Interesting Properties of Cubic Functions to Discover Graphically or Numerically and then Prove with the Power of the TI-89.

1. Consider any point P on the graph of the function $f(x) = x^3$. The tangent line at P will intersect the curve again at point Q. What is the relationship between:
   a) the *x* coordinates of P and Q?
   b) the slopes of the curve at P and at Q?
   c) the *y* intercepts of the tangent lines at P and Q?

2. The graph of any cubic function is symmetric with respect to its point of inflection.
   a) Find the *x* and *y* coordinates of the point of inflection for the general cubic polynomial function $f(x) = ax^3 + bx^2 + cx + d$
   b) Let *g* be the cubic function created by translating *f* so the point of inflection is at the origin. Establish symmetry exists by proving the function g is odd.

3. Consider a cubic polynomial function with three distinct real zeros. Where does the tangent line drawn to curve at the average of two of the three zeros intersect the curve?
   a) Does this property seem to hold no matter which two zeros you average?
   b) Is this property true for cubic functions with only one or two distinct zeros?

4. Consider the points P and Q defined in problem #1 for $f(x) = x^3$ and the region enclosed by the segment PQ and the graph of the cubic function. Divide this region into two parts using a line segment drawn from Q to the origin.
   a) Find the ratio of the areas of these two regions.
   b) Let R be the point of inflection for any general cubic polynomial function. (For part a, the origin was R.) Show the segment QR divides the region between the tangent at P and the graph of the cubic in the same ratio found in part a.

5. Consider the points P and Q defined in problem #1 for $f(x) = x^3$ and the region enclosed by the segment PQ and the graph of the cubic function. The tangent at Q will intersect the **curve** at point S. A second region lies between the curve at the line segment QS.
   a) Find the ,ratios of the areas of these two regions.
   b) Is this ratio constant for any cubic polynomial, or just for $f(x) = x^3$?

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