

3D Parametric

ID: 19034

Time Required

45 minutes

Activity Overview

In this activity, students will review the concepts of parametric and polar equations. By using the 3D graphing capabilities of the TI-Nspire handheld, students will be able to extend these ideas to the area of solids of revolution, arc length and kinematics.




Topic I: Parametric and Polar Equations

- Modeling projectile motion with parametric equations
- Using parametric equations to graph polar equations
- Finding the slope of a tangent to a polar graph at given value of θ
- Applying parametric equations that model projectile motion to a three-dimensional coordinate system
- Finding the speed and acceleration of a particle in a three-dimensional coordinate system

Topic II: Applications of Definite Integration

- Finding the volume of a solid revolved around an axis using the disk method
- Finding arc length in a three-dimensional coordinate system

Teacher Preparation and Notes

- The use of the accompanying handout is optional. Students can write their responses directly into the TI-Nspire handheld and/or on the worksheet. On self-check questions, after answering the question students can press  and select **Check Answer** (or press  + ).
- Notes for using the TI-Nspire™ Navigator™ System are included throughout the activity. The use of the Navigator System is not necessary for completion of this activity.
- To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "19034" in the keyword search box.

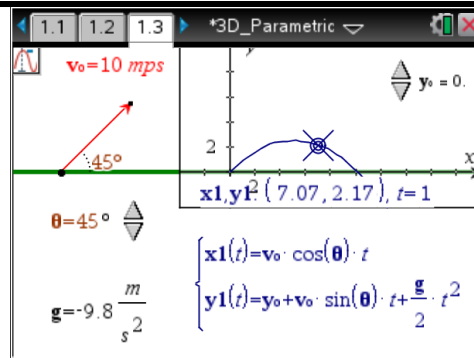
Associated Materials

- 3D_Parametric_Student.doc
- 3D_Parametric.tns

Problem 1 – Parametric Equations and Projectile Motion

TI-Nspire™ Navigator™ Opportunity: Class Capture
See Note 1 at the end of this lesson.

On page 1.3, students will see a graph depicting the position of an object based on the parametric equations $x_1(t)$ and $y_1(t)$. They may manipulate the sliders for the angle, θ , and the initial height, y_0 . Additionally, students may click inside the v_0 box and change the initial velocity.



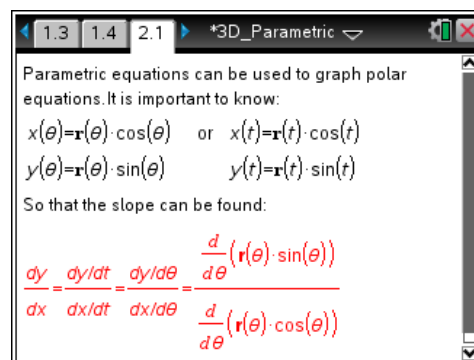
Also on page 1.3, instruct students to select **MENU > Trace > Graph Trace** to trace along the curve in order to find the x and y positions of the object at any time t . You may need to instruct them to alter the Trace Step to choose an appropriate step size.

On page 1.4, students are asked to find the location of the projectile at $t = 0.9$ seconds.

TI-Nspire™ Navigator™ Opportunity: Quick Poll
See Note 2 at the end of this lesson.

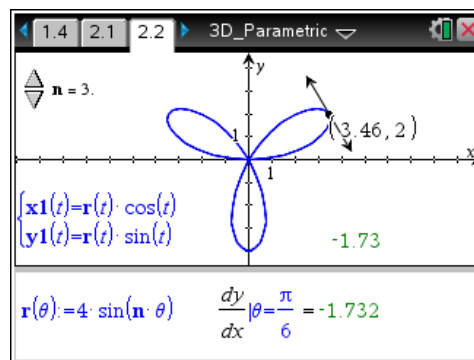
Problem 2 – Polar Graphs using Parametric Equations

Page 2.1 displays two pairs of important formulas from Calculus BC. Page 2.2 prompts students to manipulate a slider that will allow them to change the coefficient, n , of θ in the polar equation $r(\theta) = 4\sin(n\cdot\theta)$. Students are told to notice the changes in the curve as they manipulate the slider.



Additionally, you may want to direct the students' attention to the movable point on the curve and its accompanying value of the slope of the tangent line drawn to the curve at that point.

On page 2.2, students manipulate a slider to change the coefficient, n , of θ in the polar equation $r(\theta) = 4\sin(n\cdot\theta)$. Students are told to notice the changes in the curve as they manipulate the slider.

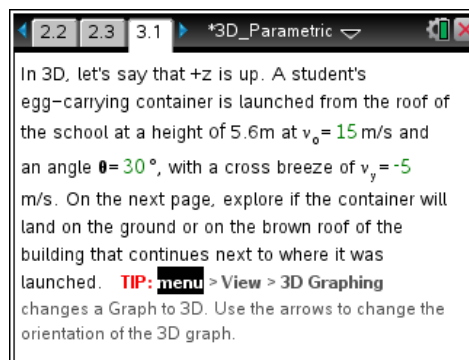


Additionally, you may want to direct the students' attention to the movable point on the curve and the accompanying value of the slope of the tangent line drawn to the curve at that point.

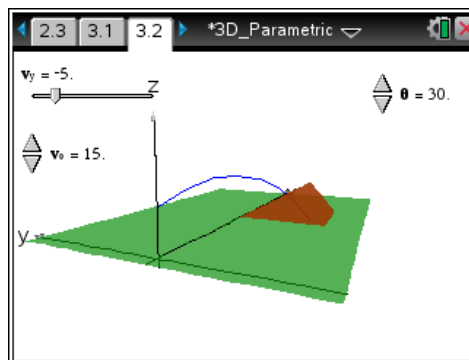
Page 2.3 asks the students find the derivative of $r(\theta) = 4\sin(3\cdot\theta)$ when $\theta = \frac{\pi}{6}$.

Problem 3 – Parametric Equations in Three Dimensions

Page 3.1 presents the students with a scenario of a student preparing to launch an egg-carrying container from the roof of a school. The conditions of the launch are given and the path of the container can be viewed graphically on page 3.2.



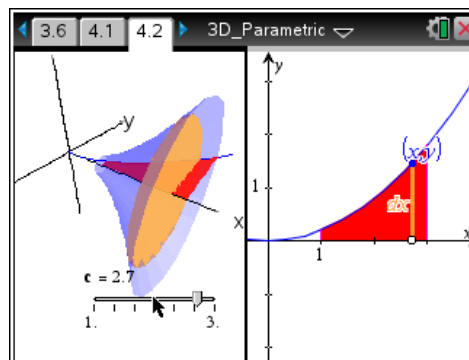
In order for the students to determine whether the container hits the brown rooftop, they may have to rotate their graph using the arrow keys on the touchpad.



Problem 4 – Volumes of Solids of Revolution

Students explore the process used to find the volume of a solid rotated about the x-axis.

They use the slider for c on the left side of the page to move the location of the orange disk. As they move the slider, they should look at how the orange rectangle on the right side of the page moves and changes. On the right side, students should now drag the open circle on the x-axis and observe the changes in the location and size of the orange disc on the left.

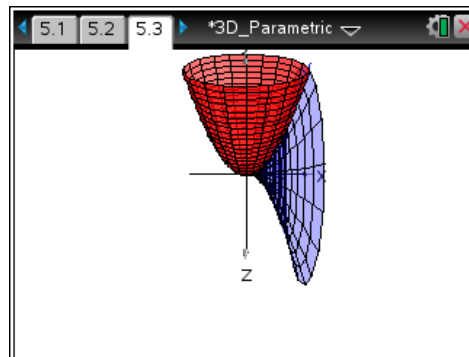


Extension

Problem 5 – More on Volumes of Solids of Revolution

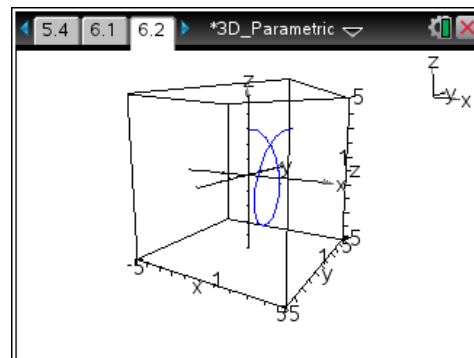
On page 5.2, students are to enter the expressions for rotating the function around the y -axis using x_{p2} , y_{p2} and z_{p2} . They will need to select Parametric graphing mode (MENU > 3D Graph Entry/Edit > Parametric).

$$\begin{cases} x_{p2}(t,u) = \sqrt{t} \cdot \cos(u) \\ y_{p2}(t,u) = t \\ z_{p2}(t,u) = \sqrt{t} \cdot \sin(u) \end{cases}$$



Problem 6 – Arc Length in 3D

On page 6.1, students read about the ways to calculate the arc length of the graph shown on page 6.2. After studying the solution for the arc length of the given curve, they should see that the calculation is greater than the value of $2\pi r$ when $r = 3$: 19.087 vs. 18.85.

**Solutions – Student Worksheet**

- | | |
|--|----------------------|
| 1. (6.364, 2.395) | 5. 9.8 |
| 2. -1.732 | 6. 6.082 |
| 3. 25.229 | 7. $\frac{64\pi}{3}$ |
| 4. $\sqrt{(x''')^2 + (y''')^2 + (z''')^2}$ | |

TI-Nspire™ Navigator™ Opportunities**Note 1****Problem 1, Class Capture**

Use Class Capture to verify that students are correctly manipulating sliders here and throughout the other interactive pages of the document.

Note 2**Problems 1–3, Quick Poll**

Use Quick Poll here or throughout the lesson to assess student understanding. The numbered questions within the document can be used as a guide for possible questions to ask.