

Activity 22

Properties of Quadrilaterals

Objective

- To investigate the properties of special quadrilaterals

Cabri® Jr. Tools










Introduction

This activity focuses on the properties of special quadrilaterals.



Part I: Investigating the Properties of Quadrilaterals

Construction

Construct the following (each as a separate Cabri Jr. figure) or use the figures you created in Activity 21.

-  **Kite** — a quadrilateral with two distinct pairs of consecutive congruent sides
-  **Trapezoid** — a quadrilateral with one pair of parallel sides
-  **Isosceles trapezoid** — a trapezoid with one pair of congruent base angles
-  **Parallelogram** — a quadrilateral with two pairs of parallel sides
-  **Rhombus** — a quadrilateral with four congruent sides
-  **Rectangle** — a quadrilateral with three right angles
-  **Square** — a quadrilateral with four congruent sides and one right angle

Exploration








-   Use various construction and measurement tools to investigate the properties of the special quadrilaterals. Be sure to investigate the relationships that exist among the sides, angles, diagonals, and area. Drag a side or vertex of your quadrilateral and observe the relationships that are true for any quadrilateral of that type.

Questions and Conjectures


1. List all the properties of the special quadrilaterals that you observed. Be prepared to demonstrate.
2. Based on the properties you have observed for the different quadrilaterals, find a way to organize/classify the quadrilaterals. Be prepared to justify your answer.

Part II: Applying the Properties of Quadrilaterals**Construction**

Construct the following quadrilaterals using the properties you discovered in Part I that are different from the properties of the minimal definitions given below.

-  **Kite** — a quadrilateral with two distinct pairs of consecutive congruent sides
-  **Trapezoid** — a quadrilateral with one pair of parallel sides
-  **Isosceles trapezoid** — a trapezoid with one pair of congruent base angles
-  **Parallelogram** — a quadrilateral with two pairs of parallel sides
-  **Rhombus** — a quadrilateral with four congruent sides
-  **Rectangle** — a quadrilateral with three right angles
-  **Square** — a quadrilateral with four congruent sides and one right angle

Exploration

-  Drag the vertices and sides of the quadrilaterals you have constructed to ensure that they retain the properties you used to construct them.

Questions and Conjectures

Write the steps you used to construct each of the quadrilaterals. Explain why each construction is valid even though it does not use the properties of the minimal definitions of each figure. Be prepared to demonstrate.

Teacher Notes



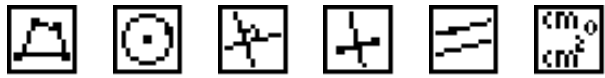
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Additional Information

For this activity, you may want to demonstrate one construction for the class so they understand what to do.

The Cabri Jr. Tools listed are there to offer suggestions to students of the tools that they might consider using. Students are not required to use every one of these tools nor are they limited to only these tools.

Part I: Investigating the Properties of Quadrilaterals

Answers to Questions and Conjectures

- List all of the properties of the special quadrilaterals that you observed. Be prepared to demonstrate.

Some of the properties of these special quadrilaterals are listed below:

Kite: a quadrilateral with two distinct pairs of consecutive congruent sides and:

- one pair of equal opposite angles
- diagonals that are perpendicular
- one diagonal that bisects the other (This is not true for both diagonals.)
- one diagonal that divides the kite into two congruent triangles (This is not true for both diagonals.)
- one diagonal that divides the kite into two isosceles triangles
- diagonals that divide the kite into two pairs of congruent triangles

A kite may be convex or concave and its area is equal to one half the product of the lengths of its diagonals.

Trapezoid: a quadrilateral with one pair of parallel sides and:

- angles sharing a common side (that is not a base) that are supplementary
- diagonals that form triangles of equal areas (see Activity 25, Investigating Area Relationships)

Isosceles trapezoid: a trapezoid with one pair of congruent base angles and:

- angles sharing a common side (that is not a base) that are supplementary
- diagonals that form triangles of equal areas
- one pair of congruent opposite sides
- two pairs of congruent base angles
- congruent diagonals
- diagonals that form congruent triangles
- diagonals that form similar and isosceles triangles

Parallelogram: a quadrilateral with two pairs of parallel sides and:

- two pairs of congruent opposite angles
- four pairs of supplementary angles
- two pairs of opposite sides that are congruent
- diagonals that bisect each other
- each diagonal dividing the parallelogram into two congruent triangles
- diagonals that divide the parallelogram into two pairs of congruent triangles

Rhombus: a quadrilateral with four congruent sides and:

- two pairs of congruent opposite angles
- four pairs of supplementary angles
- diagonals that are perpendicular
- diagonals that bisect each other
- each diagonal dividing the rhombus into two congruent isosceles triangles
- diagonals that divide the rhombus into four pairs of congruent triangles

The area of a rhombus is equal to one-half the product of the lengths of its diagonals.

Rectangle: a quadrilateral with three right angles and:

- four right angles
- two pairs of opposite sides congruent
- diagonals that are congruent
- diagonals that bisect each other
- each diagonal dividing the rectangle into two congruent triangles
- diagonals that divide the rectangle into two pairs of congruent triangles

The area of a rectangle is equal to one-half the product of the lengths of its diagonals.

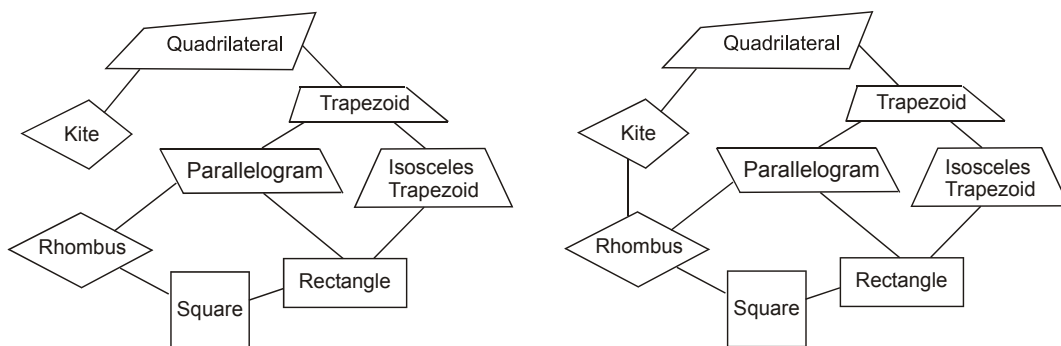
Square: a quadrilateral with four congruent sides, one right angle, and:

- four right angles
- diagonals that are congruent
- diagonals that bisect each other
- diagonals that are perpendicular
- each diagonal dividing the square into two congruent isosceles right triangles
- diagonals that divide the square into four congruent isosceles right triangles

The area of a square is equal to one-half the product of the lengths of its diagonals.

2. Based on the properties you have observed for the different quadrilaterals, find a way to organize/classify the quadrilaterals. Be prepared to justify your answer.

Here are two different ways quadrilaterals can be classified:

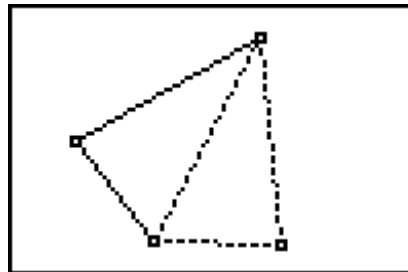


Part II: Applying the Properties of Quadrilaterals

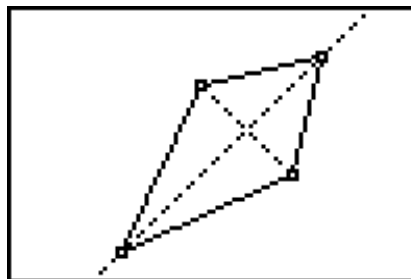
Answers to Questions and Conjectures

Write the steps you used to construct each of the quadrilaterals. Explain why each construction is valid even though it does not use the properties of the minimal definitions of each figure. Be prepared to demonstrate.

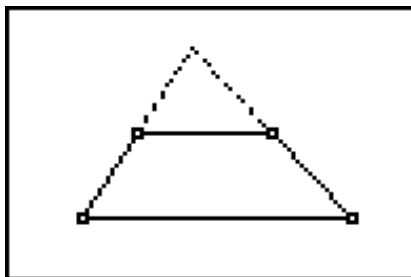
Kite: There are many ways to construct a kite. Construct any triangle and use the **Reflection** tool to reflect one of the vertices over the opposite side of the triangle. Construct the kite using the vertices of the triangle and the reflected point. This construction guarantees a kite is formed since length and angle measure are preserved in a reflection.



Another kite construction takes advantage of the properties of the diagonals of kites. In any kite, exactly one diagonal is the perpendicular bisector of the other. Draw a segment and construct its perpendicular bisector. The four vertices of the kite are the endpoints of the original segment and any two points on the perpendicular bisector.



Trapezoid: The essential property of a trapezoid is a quadrilateral with one set of opposite parallel sides. Construct any triangle and use the **Midpoint** tool to find the midpoints of two sides. Overlay a quadrilateral using these two midpoints and the two base vertices of the triangle. One side of the quadrilateral is the midsegment of the triangle and is parallel to the base. This construction satisfies the definition of the trapezoid.

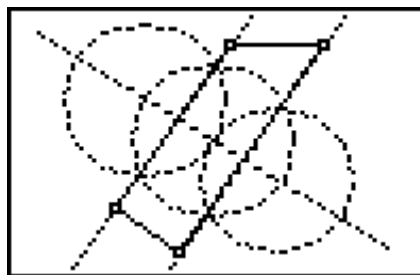


Another construction of a trapezoid uses three interconnected circles whose centers lie on a segment (or a line) and whose radii are equal.

Construct a segment. Construct a circle with its center and radius point on the segment.

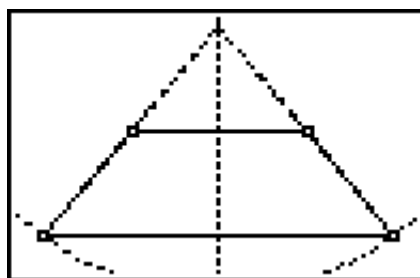
Construct a congruent circle by using the radius point of the first circle as the center of the second and the center of the first circle as the radius point of the second.

Construct a third congruent circle by using the intersection of the original segment and the second circle as the center of the third circle. Use the center of the second circle as the radius point for the third circle. (This method may be used to create any number of equal interconnected circles.) Construct lines through the intersections of the sequential circles. These lines are parallel. To construct a trapezoid, select two points on each of the parallel lines and connect them using the **Quadrilateral** tool to form a trapezoid.



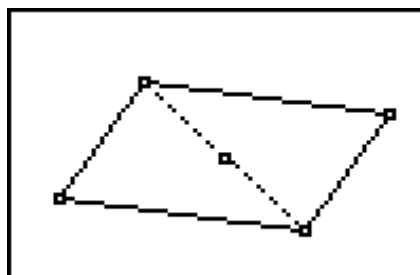
Isosceles trapezoid: If we could construct a trapezoid using the midsegment of a scalene triangle, then we can construct an isosceles trapezoid using the midsegment of an isosceles triangle.

Construct an isosceles triangle by any valid method and construct the midpoints of the congruent sides. Overlay a quadrilateral using these midpoints and the vertices of the base angles of the isosceles triangle. Using the midsegment guarantees opposite parallel sides while using an isosceles triangle guarantees that base angles of the trapezoid are equal. Using the midpoints of the two equal sides of the isosceles triangle guarantees that the non-parallel sides of the trapezoid will be equal.

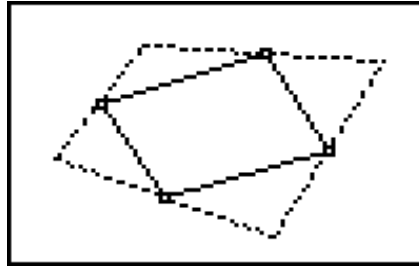


Parallelogram: There are many ways to construct a parallelogram. For example, draw any triangle and construct the midpoint of one side of the triangle. Select the **Symmetry** tool and reflect the whole triangle with respect to the midpoint of a side. The **Symmetry** tool used in this way can be seen as a “half-turn” tool.

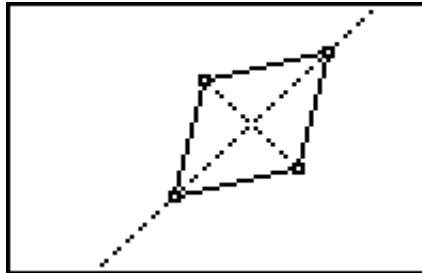
Reflecting the triangle through a point has the same effect as a 180° rotation. Since the midpoint of a side is the center of the rotation, the vertices of the rotated triangle align exactly with the vertices of the old triangle. Measurement confirms that opposite sides are equal and that opposite angles are congruent.



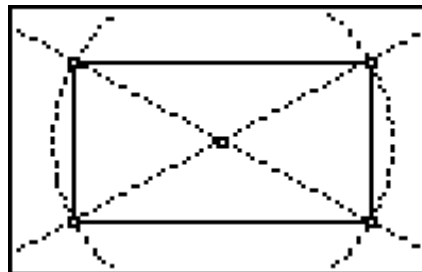
One of the most creative ways to construct a parallelogram is to use the Varignon construction. Draw any quadrilateral and construct the midpoints of the four sides. Construct a quadrilateral with its vertices at the midpoints of the sides of the first quadrilateral. The figure formed by connecting the midpoints is a parallelogram. See Activity 23, Midsegments of Quadrilaterals, for an explanation.



Rhombus: The diagonals of a rhombus bisect each other. Draw a segment and its perpendicular bisector. Construct a point anywhere on the perpendicular bisectors. Using the **Reflection** tool, reflect this point across the original segment. Construct the rhombus by connecting the endpoints of the segment with the two points on the perpendicular bisector. This construction is valid because of the properties of a perpendicular bisector and a line reflection.



Rectangle: The diagonals of a rectangle bisect each other and are congruent. Construct a circle. Using the **Line** tool, construct two diameters of the circle from a point on the edge to its center. Construct a rectangle using the four intersection points of the two diameters and the circle. This construction is valid since all diameters of circles are congruent, pass through the center of the circle, and bisect each other into congruent radii.



Square: The diagonals of a square are congruent, bisect each other, and are perpendicular. Construct a circle. Using the **Line** tool, construct a diameter of the circle from a point on the edge to its center. Construct a line perpendicular to the diameter through the center of the circle. Construct a square by overlaying a quadrilateral with vertices at the intersections of the diameter and the perpendicular to the diameter with the circle. This construction is valid because of the properties of diameters of circles and perpendicular lines.

