Estimating a Population Proportion

ID: 9986

Time required
40 minutes

Activity Overview

*In this activity, students find the confidence interval for a population proportion by first finding the critical value and the margin of error. They confirm their answers by using the **1-Prop z Interval** command found in the Statistics menu. They find confidence intervals for real-life scenarios and use those intervals to make a judgment about a claim. Finally, they use two formulas for finding the required sample size for a survey, given a confidence interval and margin of error. The first formula assumes no estimate of the sample proportion, while the second one does. An optional extension allows students to compare the formulas and explain the derivation of the first from the second.*

Topic: Sampling Distributions

- *Use the fact that the sampling distribution of a proportion $\hat{p}$ (for samples of size n, n > 30) is approximately normally distributed with mean p and standard deviation $\sqrt{\frac{p(1-p)}{n}}$ to estimate the proportion in a population.*
- *Use the fact that the sampling distribution of a proportion $\hat{p}$ (for samples of size n, n > 30) is approximately normally distribution with mean p and standard deviation $\sqrt{\frac{p(1-p)}{n}}$ to calculate a confidence interval for an estimate of p.*

Teacher Preparation

- *Students should already be familiar the concepts of margin of error and confidence intervals. They should also be familiar with the binomial distribution and its requirements.*
- *Using a confidence interval to make a decision, as done in Problem 2, is a precursor to hypothesis testing.*
- ***To download the student and solution TI-Nspire documents (.tns files) and student worksheet, go to education.ti.com/exchange and enter "9986" in the keyword search box.***

Associated Materials

- *Estimating_Population_Proportions_student.doc*
- *Estimating_Population_Proportions.tns*
- *Estimating_Population_Proportions_Soln.tns*

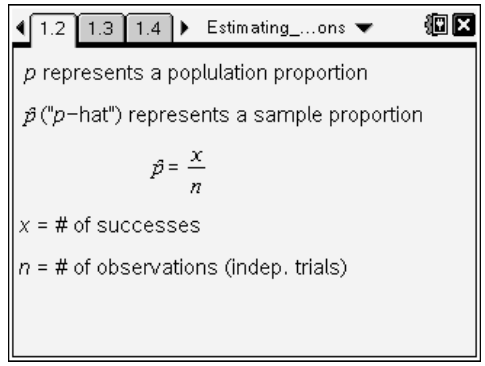
Suggested Related Activities

To download any TI-Nspire technology activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Difference Between Two Proportions — 10082*
- *Testing Claims About Proportions — 10131*
- *Claims About Two Proportions — 10259*

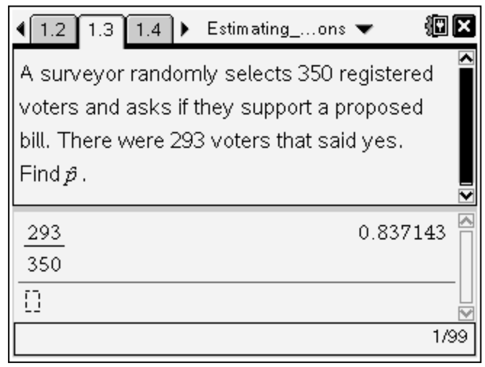
Problem 1 – Margin of Error and a Confidence Interval

Step 1: Introduce the concept of using $\hat{p}$ to estimate p and how it relates to a binomial distribution. Provided certain conditions are met, the normal distribution can be used to approximate the binomial distribution.



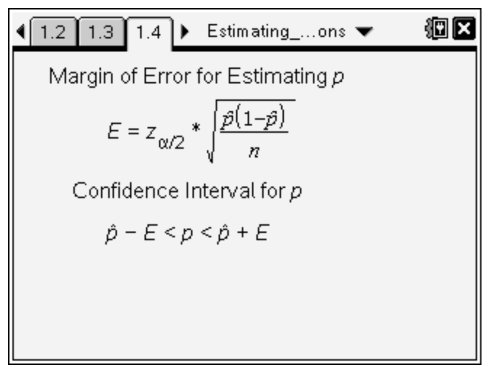
Step 2: Students are to read page 1.3 and find the sample proportion by dividing.

The sample proportion is about 0.837.



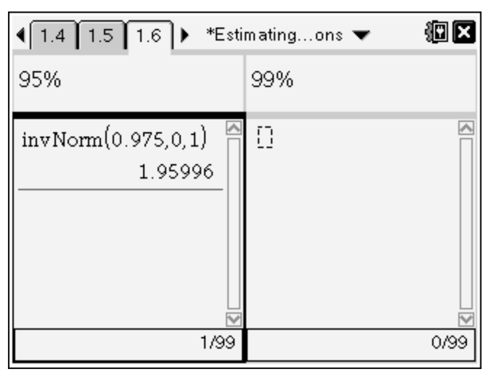
Step 3: Formulas for the margin of error and confidence interval are shown on page 1.4. Note that some books will use $\hat{q}$ in place of $1-\hat{p}$.

The confidence interval for p could also be written as $\hat{p} \pm E$ or $\left(\hat{p}-E, \hat{p}+E\right)$.



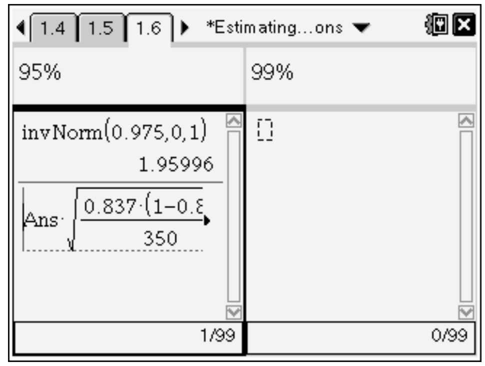
Step 4: On page 1.6, students are to find the critical value, the margin of error, and the intervals at both the 95% and 99% level.

Critical values are found by using **InvNorm** to find the area to the left of that value. The command can be typed, chosen from the Catalog, 📖, or found by pressing **MENU > Statistics > Distributions > Inverse Normal**. For a 95% interval, 5% is in both tails or 2.5% is in one tail, so 97.5% is to the left of the positive critical value.



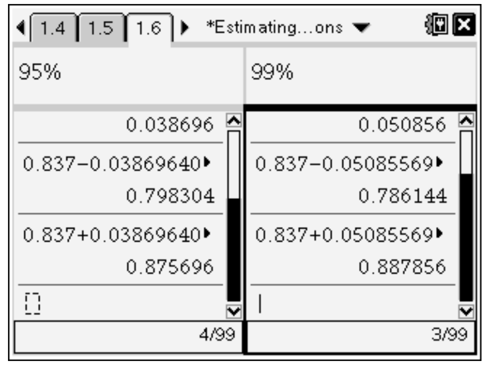
Step 5: When finding the margin of error, students can round the sample proportion to 0.837.

The margin of error is about 3.9%. Students must subtract and add this to the sample proportion: 83.7% or 0.837.



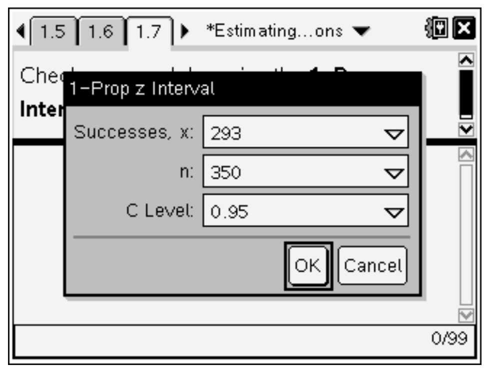
When repeating the process for the 99% interval, students will use an area of 0.995 to the left of the critical value.

This margin of error is about 5.1%. Students should explain why it makes sense that this margin of error is larger than the margin of error at the 95% level.

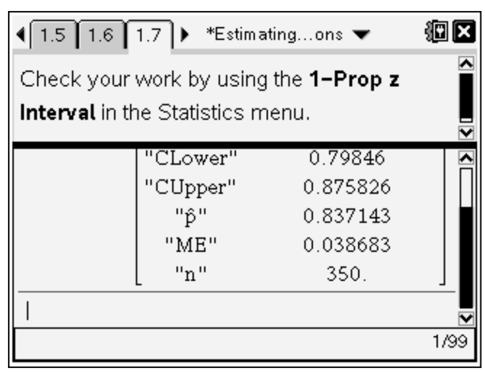


Students should state their findings in complete sentences. For example, *The percentage of voters that support the bill is about 83.7%, with a margin of error of plus or minus 3.9%*. Or, *We are 99% confident that the true percentage of voters that support the bill is between 78.6% and 88.8%.*

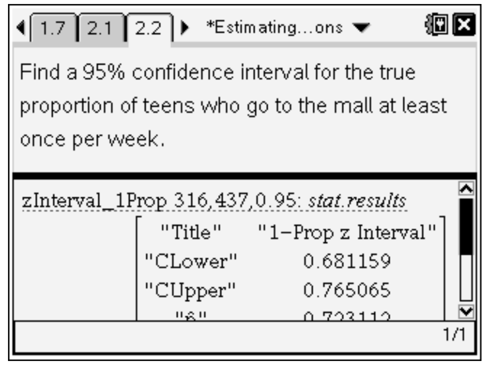
Step 6: The handheld allows this confidence interval to be found without having to first find the critical value and margin of error. On page 1.7, students can check their work by pressing **MENU > Statistics > Confidence Intervals > 1-Prop z Interval**. Then, they enter *x*, *n*, and the confidence level (C Level).



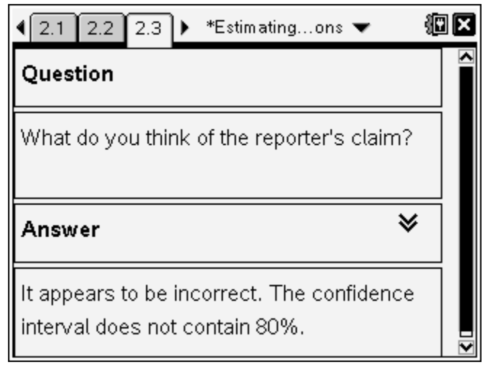
The display shows lower and upper bounds of the interval (answers are slightly different than what was found before because we rounded $\hat{p}$), $\hat{p}$, and the margin of error.



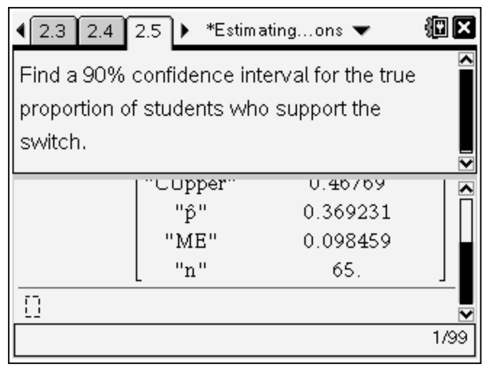
Problem 2 – Practice Problems



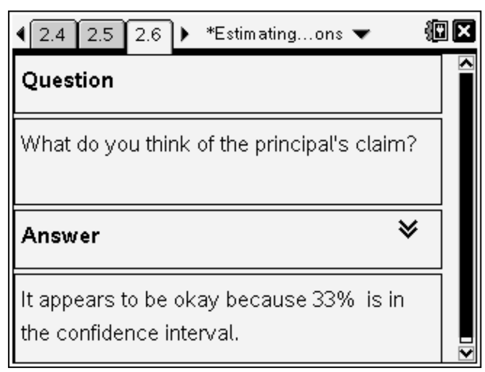
Step 1: Students are to read the scenario posed on page 2.1 and then find the 95% confidence interval on page 2.2.



They should discuss their answer to the question posed on page 2.3. Students should be wary of the reporter's claim since 80% is not within the confidence interval.



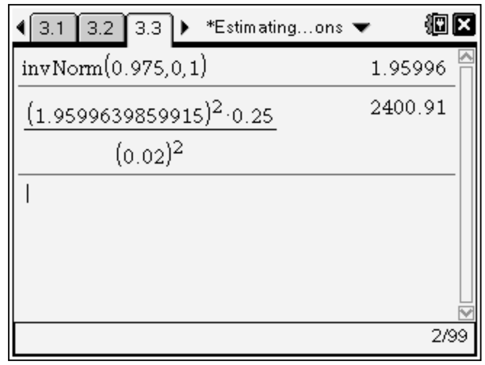
Step 2: Students are to read the scenario posed on page 2.4 and then find the 90% confidence interval on page 2.5.



They can discuss their answer to the question posed on page 2.6. Students should say that the principal's claim appears to be correct, or that there is no reason to tell the principal that he or she is incorrect, because the claimed percentage is within the confidence interval.

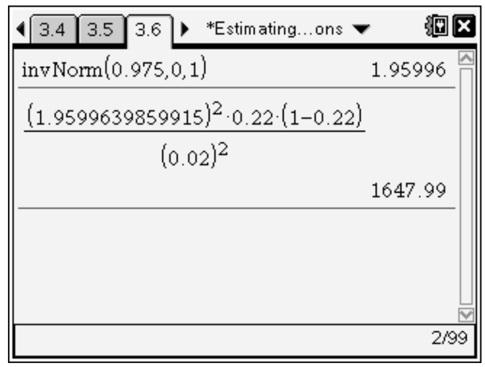
Students should look at and discuss the differences in the margins of error for the two problems and tell why the second is so much larger than the first (consider sample size).

Problem 3 – Sample Size



Step 1: Introduce the formula on page 3.1 as the formula used for estimating the sample size that must be taken to estimate a population proportion with a given margin of error. Students are to use the formula to answer the question on page 3.2.

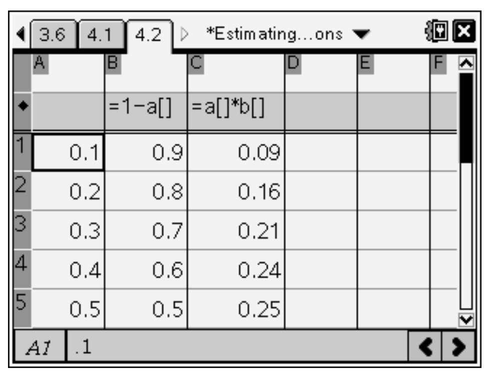
The answer of 2400.91 should be rounded up to 2401. Ask students why the sample size should *always* be rounded *up* (The larger the sample size, the smaller the margin of error is. A maximum margin of error is given.).



Step 2: Introduce the formula on page 3.4 as the formula used for estimating the sample size when there is an estimate of the sample proportion. (An estimate might be known due to previous studies or expert opinion.) Students are to use the formula to answer the question on page 3.5.

Discuss why the sample size needed for this situation is less than the sample size when an estimate was not given (an estimate for success is now given, now $\hat{p}\left(1-\hat{p}\right)$ is less than 0.25).

Problem 4 – Extension



Students are to use the spreadsheet to find various products of $\hat{p}$ and $1-\hat{p}$. They should explain how the two formulas for sample size are related and why 0.25 replaces $\hat{p}\left(1-\hat{p}\right)$ and why.

(The maximum value of the product $\hat{p}\left(1-\hat{p}\right)$ is 0.25. It gives the greatest value of *n* that could be needed. It is always safer to survey more people than needed, rather than not enough. Students may think then that the first formula should always be used. Remind them of the practical limitations of time and money.)

