Expecting a Win

Answers & Notes

7 8 9 10 **11** 12

TI-Nspire™ Investigation Student 50 min

Introduction

Many board games require players to land precisely on the last square of the board in order to complete the game. This investigation involves the simplest of board games. Players take it in turns to roll a die, each time advancing toward the final square. Players must finish precisely on the last square in order to finish the game. So far the game sounds simple and fair. There is however one catch! Players can nominate the number of 'sides' on their die. As the die is virtual (simulated by the calculator), players can nominate any quantity between 1 and 20. There are 20 squares on the board, so exactly 20 squares must be advanced and the first player to do so is the winner!

Teacher Notes: Implicit and explicit Activity Objectives

- Understand that simulations represent a valid methods for solving challenging problems
- A single game or relatively small sample of games is not sufficient to determine which die is best particularly where a significant variation exists in the data.
- Larger dice (numbers) lead to a larger variation (standard deviation) in the results. Students should identify this in the discussion questions, particularly where they have been required to graph the results.
- Concept of a sampling distribution and an informal understanding of the Central Limit Theorem and the Normal Distribution.
- For a sampling distribution, larger sample sizes have a smaller standard deviation. (Informal treatment)
- A good understanding of statistics is required to make valid decisions. The majority of the activity focuses on using the 'mean' number of moves required to complete the game. This is not the ideal measure when the focus is on winning! The median and mode are less sensitive to outliers or long 'tails' in the distribution of the data. Dot plots or box and whisker plots are useful tools.

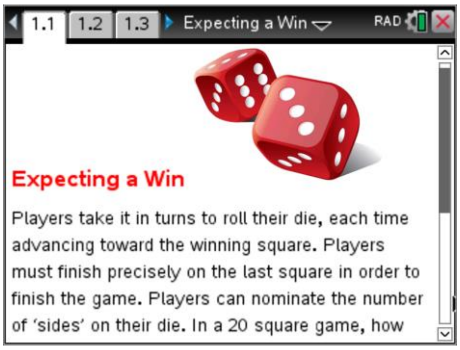
Instructions

Open the TI-nspire file "Expecting a Win".

Page 1.2 includes a game simulation where you can simulate a single game at a time. Game updates are displayed on screen as:

- Current Square
- Dice Roll
- New Square
- Number of rolls

To run the game, select "Game" from the variable menu.



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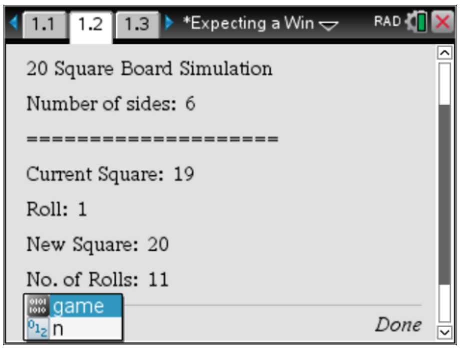
Explore a number of games using a different number of sides each time. Carefully observe the "Current Square" and the dice roll.



Question: 1.

Discuss the advantages and disadvantages of using a 20 sided die to play the game compared with a 4 sided.

Answer: Initial progress may be quite rapid, potentially reaching the final square in just one roll, however, as a player gets close to the finish line the probability of rolling the exact number is less likely than if they are using a die with fewer sides/numbers.

Question: 2.

How many sides do you think would give you the maximum chance of winning?

Answer: Answers will vary. Students should provide a level of justification (refer previous answer).

Teacher Notes:

The intention of this question is to generate a level of investment in the problem, determining whether or not their response is correct. Students should recognise that calculating the 'expected' value for even a 20 square board is relatively complex and therefore 'simulation' is an appropriate alternative for approximating the answer.

The expected value can be calculated using Markov Chains, but is the 'expected values' for the number of moves the correct approach?

Important: Before students start their simulation they should set the Random Seed to ensure each student obtains different 'random numbers'. **Menu > Probability > Random > Seed** … then enter a four digit number.

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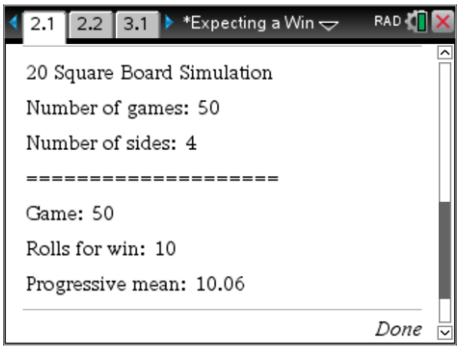
Run the GameSim program. This program plays the requested number of games using the corresponding number of sides input for the die. Updates displayed on screen:

- Game Number
- Rolls to Win
- Progressive Mean



Question: 3.

Play 50 games with a 4 sided die. Use a dot plot (Page 2.2) to display the distribution of results.

Answer: Answers will vary. Students can use a 'screen shot', teachers can extract them if using TI-Navigator or students can simply draw a scatterplot by hand.

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A record of the simulations can be made by storing the most recent results into a different variable. The variable “R” is the list where the original results are stored for each individual game. To transfer the results to R1:

R1: = R

Additional data sets can be named: R2, R3, R4 … OR … R10 for 10 sided, R4 for 4 sided.

Question: 4.

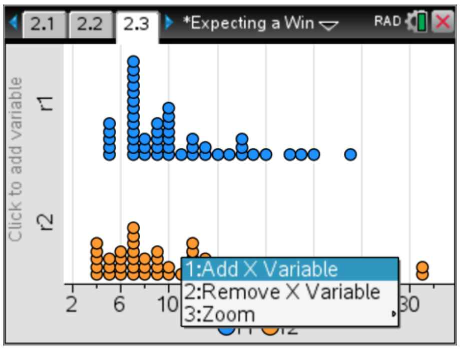
Play 50 games with a 10 sided die. Use a dot plot to display the distribution of results.

Answer: Answers will vary. Students can use a ‘screen shot’, teachers can extract them if using TI-Navigator or students can simply draw a scatterplot by hand.

In the example shown opposite two data sets have been saved as R1 and R2 where R1 = 50 games with a 6 sided die and R2 = 50 games with a 9 sided die.

To add a second dot plot press:

Menu > Plot Properties > Add X Variable



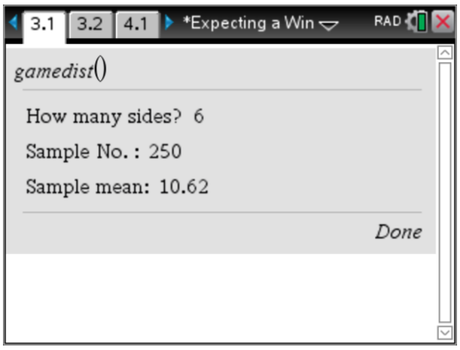
Question: 5.

Discuss differences between the distribution of results for the 10 sided die and the 4 sided.

Answer: The distribution for the 4 sided dice has a much smaller spread (standard deviation) than that of the 10 sided die. Students may also consider changing the plot type to a Box Plot to see that in most cases students will see that the median for the 10 sided die is lower than that of the 4 sided.

Navigate to Page 3.1

The GameDist program in this problem generates the requested number of samples, each consisting of the specified number of games, and corresponding quantity of sides on the die.



Large combinations of simulations such as 500 samples each consisting of 100 games will most likely involve approximately half a million rolls of a die. Choose your quantities wisely to avoid lengthy simulations!

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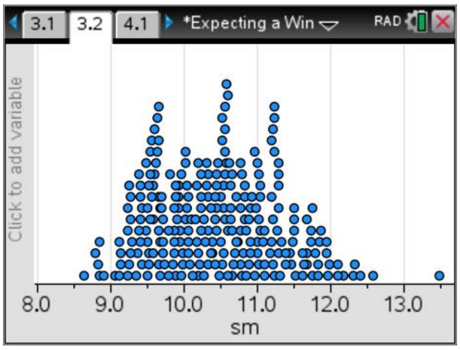
Once the sample means have been generated, they can be graphed as a dot plot.

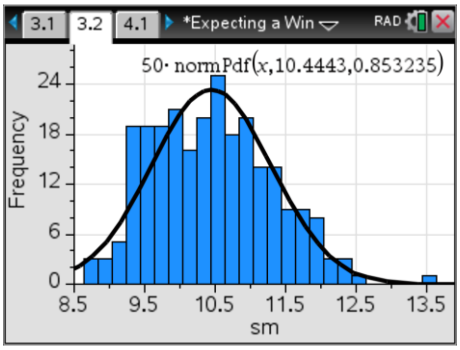
The variable 'sm' contains a list of the sample means calculated from each sample.

The plot shown represents 250 sample means where each sample consisted of 25 games using a 6 sided die.



Changing the plot type to a histogram activates the **Show Normal** option in the **Analyse** menu.

The mean: 10.44 and standard deviation: 0.853 are displayed for this distribution of sample means.



Question: 6.

Generate 200 samples of 25 games for an 8 sided die.

a) Graph the results using a dot plot and describe the approximate shape of the distribution of the sample means. (Sample means are stored in a variable: sm)

Answer: The distribution is approximately normal.

b) Change the graph type to a histogram and include an approximate Normal Distribution curve. Discuss and compare the differences (if any) between the mean and standard deviation of your data with the example shown above.

Answer: The mean should be approximately 11.2 with a standard deviation of approximately 1.6. The mean appears higher but the standard deviation is comparatively a lot higher. The larger number of sides on the die results in a lot more variability in the number of rolls required to complete the game, therefore a larger variation in the sample means also exists.

Question: 7.

Determine an appropriate number of samples and games and complete the table below including the computed mean and standard deviation[1] of each sampling distribution.

All answers below are approximate only for 200 samples each consisting of 50 games. Students should notice a distinct trend, as the number of sides on the die is increased there is an increased variation in the number of rolls required to complete the game. Students should also notice the slight change in the average number of rolls required to land on the final square with the minimum being for a 6 sided die.

Number of Sides	2	4	6	8	10	12	14	16	18
Dist. Mean: $\mu_{\bar{x}}$	14.2	10.8	10.5	11.2	12.3	13.7	15.5	17.4	19.2
Dist. Standard Deviation. $\sigma_{\bar{x}}$	0.28	0.51	0.77	1.04	1.43	1.60	1.93	2.02	2.63

[1] Standard Deviation of Sampling Distribution – This is also referred to as the "Standard Error".

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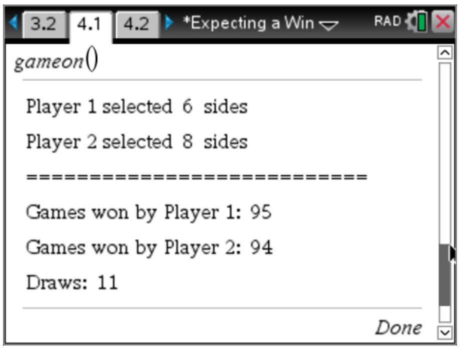
Question: 8.

Which die would you chose to give you the best opportunity of winning?

Answer: This is a 'loaded' question if students only use the table of values to select the best number. The result would be approximately 6 sides. If students however looked closely at the distributions in Questions 3 to 5 they may have realised that the 'mean' is not the best statistical measure the aim is to win games.

Teacher Notes:

The programs can be easily modified to measure the median of each sample and therefore generate sampling distributions for the sample 'medians' … this too will be approximately normal.



Navigate to page 4.1.

Now is your chance to simulate games where players can determine how many sides they would like to have on their die.

The GameOn program simulates the desired number of games using the selected number of sides for each player. Try simulating some games using your table as a guide.

In the example shown, 200 games were simulated with player 1 selecting a 6 sided die and player 2 selecting an 8 sided die. Note that a draw is counted when both players take exactly the same number of rolls to reach the finish.

Question: 9.

Using the GameOn program, play 200 games where Player 1 has a 4 sided die and Player 2 has a 8 sided die. Who would you predict to win most often?
[Try this simulation a couple of times or try playing a larger quantity of games.]

Note: Data for each Player 1 and Player 2 are stored in T_1 and T_2 respectively.

Based on the table only, it would appear that Player 1 should win more often with an average number of moves 10.8 compared with 11.2, however when students explore this using the program, Player 2 generally wins more often! Students may accept this as simply an anomaly, something that happened purely by chance. The reality is that the 8 sided dice does have a better chance at winning.

Teacher Notes:

This is a great opportunity to informally discuss hypothesis testing and Type I and Type II errors. If we believe that the 4 sided die has a better chance of winning (based on tabulated information collected previously) then we could put the number of wins for the 8 sided die down to chance. How confident are we that this has occurred by chance? What is really happening? Our 'assumption' with regards to a lower 'mean' will result in more game wins is flawed. The distributions for the die with more sides have much longer tails which effectively skews the mean. The mode and arguably the median, represent better ways to determine the quantity of sides for a die that creates the greatest probability of winning.

History is filled with examples of false assumptions leading to flawed acceptance or rejection of statistical results. This is why it is so important to **understand** what is being measured, how it is being measured and what are the implications of the results.

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Question: 10.

Review your exploration so far. Discuss the outcomes to date and continue your exploration as necessary to determine the optimal number of sides for a dice in this game consisting of a 20 square board.

Answer: Answers will vary. Students should however recognise that the mean is not the ideal statistical measure for determining which die is best. Recommendations and future explorations should use the mode or median noting that for small samples the mode will be too volatile.

Teacher Notes:

Further exploration for this activity could include a smaller board making it possible to calculate theoretical values much easier.

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