## Math Objectives

- Students will recognize that the shape of the distribution for a single sample varies from sample to sample, even when all the samples come from the same population.
- Students will recognize that the variability in shape from sample to sample decreases as sample size increases.
- Students will recognize that larger samples from an approximately normal population will more often "look" normal than will smaller samples from the same population.
- Use appropriate tools strategically (CCSS Mathematical Practices).

## Vocabulary

- dot plot
- histogram
- normal distribution
- normal probability plot (NPP)
- population
- sample

## About the Lesson

- This lesson involves examining multiple samples taken from a single approximately normal population.
- As a result, students will:
  - Compare dot plots, histograms, and normal probability plots to determine which representation best provides information about the normality of a distribution.
  - Select multiple samples of the same size to discover that not all samples from a normal population actually look normal.
  - Change sample size to compare the shapes of larger samples to those of smaller samples.

## Prerequisites

- familiarity with normal distributions
- familiarity with normal probability plots and their use
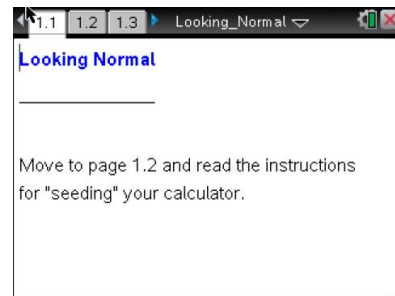


**TI-Nspire™ Technology Skills:**

- Download a TI-Nspire document
- Open a document
- Move between pages
- Grab and drag a point

**Tech Tips:**

- Make sure the font size on your TI-Nspire handhelds is set to Medium.
- You can hide the function entry line by pressing [ctrl] [G].

**Lesson Files:**
*Student Activity*
Looking_Normal_Student.pdf
Looking_Normal_Student.doc

*TI-Nspire document*
Looking_Normal.tns

Visit www.mathnspired.com for lesson updates and tech tip videos.

## Related Activities

- Histograms
- Normal Probability Plot

## TI-Nspire™ Navigator™ System

- Transfer a File.
- Use Screen Capture to examine a variety of individual samples from a given population, both for a fixed sample size and to compare across sample sizes.

## Discussion Points and Possible Answers

**Teacher Tip:** Several hypothesis tests and confidence procedures for means of populations (t-procedures) assume that the population under question is normally distributed. It is usually impossible to check a real population, so it is necessary instead to examine the sample to see whether it provides information about whether the population itself might reasonably be assumed to be normally distributed.

**Tech Tip:** Page 1.2 gives instructions on how to seed the random number generator on the handheld. Page 1.3 is a Calculator page for the seeding process. Ensuring that students carry out this step will prevent students from generating identical data. (Syntax: RandSeed #, where # is a number unique to each student.)

**Teacher Tip:** Once students have seeded their random number generators, they do not have to do it again unless they have cleared all of the memory. But it is important that this be done if the memory has been cleared or with a new device. This prevents the "random" numbers from being the same as those on other similarly-cleared devices.

**Move to page 2.1.**

1. The graph on the top of Page 2.1 represents a population of head circumferences measured in centimeters that is approximately normally distributed. Suppose you were to select a random sample from this population. Describe how you would expect the points representing your sample values to be distributed. Explain briefly.
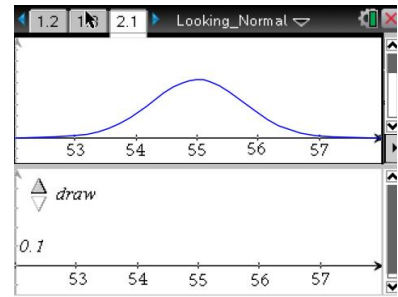
   **Sample Answers:** Since the population is normally distributed, I would expect more of the measurements to fall near the center than toward the ends, with rough symmetry around that center.

2. Click on the arrow on Page 2.1 to generate a random sample of size $n = 50$ from the population. Is your sample distributed approximately as you predicted?

   **Sample Answers:** Somewhat, but not as much as I expected. Many of the points are near the center, but they don't taper off toward the ends as much as I expected them to.

**Move to page 2.2.**

3. a. Page 2.2 displays a histogram of the same sample values you generated on Page 2.1. How well does it capture the shape you saw in the dot plot?
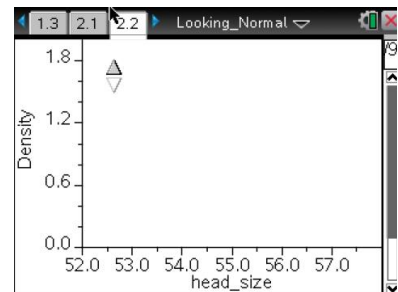
   **Sample Answers:** It does a good job. The shape seems to be about the same as I saw in the dot plot.

   b. Click the arrow at the top left of the page to overlay a normal density curve. Does the histogram seem to fit the normal curve reasonably well?
   **Sample Answers:** Yes, reasonably well, but the histogram is not as smooth and not quite as concentrated with a clear peak in the middle.

   > **Tech Tip:** On Page 2.3, click in the "white space" of the window to remove the equation of the displayed line.

**Move to page 2.3.**

4. The graph on Page 2.3 is a Normal Probability Plot, or NPP, of your sample data. Recall that linearity in an NPP indicates normality in the distribution of the data it represents. Based on that principle, would you say that your sample appears to be approximately normally distributed?
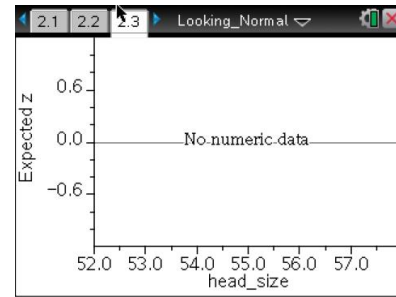
   **Sample Answers:** Yes. The NPP is almost perfectly linear, with just a little "wiggle" near the ends, so my data are very nearly normally distributed.

5. Which of your graphs from Pages 2.1, 2.2, and/or 2.3 makes it easiest to determine "how normal" your sample is? Explain briefly.

   **Sample Answers:** They all work pretty well. In each case, the shape is pretty clear, but the NPP seems simplest.

6. Compare your graphs in Pages 2.1, 2.2 and 2.3 to those of your classmates. Describe how corresponding graphs seem to differ from student to student, and how often someone's graph looks "really non-normal."

   **Sample Answers:** The graphs do not look drastically different. Some graphs (dot plots and histograms) seemed a little skewed one way or the other but not very badly. And sometimes there were "low spots" in the plot where it should have been taller. Overall, though, they all looked pretty much normal.

   > **Teacher Tip:** As an alternative to comparing graphs from everyone in the class for Question 6, especially if you do not have a TI-Navigator system, you can have each student click the arrow to draw multiple samples of size $n = 50$ and compare a number of graphs of their own.

   **TI-Nspire Navigator Opportunity:** *Screen Capture*
   **See Note 1 at the end of this lesson.**

**Move to page 3.1.**

7. a. The "sample size" arrow in Page 3.1 is set to *n* = 35. Click on the "draw" arrow to select a random sample of size 35 from the given population. Compare the shapes of the graphs on Pages 3.1, 3.2, and 3.3 to those you saw in your and your classmates' Problem 2.

   **Sample Answers:** The plots don't seem nearly as "normal" as before. The histogram and dot plot seem less mound-shaped and more jagged. Only the NPP still seems relatively like it was in Problem 2.
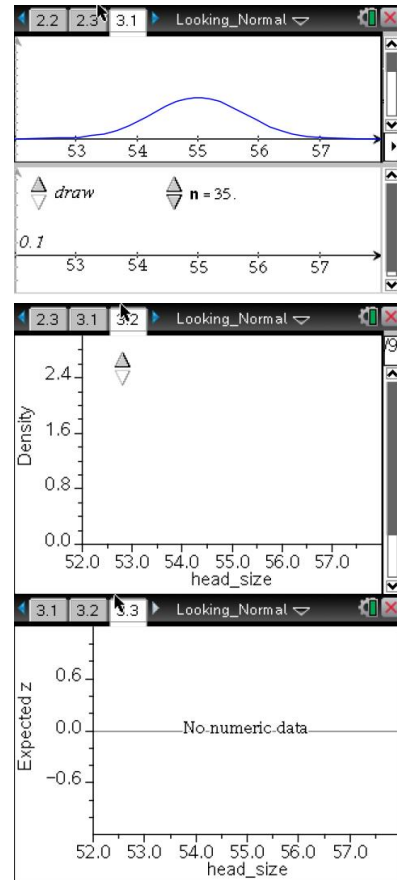
b. Look back at your prediction about shape in Question 1. Does your sample of size *n* = 35 seem to fit your prediction pretty well?

   **Sample Answers:** Not as well as for *n* = 50, but still somewhat OK.

**Move back to page 3.1.**

8. a. Use the "sample size" arrow to decrease the sample size, then click the "draw" arrow to get a new sample. Look at Pages 3.1, 3.2, and 3.3, and describe how well this new sample fits the shape you predicted back in question 1.

   **Sample Answers:** I used *n* = 15, and now my graphs don't look very normal at all. Even the NPP seems to curve around more.
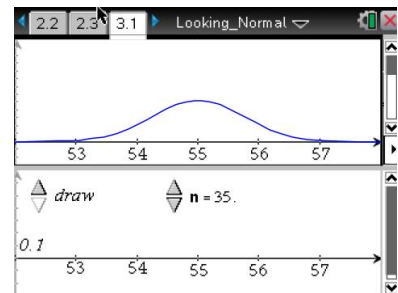
b.   Which of the three graphs is easiest to interpret in judging the normality of your sample now?
     Explain why you think that is the case.

**Sample Answers:** With only 15 values, the histogram is spread out so much that only a few bars
are more than one unit tall. The same is true for the dot plot. Only the NPP still has a clearly
visible overall shape.

> **Teacher Tip:** Assign certain students to set the sample size at 35, some at
> 30, 25, 20, and 15. Then examine the sample histograms in descending
> sample size order to help reinforce the idea that larger sample sizes look
> more "normal." Repeat for NPPs for various sample sizes.

> **TI-Nspire Navigator Opportunity:** *Screen Capture*
> **See Note 2 at the end of this lesson.**

## Wrap Up

Upon completion of the discussion, the teacher should ensure that students understand:

*   The shape of the distribution for a single sample varies from sample to sample, even when all the
    samples are selected randomly from the same normal population.

*   The variability in shapes of individual samples from a normal population decreases as sample size
    increases. Large samples will generally appear "roughly normal" when plotted, while small samples
    frequently appear very "non-normal" when plotted.

## Assessment

Suppose you had to check the assumption that a set of sample data came from a normal distribution.
What kind(s) of graph would you look at, and what would you look for in that (those) graph(s)? Explain
briefly.

**Sample answer:** I would probably use a normal probability plot (NPP) and look for approximate
linearity, which would be an indication that the sample was approximately normally distributed,
supporting the possibility that the population was normally distributed. If the NPP turned out to
seem nonlinear, then I would think that the population from which the sample was taken was
probably not normally distributed. But I would tolerate a bit more non-linearity in smaller samples
than in larger ones.

## TI-Nspire Navigator

**Note 1**

**Question 6,** *Screen Capture*

Screen captures for question 6 provide the opportunity to discuss data that are approximately normally distributed. Point out that no finite set can ever be exactly normally distributed. Focus here on the amount of variation from student to student (sample to sample) when it is known that all samples are of the same size and from the same population. Be sure to look across the class at each graph type to see the variability that is possible among dot plots, then among histograms, and finally among NPPs.

**Note 2**

**Question 8,** *Screen Capture*

Screen captures for question 8 provide the opportunity to discuss the amount of variation in the shapes of different samples. Focus on how the departures from "looks normal" change as sample size decreases. Assign certain students to set the sample size at 35, some at 30, 25, 20, and 15. Then arrange the sample histograms on the screen capture screen in descending sample size order to help reinforce the idea that larger sample sizes look more "normal." Repeat the captures using NPPs for the various sample sizes.

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