ball swings in a line directly away from the Motion Detector.
8. With the ball swinging properly, select **(Start)** to begin data collection. Data collection will run for five seconds.
9. When data collection is complete, a graph of distance versus time will be displayed. Examine the distance versus time graph, which should be sinusoidal. Check with your teacher if you are not sure whether you need to repeat the data collection. To repeat data collection, select **(Main)** and repeat Step 8.


Once you have a good run, exit EasyData by selecting **(Main)**, selecting **(Quit)**, and then selecting **(OK)**.

## ANALYSIS

1. Change the angle calculation mode on the calculator to radians, or confirm that it is set to radians.
2. Compare the Motion Detector data to the cosine model of  $y = A \cos(B(x - C)) + D$ . The setup measurements will allow you to determine the parameters  $A$ ,  $B$  and  $D$ . To determine  $C$ , redisplay the graph of the Motion Detector data.
  - a. Press  $\text{2nd}$  [STAT PLOT].
  - b. Press  $\text{ENTER}$  to select Plot1 and press  $\text{ENTER}$  again to select On.
  - c. Press  $\text{ZOOM}$ .
  - d. Press  $\text{V}$  until ZoomStat is highlighted; press  $\text{ENTER}$  to display a graph with the  $x$  and  $y$  ranges set to fill the screen with data.
  - e. Press  $\text{TRACE}$  to determine the coordinates of a point on the graph using the cursor keys.
3. Since the cosine function starts at a maximum value when its argument is zero, you can use the location of a maximum to determine the value of  $C$ , which represents the horizontal shift of the data. Trace across the data to any maximum and read the time ( $x$ ) value. Record this value as  $C$  in the Data Table.
4. Enter the model equation in the equation editor of the calculator.
  - a. Press  $\text{Y=}$ .
  - b. Press  $\text{CLEAR}$  to remove any existing equation.
  - c. Enter the expression  $A \cdot \cos(B \cdot (X - C)) + D$  in the  $Y_1$  field.
  - d. To make the model equation plot with a bold line, press  $\text{2nd}$  until the diagonal line to the left of  $Y_1$  is highlighted. Press  $\text{ENTER}$  until a bold diagonal line is shown in order to display your model with a thick line.
  - e. Press  $\text{2nd}$  [QUIT] to return to the home screen.
5. The parameter  $A$  represents the amplitude of the motion of the bob. This distance is measured horizontally from one extreme of motion to the center position. By always starting the pendulum bob from the same position, you have the same amplitude each time. Enter the value for the parameter  $A$  in the calculator's memory. To do this, enter the value for the parameter  $A$ . Press  $\text{STO}$   $A$   $\text{ENTER}$  to store the value in the variable  $A$ .
6. You have measured the time for ten complete cycles of the pendulum. Use this value to find the period of the motion, which is the time for one complete cycle. Enter this value in the Data Table on the *Data Collection and Analysis* sheet.
7. The sinusoidal model has a parameter  $B$  that represents the number of cycles the function makes during the natural period of the sine function. Find  $B$  by taking  $2\pi$  (the natural period of the cosine function) divided by the period of the pendulum (the time for one cycle). Store this value in the calculator after performing the division by pressing  $\text{STO}$   $B$   $\text{ENTER}$  to store the value in the variable  $B$ . Record the value in the Data Table.
8. Enter the value for  $C$ , the time offset, from your Data Table. Press  $\text{STO}$   $C$   $\text{ENTER}$  to store the value in variable  $C$ .
9. The parameter  $D$  represents the offset of the cosine function from the distance axis. The distance from the Motion Detector to the pendulum bob at rest is the value of  $D$ . Enter the value from the Data Table and store it by pressing  $\text{STO}$   $D$   $\text{ENTER}$  to store the value in the variable  $D$ .

## Activity 25

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10. You have now entered the model equation and values for the four parameters  $A$ ,  $B$ ,  $C$ , and  $D$ . Press  to display a graph of the Motion Detector data and the model equation.

⇒ Answer Questions 1–4 on the *Data Collection and Analysis* sheet.

### CALCULUS EXTENSION

Once you have an equation for the position versus time graph of the pendulum motion, take the derivative of the equation. This represents the velocity of the pendulum at any time  $t$ . How does the velocity versus time graph compare with the position versus time graph? When during the pendulum motion is the velocity zero? When is the velocity a maximum?

The derivative of velocity is *acceleration*. Take the second derivative of the position equation. Describe the position and velocity when the acceleration is a maximum. Do the same when the acceleration is zero.

Give a general description of the pendulum's position, velocity, and acceleration when the bob is passing through the at-rest position and when it is farthest from the detector.

**DATA COLLECTION AND ANALYSIS**

Name \_\_\_\_\_

Date \_\_\_\_\_

**DATA TABLE**

<b><i>A (m)</i></b>	
<b><i>B</i></b>	
<b><i>C (s)</i></b>	
<b><i>D (m)</i></b>	
<b><i>Time for ten cycles (s)</i></b>	
<b><i>Period (s)</i></b>	

**QUESTIONS**

1. How well does the model equation fit the data? If the fit is acceptable, write the model equation below, and suggest explanations for any discrepancies. If the fit of the model is not acceptable, deduce which of the parameters is producing the problem. Make changes as necessary to the parameters, and discuss why the changes were necessary. Write out the equation that produced a good fit.

### Activity 25

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- How would the parameters  $A$ ,  $B$ ,  $C$ , and  $D$  change if you were to use the sine function  $y = A \sin(B(x-C))+D$  instead of the cosine function? Predict the values below and explain your reasoning for each.
  
  
  
  
  
  
  
  
  
  
- Test your predictions by storing any changed values in the four parameters using the same method you used above. Also using the same method as above, change the model equation to a sine function. Redisplay the graph to compare the data and sine model. How well does the sine model fit the data? Explain any discrepancies.
  
  
  
  
  
  
  
  
  
  
- Give a physical interpretation of each of the parameters  $A$ ,  $B$ ,  $C$ , and  $D$  from the model  $y = A \cos(B(x-C))+D$  in terms of the pendulum.