

Winter Conics

ID: 10282

Time required
45 minutes

Activity Overview

Students will explore conic graphing using the polar notation equation $r = \frac{ed}{1 \pm e \cos(\theta)}$.

Specifically, students will explore the effects the various variables have on the resulting graph of the conic.

Topic: Parametric Equations & Polar Coordinates

- Graph the equation of any conic expressed in parametric form and identify its properties.

Teacher Preparation and Notes

- Students should already be familiar with graphing equations in polar notation.
- This activity is designed to be teacher-led.
- Information for an optional extension is provided at the end of the activity. Should you not wish students to complete the extension, you may delete Problem 4 of the student TI-Nspire document and have students disregard that portion of the student worksheet.
- **To download the student TI-Nspire document (.tns file) and student worksheet, go to education.ti.com/exchange and enter "10282" in the quick search box.**

Associated Materials

- *WinterConics_Student.doc*
- *WinterConics.tns*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *Eccentricity of Polar Equations of Conics (TI-Nspire CAS technology)* — 10166
- *Conic Sections (TI-84 Plus and TI-Navigator)* — 1909
- *Conics in Standard and Parametric Form (TI-84 Plus)* — 6644
- *Conics in Geometry (TI-84 Plus)* — 1281

Conic equations in polar notation

A conic is defined as the locus of points in a plane whose distance from a fixed point (focus) and a fixed line (directrix) is a constant ratio. This ratio is called the eccentricity, e , of the conic. The polar notation for the ellipse, hyperbola, and parabola is given by the equation:

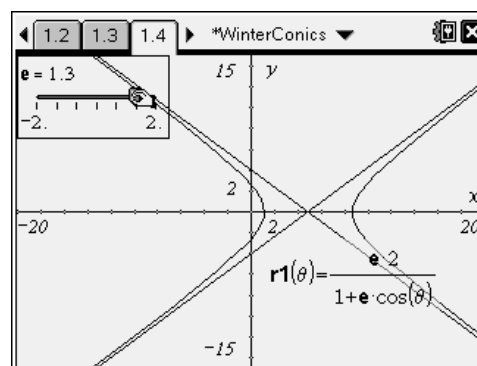
$$r = \frac{ed}{1 \pm e \cos(\theta)}, \text{ or } r = \frac{ed}{1 \pm e \sin(\theta)}$$

where e is the eccentricity and d is the distance from the origin to the directrix.

Which conic is it?

Students will explore how the eccentricity affects the graph of a conic section in polar coordinates. They should drag the pointer on the slider to change the value of e and observe what happens to the graph.

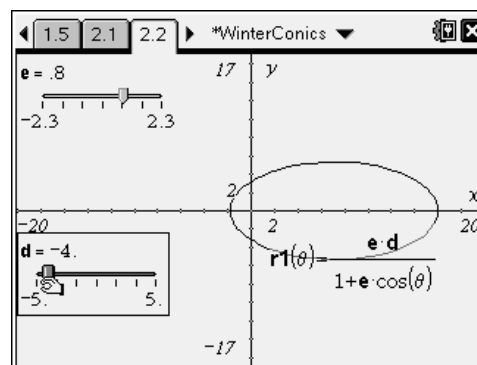
- Parabola: $e = 1$
- Hyperbola: $|e| > 1$
- Ellipse: $|e| < 1$



The d variable

Next, students will explore how the distance from the conic section to the directrix affects the graph. On page 2.2, they will drag the slider that controls the variable d in the equation. To see how d affects different conic sections, students should also change the value of e and then change the value of d again.

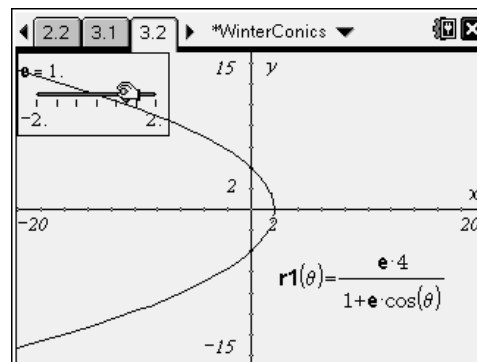
- The d variable controls the width of the conic sections. In addition to the width, the d variable also translates the hyperbola left and right.



The other stuff

Students will observe how other changes with the equation will affect the graph. Students should click on the equation on page 3.2 twice to edit the equation. They should observe what happens when the plus sign becomes a minus sign and what happens when cosine becomes sine.

- The plus/minus sign reflects the graph about the y -axis. Using cosine results in the graph having a horizontal orientation, while sine results in a vertical orientation.

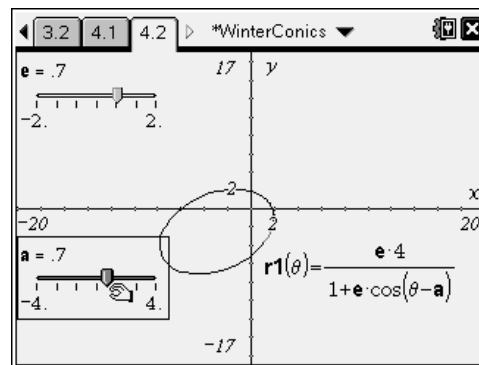


Extension

Students will explore what happens to the graph of a conic section when a phase shift is added to the cosine part of the equation. We can represent this situation by the following equation:

$$r = \frac{ed}{1 \pm e \cos(\theta - a)}$$

- The a variable rotates the conic about the origin. It needs to be noted that this variable's unit is radians, not degrees. This can also be thought of as rotating the polar axis.



Solutions – Student Worksheet Exercises

Determine the conic section for each equation listed below

1. Parabola
2. Hyperbola
3. Ellipse