Up and Down: Vertical Oscillations – ID: 10541

By Irina Lyublinskaya

Time required
45 minutes

Topic: Circular and Simple Harmonic Motion

- *Use the equations of motion to find frequency, period, force, energy, velocity, and acceleration of objects in simple harmonic motion.*

Activity Overview

In this activity, students explore vertical oscillations produced by a mass attached to an elastic spring and collect data on the force of the spring as a function of time. Students then explore the relationship between the period of oscillations and the magnitude of the oscillating mass, fit a model to their collected data, and apply the model to solve problems.

Materials

To complete this activity, each student will require the following:

- *TI-Nspire™ technology*
- *Vernier Dual-Range Force sensor*
- *Vernier EasyLink™ or Go!®Link interface*
- *stand (rod and base) and clamp*
- *elastic spring*
- *slotted weight set with hanger*
- *copy of student worksheet*
- *pen or pencil*
- *blank sheet of paper*

TI-Nspire Applications

Lists & Spreadsheet, Graphs & Geometry, Data & Statistics, Notes, Calculator

Teacher Preparation

Students should be familiar with the concept of simple harmonic motion and with the fact that, in the absence of air resistance, small oscillations of a mass attached to an elastic spring represent simple harmonic motion.

- *Before carrying out this activity, you should review with students Newton's second law and Hooke's law. This will help students set up the equation describing the motion of the spring-mass system. You may also want to review the equation for simple harmonic motion.*
- *The screenshots on pages 2–8 demonstrate expected student results. Refer to the screenshots on page 9 for a preview of the student TI-Nspire document (.tns file). The student worksheet is shown on pages 10–12.*
- ***To download the .tns file and student worksheet, go to education.ti.com/exchange and enter "10541" in the search box.***

Classroom Management

- *This activity is designed to be **student-centered**, with the teacher acting as a facilitator while students work cooperatively. The student worksheet guides students through the main steps of the activity and includes questions to guide their exploration. Students may record their answers to the questions on blank paper or answer in the .tns file using the Notes application.*
- *The ideas contained in the following pages are intended to provide a framework as to how the activity will progress. Suggestions are also provided to help ensure that the objectives for this activity are met.*
- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

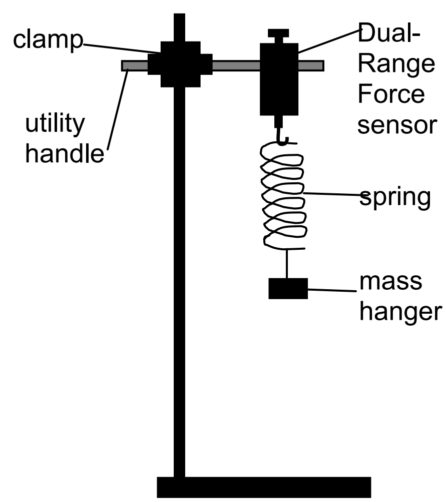
The following questions will guide student exploration during this activity:

- What are the forces that act on the oscillating mass? What causes oscillations in the spring-mass system?
- What is the relationship between the period of oscillations and the oscillating mass? Can you model this relationship mathematically? Does your model agree with the theoretical results?

The purpose of this activity is to allow students to observe the vertical oscillations of a mass attached to an elastic spring and explore the dynamics and kinematics of this case of simple harmonic motion. Students collect data on the force of the spring as a function of time, analyze these data, and measure the period of oscillations for various masses. They observe changes in the spring force for each mass and determine the period of oscillations by either tracing the graph or by using the regression capabilities of TI-Nspire technology. Students then develop a mathematical model for the relationship between mass and period and apply it to solve problems.

If time allows, students can determine the spring constant of the spring used in the activity by measuring displacement as mass is added to the spring at rest. The force constant is the slope in the graph of force (*mg*) vs. displacement. Otherwise, you should provide students with the value of the spring constant so that they can compare their mathematical models with the theoretical formula for the period of oscillation.

Problem 1 – Collecting force data



Step 1: Students will use a Vernier EasyLink (if using a handheld) or Go!Link (if using a computer) interface connected to a Vernier Dual-Range Force sensor to collect force data. Make sure the switch on the force sensor is in the ±10N position. Students should set up the experiment as shown to the right and explained in the worksheet. They should practice pulling and releasing the spring several times to initiate vertical linear oscillations before collecting data.

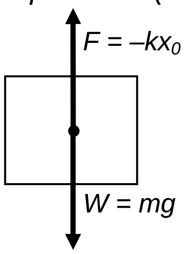
Step 2: Students should connect the Dual-Range Force sensor to the EasyLink or Go!Link interface. Once the experimental setup is complete, students should answer questions 1 and 2.

Q1. What are the forces acting on the oscillating mass?

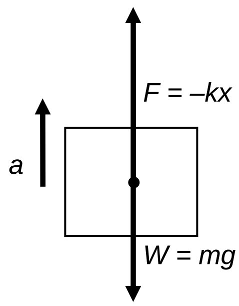
A. *the force of gravity (the weight of the spring and mass) and the spring force (restoring force of the spring)*

Q2. Draw a free-body diagram for the spring-mass system, and use Newton's second law to set up the equation describing its oscillation.

A. *A sample free-body diagram for a mass in equilibrium (i.e., not moving) is shown below:*



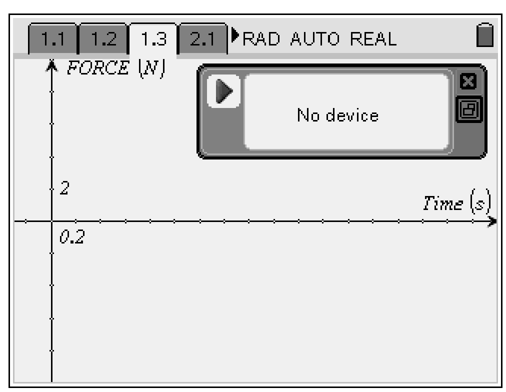
A free-body diagram for a mass in motion is shown below:



If x$_0$ *is the position of the bottom of the spring at equilibrium,* x *is the displacement of the bottom of the spring from equilibrium,* k *is the spring constant,* m *is the total mass of the spring-mass system,* W *is the weight of the system, and* a *is the acceleration of the spring, then according to Newton's second law,* $-kx - kx_0 + W = ma$. *Because* $W = kx_0$, *the final equation for the motion of the spring is the following:*

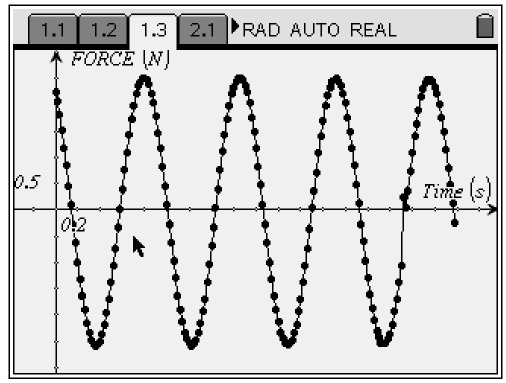
$$-kx = ma, \text{ or } a + \frac{k}{m}x = 0$$

Step 3: Students should open the file **PhyAct_10541_UpDown.tns**, read the first two pages, and then move to page 1.3. When students reach page 1.3, they should connect the EasyLink or Go!Link interface to their handheld or computer. This should activate the force sensor, and a force display should appear in the data collection box on page 1.3.



Step 4: Before students carry out data collection, they should clear any data stored in the device (**Menu > Data > Clear All Data**) and zero the sensor (**Menu > Sensors > Zero**). Note that the spring-mass system should be in equilibrium (i.e., not moving) before students zero the sensor.

Step 5: The “play” button (▶) should be highlighted in the data collection box. If this is the case, students need only click (press) to begin data collection. If the play button is not highlighted, students should press tab until it is selected. They should then pull the mass down slightly, release it, and then begin the data collection. When data collection has finished, the scatter plot will be displayed on the graph. Students should then answer questions 3–6.



Q3. Describe the shape of the graph.

A. *The graph has the shape of a sine or cosine function.*

Q4. What is the meaning of a positive force on this graph? What is the meaning of a negative force?

A. *A positive force represents a stretching (extension) of the spring from the equilibrium position. A negative force represents compression of the spring from the equilibrium position.*

Q5. Describe the motion of the mass when force has its maximum value and minimum and zero values. In which direction does the mass move in each instance? How is the slope of the curve related to the motion of the mass?

A. *The mass is at rest at the bottom position when force reaches maximum and is at the top position when force reaches minimum. When force is zero, the mass is moving through the equilibrium position. When the slope of the force vs. time graph is positive, the mass is moving down, and when slope is negative, the mass is moving up.*

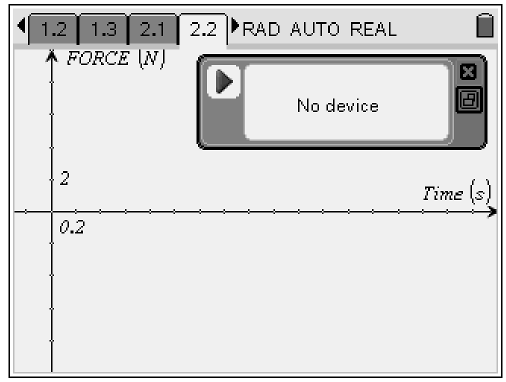
Q6. What is the period of oscillations? What mass is attached to the spring?

A. *Answers will vary. Students can trace the graph to determine the period of oscillations.*

Problem 2 – Modeling the period of oscillations

Q7. What will happen to the period of oscillations if you increase the mass attached to the spring?

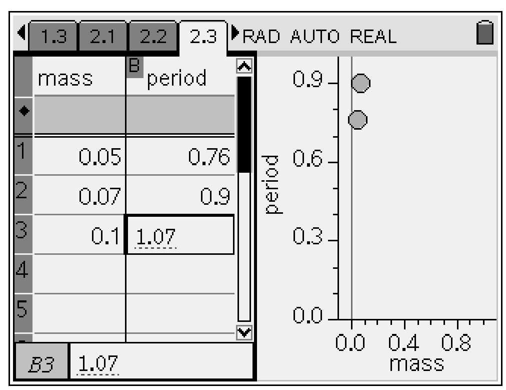
A. *Student answers will vary, but many students will probably predict that a larger mass will lead to a larger period of oscillation. Encourage students to discuss and justify their predictions, and encourage more specific predictions than simply "period will increase." An example of such a prediction is "the period will increase in direct proportion to the increase in mass."*



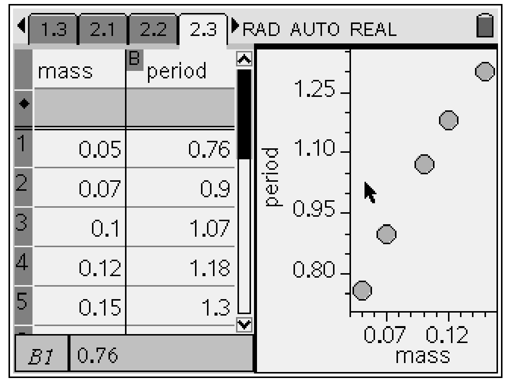
Step 1: Next, students should read the text on page 2.1 and then move to page 2.2. Page 2.2 shows a *Graphs & Geometry* page with a data collection display. If the data collection display is not present, students can insert it by pressing ctrl D.

Step 2: Students should place additional mass on the hanger, record the total mass of the spring-mass system, and repeat steps 4 and 5 from problem 1. They should record the mass and the oscillation period on their worksheets.

Step 3: Students should repeat step 2 for several more masses. They should record data for at least five masses. They can use the data from problem 1 as one of the data points.



Step 4: Next, students should move to page 2.3, which contains a *Lists & Spreadsheet* application and a *Data & Statistics* application. Students should record the data they collected in steps 2 and 3 in the *Lists & Spreadsheet* application (mass in column A and period in column B). A corresponding scatter plot will appear on the graph to the right of the data. If all the data do not appear in the *Data & Statistics* application, students can resize the window (**Menu > Window/Zoom > Zoom-Data**). Students should study the graph and then answer questions 8–10.

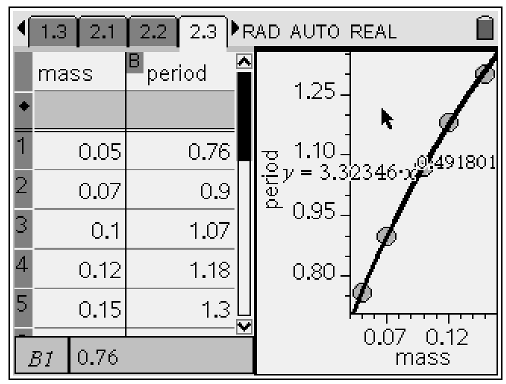


Q8. What mathematical model would you use to best fit these data?

A. *The data are consistent with a power relationship (i.e., $y = x^k$). Many students will not correctly identify the relationship. Encourage students to try several different regressions in order to identify the correct fit.*

Q9. Find the mathematical model for the period of oscillations.

A. *For the sample data shown, the best-fit equation is $y = 3.320x^{0.492}$. Students should use the **Power Regression** tool (**Menu > Analyze > Regression > Show Power**) to determine the best-fit equation for their data.*



Q10. Use the spring constant of your spring to determine the theoretical relationship between period and mass. How close are your theoretical results to your experimental results?

A. *The spring used to collect the sample data has a spring constant of 3.4 N/m. The formula describing the theoretical relationship between period (*T*), mass (*m*), and spring constant (*k*) is* $T = 2\pi\sqrt{\frac{m}{k}}$ *. Substituting for the value of* k, *we get* $T = \frac{2\pi\sqrt{m}}{\sqrt{3.4}} = 3.4m^{0.5}$ *. The error is about 2% between the experimental and theoretical results, as shown below:*

$$\frac{|3.4 - 3.32|}{3.4} \cdot 100\% = 2.3\%$$

$$\frac{|0.5 - 0.492|}{0.5} \cdot 100\% = 1.6\%$$

Problem 3 – Applications and problem solving

Step 1: Next, students should answer questions 11 and 12. A *Calculator* application is given on page 3.1 for students to use to solve these problems.

Q11. An elastic cord vibrates with a frequency of 3.0 Hz when a mass of 0.60 kg is hung from it. What is its frequency if only 0.38 kg hangs from it?

A. *The period of oscillations is proportional to* $\sqrt{m}$ *. Therefore, frequency is inversely proportional to* $\sqrt{m}$ *. This leads to the equation* $\frac{f_2}{f_1} = \frac{\sqrt{m_1}}{\sqrt{m_2}}$ *. Solving this equation for* f_2 *yields*

$f_2 = 3.8$ *Hz.*

Q12. At equilibrium, a mass of 0.262 kg stretches a vertical spring 0.315 m. If the spring is stretched an additional 0.130 m and released, how long does it take to reach the equilibrium position again?

A. *The spring constant can be found from the equilibrium condition, when the spring force (*kx*) is equal to the weight (*mg*), as shown below:*

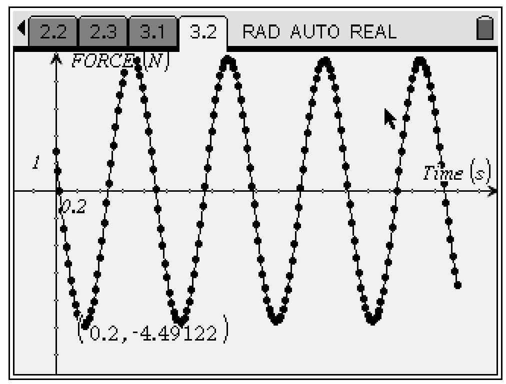
$$kx = mg$$

$$k = \frac{mg}{x} = \frac{(0.262\ kg)(9.8\ m/s^2)}{0.315\ m} = 8.15\ N/m$$

The period of oscillation is therefore

$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.262\ kg}{8.15\ N/m}} = 1.13\ s^{-1}$*. It takes one-quarter of a period for the spring to return to the equilibrium position, so the time is 0.28 sec.*

Step 2: Next, students should move to page 3.2, which is set up for data collection. They should connect the *Dual-Range Force* sensor, zero it, and collect data to answer question 13. Students may also use the *Calculator* application provided on page 3.1.



Q13. Collect another set of data for vertical oscillations. Determine the amplitude of the oscillations. What is the total distance the mass travels in one period of oscillations?

A. *The sample data shown above are for m = 0.050 kg. To determine the amplitude of oscillations, first find the maximum force by tracing the curve. The maximum force is the average of the minimum and maximum points on the curve:* $\frac{0.16 + 0.18}{2} = 0.17\ N$*.*

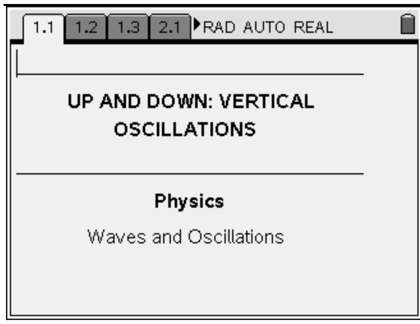
*According to Hooke's law, F = kx; therefore, the maximum displacement (*x*), or amplitude, of the oscillation is given by* $x = \frac{F}{k}$*. Solving this equation with the given data yields the following:*

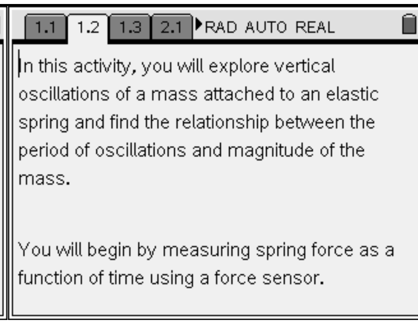
$$x = \frac{F}{k} = \frac{0.17}{3.4} = 0.05\ m$$

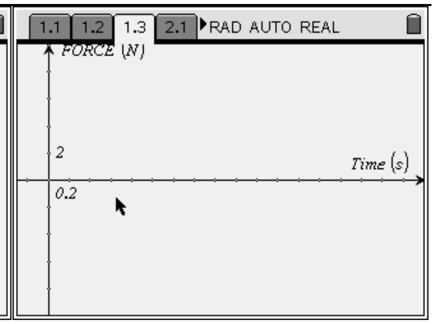
The amplitude of the oscillation is 0.05 m. In one period, the mass travels four amplitudes' worth of distance (two up and two down), so the total distance traveled is 0.2 m. As a point of interest, you may wish to explain that tides follow the same harmonic pattern and that scientists use measurements of the height of the water level to examine tides and the various phenomena that influence tides, such as hurricanes and winter storms. Real-world data and more information can be found on the Internet.

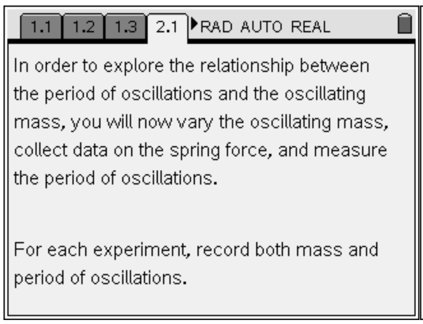
Up and Down: Vertical Oscillations – ID: 10541

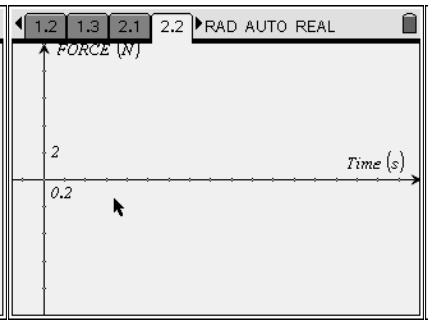
(Student)TI-Nspire File: *PhyAct_10541_UpDown.tns*

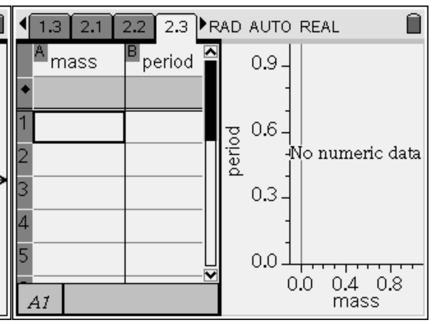


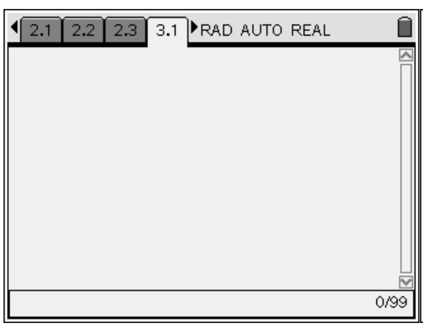


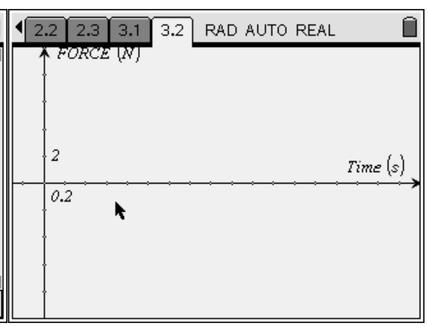












Up and Down: Vertical Oscillations

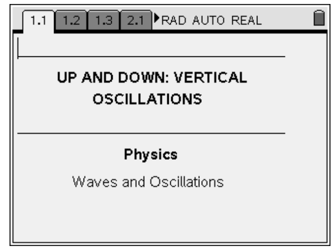
ID: 10541

Name ____________________

Class ____________________

In this activity, you will explore the following:

- *how the force exerted by an elastic spring changes with time during vertical oscillations of a mass attached to the spring*
- *how the period of oscillations depends on the mass attached to the spring*
- *the mathematical model of simple harmonic motion*

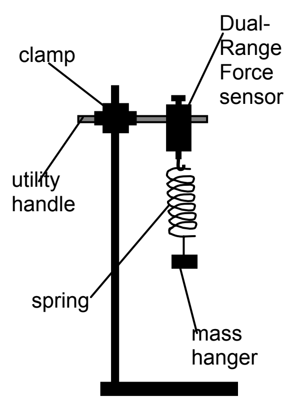


In this experiment, you will study simple harmonic motion. Consider a mass attached to an elastic spring. If you pull the mass down slightly and then release it, the mass will move up and down (oscillate). If resistance is small (as it generally is for highly elastic springs), these oscillations will continue for a long time. We call these oscillations "free oscillations." The period of oscillations, *T,* is the time it takes for the mass to complete a full cycle and return to the same point, moving in the same direction. The period of oscillations, *T,* is the reciprocal of the frequency of oscillations (that is, $f = \frac{1}{T}$). The motion of the mass on an elastic spring is called simple harmonic motion, and it is described by the sine or cosine function.

In this activity, you will collect spring force data with a force sensor, analyze these data, and measure the period of oscillations for various masses. You will then develop a mathematical model for the period of oscillations and apply it to solve conceptual problems.

Problem 1: Collecting force data

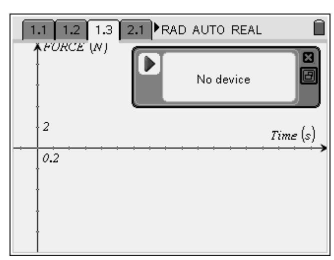
Step 1: Use the clamp to mount the utility handle of the Dual-Range Force sensor (the threaded aluminum rod) so that the utility handle is horizontal. Mount the force sensor on the utility handle with the hook pointing down. Tighten the thumb screw to fix the sensor in position. Attach an elastic spring to the hook at the bottom of the force sensor. Place the mass hanger on the other end of the spring, as shown in the diagram to the right.



Step 2: If you are using the TI-Nspire handheld for data collection, connect the Dual-Range Force sensor to the EasyLink interface. If you are using TI-Nspire computer software to collect data, connect the Dual-Range Force sensor to the Go!Link interface. Do not connect the EasyLink or Go!Link interface to the handheld or computer yet.

Q1. What are the forces acting on the oscillating mass?
Q2. Draw a free-body diagram for the spring-mass system, and use Newton's second law to set up the equation describing its oscillation.

Step 3: Open the file **PhyAct_10541_UpDown.tns** on your handheld or computer. Read first two pages and move to page 1.3. Connect the interface to your handheld or computer. This should activate the force sensor, and a force display should appear in the data collection box on page 1.3.



Step 4: Before you carry out the data collection, clear any data stored in the device by pressing (menu) and selecting **Data > Clear All Data**. Next, zero the sensor. To do this, make sure the spring is in equilibrium (not moving), and then press (menu) and select **Sensors > Zero**.

Step 5: The "play" button (▶) in the data collection box should be highlighted. If it is not, press (tab) until it is highlighted. Then, pull the mass down slightly, release it, and click (press (click)) to begin the data collection. When the data collection has finished, a scatter plot will be displayed on page 1.3. After data are collected, close the data collection box.

Q3. Describe the shape of the graph.

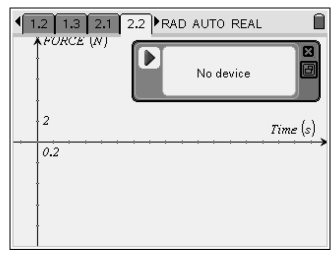
Q4. What is the meaning of a positive force on this graph? What is the meaning of a negative force?

Q5. Describe the motion of the mass when force has its maximum value and minimum and zero values. In which direction does the mass move in each instance? How is the slope of the curve related to the motion of the mass?

Q6. What is the period of oscillations? What mass is attached to the spring?

Problem 2: Modeling the period of oscillations

Q7. What will happen to the period of oscillations if you increase the mass attached to the spring?



Step 1: Read the text on page 2.1, and then move to page 2.2. Page 2.2 shows a *Graphs & Geometry* application with a data collection display. If the data collection display does not appear, press (ctrl)(D) to insert it.

Step 2: Place additional mass on the hanger, record the total mass, and repeat the data collection (steps 4 and 5 in problem 1).

Step 3: Repeat step 2 for several more masses. Record your data in the spaces below.

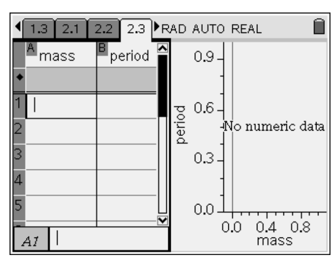
m_2 = ______ kg, T_2 = ______ s

m_3 = ______ kg, T_3 = ______ s

m_4 = ______ kg, T_4 = ______ s

m_5 = ______ kg, T_5 = ______ s

Step 4: Move to page 2.3, which contains a *Lists & Spreadsheet* application and a *Data & Statistics* application. Record the masses and periods you measured in steps 2 and 3 above in the *Lists & Spreadsheet* application. Record masses in column A and periods in column B. As you enter the data, a scatter plot will appear in the *Data & Statistics* application.



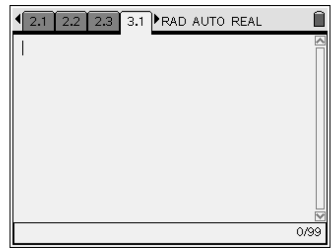
Q8. What mathematical model would you use to best fit these data?

Q9. Find the mathematical model for the period of oscillations.

Q10. Use the spring constant of your spring to determine the theoretical relationship between period and mass. How close are your theoretical results to your experimental results?

Problem 3: Applications and problem solving

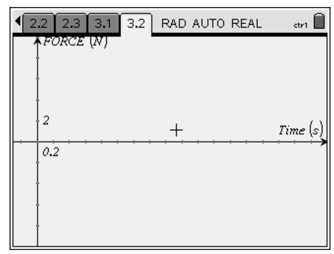
The equation of motion for the spring-mass system is $-kx = ma$, or $a + \frac{k}{m}x = 0$, where *k* is the spring constant, *x* is displacement from equilibrium, *m* is mass, and *a* is acceleration. The period of motion (*T*) for the system is given by the equation $T = 2\pi\sqrt{\frac{m}{k}}$. In this part of the activity, you will compare your mathematical model for the period of oscillations with this formula and apply what you learned about oscillations to problem solving.



Step 1: Page 3.1 contains a *Calculator* application that you can use to solve questions 11 and 12.

Q11. An elastic cord vibrates with a frequency of 3.0 Hz when a mass of 0.60 kg is hung from it. What is its frequency if only 0.38 kg hangs from it?

Q12. At equilibrium, a mass of 0.262 kg stretches a vertical spring 0.315 m. If the spring is stretched an additional 0.130 m and released, how long does it take to reach the equilibrium position again?



Step 2: Next, move to page 3.2, which contains a *Graphs & Geometry* page. Insert a data collection box by pressing ctrl D. Zero the force sensor and then collect the data necessary to answer question 13.

Q13. Collect another set of data for vertical oscillations. Determine the amplitude of the oscillations. What is the total distance the mass travels in one period of oscillations?