Properties of an Ellipse

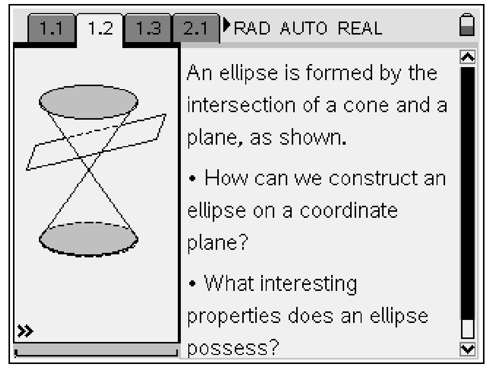
PrecalcWeek21_PropOfEllipse.tns

Name ______________________

Class ______________________

Introduction

All conic sections can be created by intersecting a plane and a right circular cone and simply changing the orientation of the plane. The diagram shown at right illustrates how to orient the plane to create a closed curve called an **ellipse**. How might you intersect a plane and a cone to create a circle, hyperbola, or parabola?



In this activity, you will focus on ellipses drawn on a coordinate plane, by answering the two questions shown at right.

Problem 1 – Investigating the definition of an ellipse

Advance to page 1.3. You will see two segments that share an endpoint, with the other (fixed) endpoints on the *x*-axis. The length of these segments *PF1* and *PF2*—as well as the sum of these lengths—are also displayed.

We are interested in the path that point *P* traces out as it is dragged. Grab point *P* and move it in either a clockwise or counterclockwise direction. Notice the scatter plot that is displayed on the screen as point *P* moves. While dragging *P*, pay close attention to the measurements on the screen, including the sum of *PF1* and *PF2*.

The shape that is traced out is called an ***ellipse***.

- Describe what you observed about *PF1*, *PF2*, and *PF1* + *PF2* as you made this trace.

The two fixed points *F1* and *F2* are called ***foci*** (singular: ***focus***). The definition of an ellipse is based on the relationship between these two points and the set of points (called a ***locus***) that you traced out.

- Use your observations about these points and measurements to write a definition of an ellipse.

Problem 2 – An interesting property of an ellipse

Advance to page 2.1. Here you see an ellipse with a point *P* located on the ellipse. Recall that the foci of an ellipse are fixed points. Drag point *P* and observe what happens to *F1* and *F2*.

- Based on your observations, can you say that *F1* and *F2* are the *actual* foci for this ellipse?

- If your answer to the previous question is *no*, drag *F1* to a correct position for the focus. For the foci *F1* and *F2* to be in the correct place, what must happen to their position when dragging point *P*?

- Based on these observations, describe how to locate the foci of an ellipse.

Look closely at the vector that begins at *F1* and ends at *P*, as well as the vector that begins at *P* and ends at *F2*.

- What is the relationship between these vectors? If you are unsure, select **MENU > Actions > Hide/Show** to reveal some hidden objects. In particular, look at the construction located near point *P*. (Press (esc) when you are finished.)

Suppose these rays represent sound waves that originate at *F1* and land at *F2*. Also consider that sound waves generally emanate in many directions, not just along a single ray.

- Assume a person is standing at *F1* and whispers quietly. Where do you think a second person (not within earshot of the first person) would need to stand to hear this whisper clearly?

- Summarize this property and explain how it relates to the foci and *any* point on the ellipse.

Problem 3 – Another interesting property of an ellipse

On page 3.1, you will explore the role that the location of the foci plays in determining the *shape* of an ellipse. In this diagram, point *F1* can be dragged left and right along the *x*-axis.

- Drag *F1* to the left. What happens to the shape of the ellipse as you do so? What happens when you drag *F1* to the right?

- Drag *F1* so that it coincides with *F2*. What does the figure now resemble? Does the definition of an ellipse still hold true for this situation?