NUMB3RS Activity: Regular Polygon Centroids
Episode: "Burn Rate"

Topic: Geometry, Points of Concurrency **Grade Level:** 9 - 10
Objective: Students will be able to find the centroid of regular polygons.
Time: 20 minutes
Materials: cardboard, scissors, and a TI-83 Plus/TI-84 Plus graphing calculator

Introduction

In "Burn Rate," when mail bombs kill a series of seemingly unrelated people, Don asks Charlie to help uncover the source and the link that connects the people. Don's team has determined the locations where the letter bombs were mailed, where the envelopes were purchased, a hardware store where some components were bought, etc. Using geo-profiling, Charlie is able to find the likely starting point where the bomber set out to buy his bomb materials.

Geo-profiling is an investigative technique used by law enforcement that uses the locations of connected crimes to determine the most probable area of offender residence. In this activity, students will do their own geo-profiling by finding the centroid of a polygon where the vertices of the polygon will represent the locations that Don's team has discovered. Four different methods will be described.

The extension of this activity explores other lesser known points of concurrency, two of which do not appear in most traditional geometry textbooks.

Discuss with Students

A *centroid* is defined as the center of mass of an object, or the point upon which a cutout of the solid would balance on a pin. It is not necessary for students to be familiar with centroids. Students may have found centroids of uniform plane figures in science class by drawing plumb lines from each vertex and observing the intersections. This activity focuses on finding the centroid mathematically.

Another term used in the activity is *midsegment*. For this activity, a midsegment will be defined as the line segment connecting midpoints on the sides of a polygon which are directly opposite of each other.

The order in which the methods are presented is done purposely. The first two deal with an iterative process of reducing an area into a smaller, similar shape upon which the described method must be repeated upon.

The last two methods determine the exact location of the centroid—depending upon the number of sides the regular polygon has.

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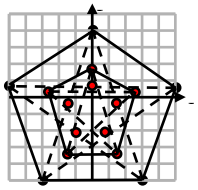
Student Page Answers:

1. *(3, 1.7)*

2.



Extension Page Answers:

1. *The four points are on top of each other.*

2. *circumcenter*

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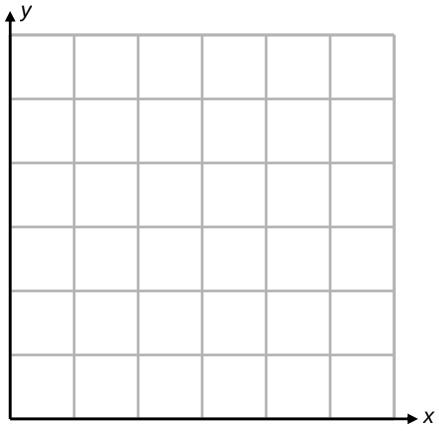
Name: _____ Date: _____

NUMB3RS Activity: Regular Polygon Centroids

In “Burn Rate,” when mail bombs kill a series of seemingly unrelated people, Don asks Charlie to help uncover the source and the link that connects the people. Don’s team has determined the locations where the letter bombs were mailed, where the envelopes were purchased, a hardware store where some components were bought, etc. Using geo-profiling, Charlie is able to find the likely starting point where the bomber set out to buy his bomb materials.

This likely starting point is known as the *centroid*—the center of mass of an object, or the point upon which a cutout of the solid would balance on a pin.

Suppose Don has determined that the locations associated with the mail bomber are (0, 0), (6, 0), and (3, 5.2) when laid out on a map.

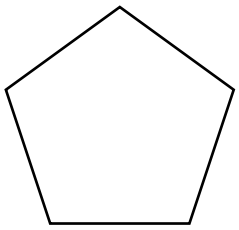


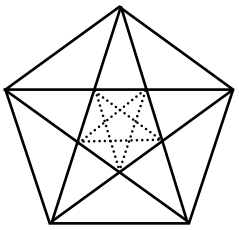
1. Plot the points and find the location of the centroid using the formula $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

This is one way to find the centroid. This activity explores other ways to find the centroid.

The Diagonal Method

The first method to explore is called the Diagonal Method. Trace the regular pentagon on Student Page 3 onto the supplied cardboard and cut the traced regular pentagon out. Each vertex on the pentagon represents a location on a map that Don’s team has identified the mail bomber used. To find the centroid of this polygon, draw all the diagonals. This results in a similar, smaller region that the mail bomber (centroid) resides in. To further narrow in the mail bomber’s location, repeat the process with this new region. This results in a still smaller region. Continue with this process (called an *iteration*) until the smaller region is the size of your finger tip. Test the validity of this location for the centroid by balancing the cardboard on your finger at this location.





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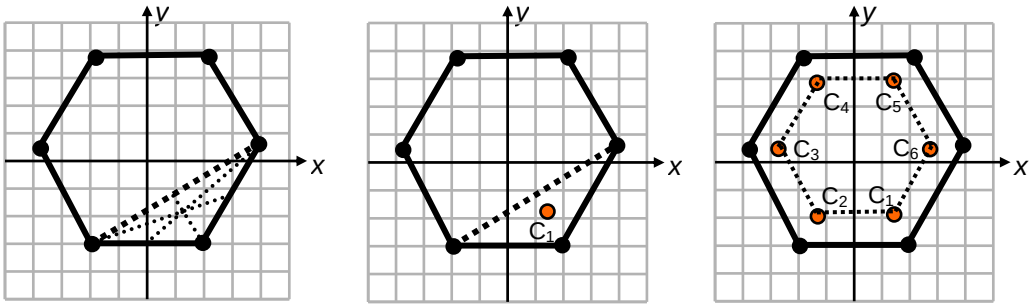
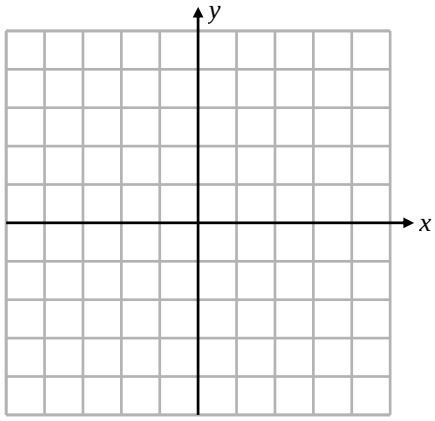
The Triangular Method

The second method to explore is called the triangular method. A regular hexagon is plotted on the left grid below. Using two consecutive sides, draw a third to form a triangle. Find the centroid of this triangle by finding the intersection of the three medians and mark it C_1. Continue this with the other sides. Connect points C_1 – C_6 to form a new hexagon, as shown on the grid on the right. The centroid is located somewhere within this new region. The process is to continue with more iterations until the centroid is found.



2. Plot the following points on the grid below: (0, 4), (3, –5), (–3, –5), (5, 0.6), (–5, –0.6). Find the centroid of the pentagon using the triangular method.



The Midsegment Method (even number of sides)

To find the centroid of an object with an even number of sides, draw the midsegments for opposite sides. These midsegments will all intersect at the centroid. Try this with the regular hexagon on the next page.

The Median Method (odd number of sides)

To find the centroid of an object with an odd number of sides, draw the line segments from a vertex to the midpoint of the side directly opposite. These "medians" will all intersect at the centroid. Try this with the regular pentagon on the next page.

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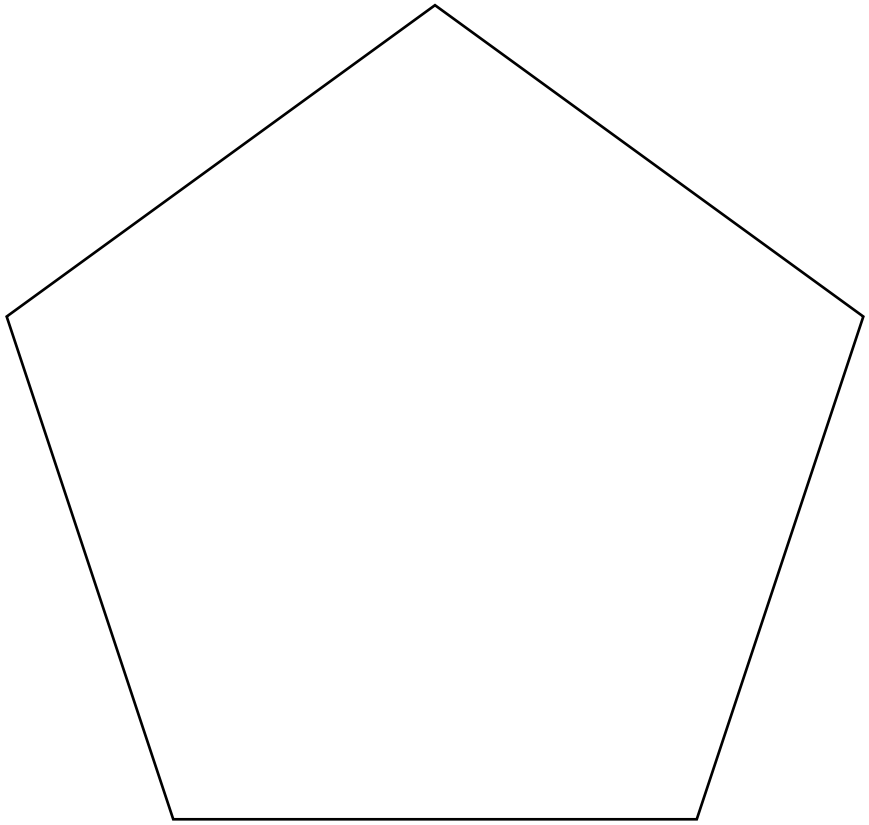
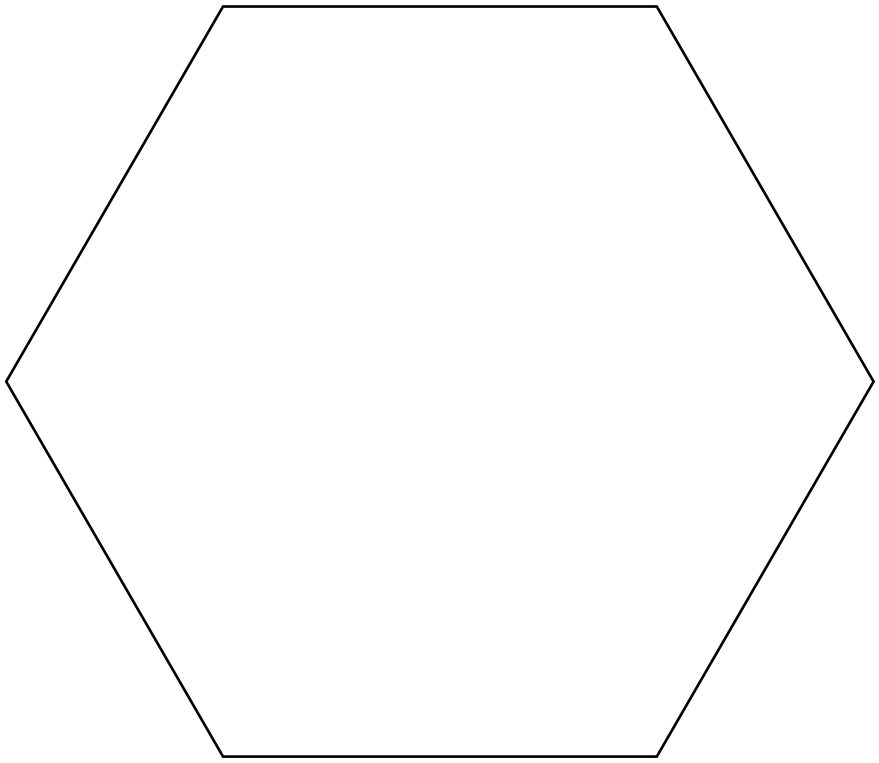




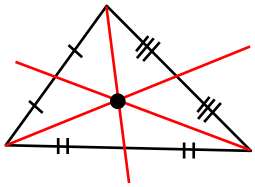
The goal of this activity is to give your students a short and simple snapshot into a very extensive mathematical topic. TI and NCTM encourage you and your students to learn more about this topic using the extensions provided below and through your own independent research.

Extension: Points of Concurrency

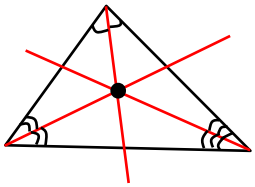
Introduction

The centroid is but one of a class of points known as points of concurrency in triangles. The others are:

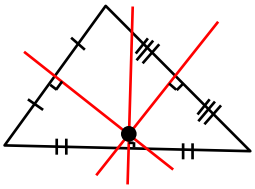
- **Incenter** – Found at the intersection of the three angle bisectors
- **Circumcenter** – Found at the intersection of the three perpendicular bisectors
- **Orthocenter** – Found at the intersection of the three altitudes
- **Nagel Point** – Found at the intersection of the three segments connecting each vertex to its semi-perimeter point
- **Fermat Point** – the point that minimizes the sum of the distances from it to the vertices



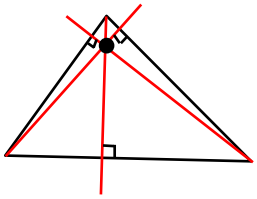
Centroid



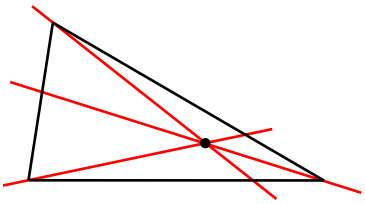
Incenter



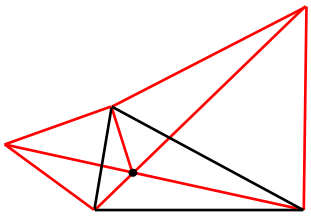
Circumcenter



Orthocenter



Nagel Point



Fermat Point

1. The Euler line is the line formed by the circumcenter, orthocenter and centroid. Why is there no Euler line for regular polygons?
2. In an isosceles right triangle, which point of concurrency would lie on the midpoint of the hypotenuse?

Additional Resources

To learn more about Geo-profiling, visit the Environmental Criminology Research Web site at: **http://www.ecricanada.com/index.html**

Explore an excellent use of points of concurrency to solve real world problems at the Web site:
http://math.kennesaw.edu/mathed/NMMC/Points%20of%20Concurrency%20in%20Triangles%2010-08-06%20Student.doc

An animated proof of the Nagel point can be found at the Web site:
http://agutie.homestead.com/files/nagel_point1.htm

Alternative diagrams for these and other points of concurrency can be found at the Web site: **http://agutie.homestead.com/files/geometry_help_online.htm**

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