Nuclear Binding Energy – ID: 9521

By Charles W. Eaker

Time required
45 minutes

Activity Overview

In this activity, students explore the energy that results from the strong force in nuclei. The activity begins with the unexpected result that the mass of a nucleus is less than the sum of the masses of the protons and neutrons in the nucleus. Students use the equation $E = mc^2$ to calculate the energy equivalent of mass. In part 2 of the activity, students analyze a graph to determine the ratio of neutrons to protons in stable nuclei. Then, they investigate how fission and fusion of nuclei can be accounted for in terms of the binding energy per nucleon.

Concepts

- *Nuclear binding energy*
- *Ratio of neutrons to protons in stable nuclei*
- *Fission and fusion of nuclei*

Materials

To complete this activity, each student will require the following:

- *TI-Nspire™ technology*
- *pen or pencil*
- *blank sheet of paper*

TI-Nspire Applications

Calculator, Graphs & Geometry, Notes, Lists & Spreadsheet, Data & Statistics

Teacher Preparation

Before carrying out this activity, review the relationships between protons, neutrons, atomic number, and mass number. Go over nuclear symbols, such as He-4 and $^{4}_{2}He$. Distinguish between mass number, atomic mass, and nuclear mass (all the masses given in this activity are for the nuclei, not the atoms). Review the atomic mass unit and Einstein's equation $E=mc^2$.

- *The screenshots on pages 2–7 demonstrate expected student results. Refer to the screenshots on pages 8–10 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the .tns file, go to education.ti.com/exchange and enter "9521" in the search box.***

Classroom Management

- *This activity is designed to be **teacher-led** with students following along on their handhelds. You may use the following pages to present the material to the class and encourage discussion. Note that the majority of the ideas and concepts are presented only in **this** document, so you should make sure to cover all the material necessary for students to comprehend the concepts.*
- *Students may answer the questions posed in the .tns file using the Notes application or on blank paper.*
- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

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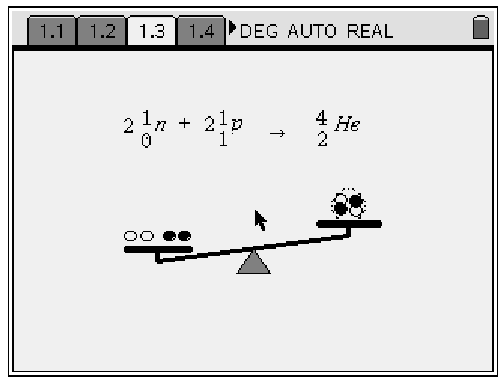
The following questions will guide student exploration in this activity:

- Is mass conserved in nuclear reactions?
- What is the ratio of neutrons to protons in stable nuclei?
- How is it that both fusion (combining nuclei) and fission (splitting nuclei) release energy?

Students will calculate the mass loss in the formation of nuclei and relate this mass loss to nuclear binding energy. Students will construct graphs of neutrons vs. protons for stable nuclei and of binding energy per nucleon vs. mass number. They will then calculate the mass loss and energy released in nuclear fission and fusion reactions.

Problem 1 – Nuclear binding energy

Step 1: Students should open the file **PhyAct23_nuclearbinding_EN.tns** and read the first two pages. Page 1.3 illustrates that the He-4 nucleus has less mass than the sum of the masses of the neutrons and protons that comprise it. Students should examine page 1.3 and then answer questions 1 and 2.



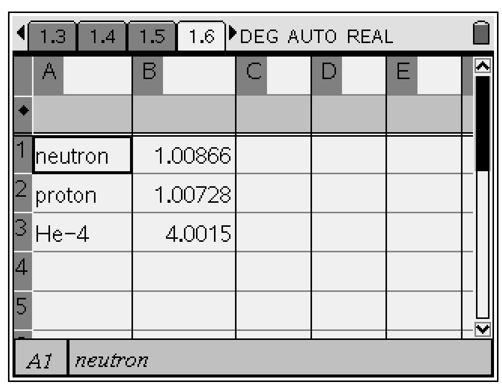
Q1. What is surprising about the mass of the He-4 nucleus compared with the masses of the neutrons and protons that comprise the nucleus?

A. *The mass of the nucleus is less than the sum of the masses of the neutrons and protons.*

Q2. Is mass conserved in this nuclear reaction?

A. *No. Mass is lost.*

Step 2: Next, students should read the text on page 1.5 and then examine the data on page 1.6, which gives the mass in amu of a neutron, a proton, and a He-4 nucleus. After students have examined the data on page 1.6, they should answer question 3.



Q3. How much mass is lost in the formation of a He-4 nucleus from two neutrons and two protons?

A. *To answer this question, subtract the actual mass of the He-4 nucleus from the total mass of two protons and two neutrons, as shown below:*
mass loss = 2(1.00866) + 2(1.00728) – 4.0015
= 0.03038 amu

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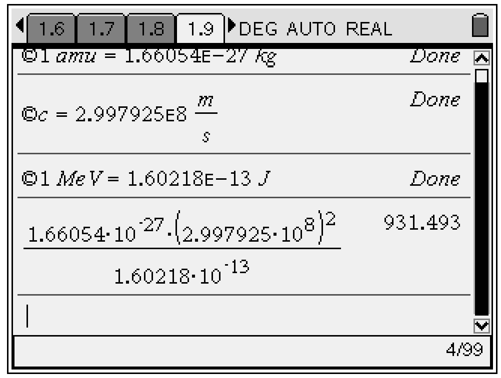
Step 3: Next, students should read the text on page 1.8 and then move on to the *Calculator* application on page 1.9. The calculator application should already be displaying the values of the various constants. Students should use the equation $E = mc^2$ to calculate the energy equivalent of 1 amu in MeV. Then, they should answer questions 4 and 5. Finally, they should read the text on page 1.11.



Q4. What is the energy equivalent in MeV of 1 amu?

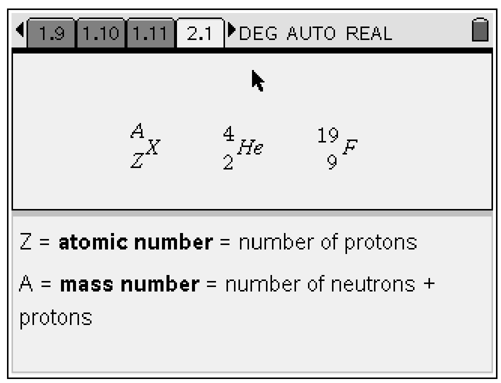
A. *931.49 MeV*

Q5. The amount of energy released in the formation of a He-4 nucleus is 28.3 MeV. Does this value agree with the mass loss you calculated in question 3?

A. $0.03038\ \cancel{amu} \bullet \frac{931.49\ MeV}{1\ \cancel{amu}} = 28.3\ MeV$ *; yes, the values agree.*

Problem 2 – Neutrons and protons in stable nuclei

Step 1: Next, students should move to page 2.1, which shows the symbols used to indicate the atomic number and mass number of nuclei. The mass number is equal to the number of neutrons and protons in the nucleus. The atomic number is equal to the number of protons. Student should use this information to answer questions 6–8.



Q6. What is the mass number of F-19?

A. *19*

Q7. How many neutrons are in an F-19 nucleus?

A. *10*

Q8. Write an equation for the number of neutrons in terms of *A* and *Z*.

A. *number of neutrons = A – Z*

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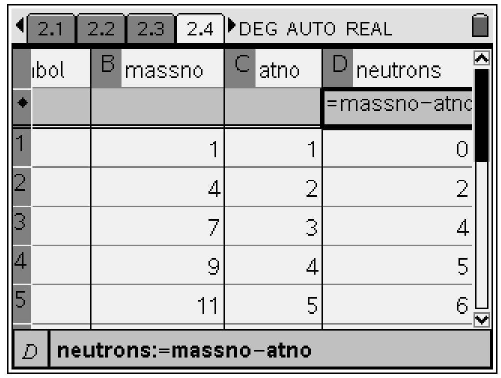
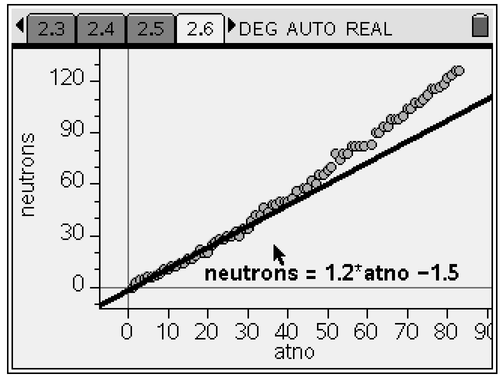
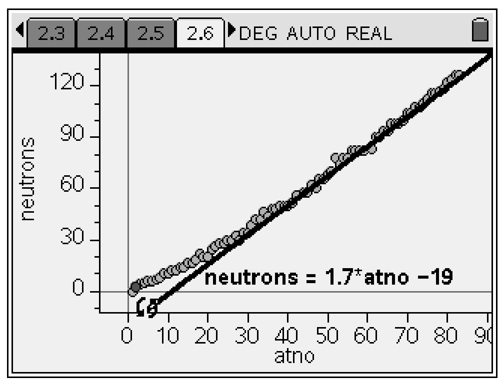
Step 2: Next, students should read the text on page 2.3 and then move to page 2.4. The *Lists & Spreadsheet* application on page 2.4 contains data on the number of protons and neutrons in 81 assorted stable nuclei. Students should enter the equation they derived in question 8 into column D of the spreadsheet.



Step 3: Next, students should read the text on page 2.5 and then make a plot of **neutrons** vs. **atno** on the *Data & Statistics* application on page 2.6. Then, students should use the **Movable Line** tool **(Menu > Actions > Add Movable Line)** to determine the ratio of neutrons to protons for stable nuclei in two regions: (1) light nuclei ($Z < 20$) and (2) heavy nuclei ($Z > 60$). Students should then answer questions 9 and 10 using this information.





Q9. For the stable nuclei, what can you say about the ratio of neutrons to protons as Z increases?

A. *As Z increases, the ratio of neutrons to protons increases.*

Q10. Explain this trend by taking into account the strong nuclear force and the electric force.

A. *As Z increases, the number of protons increases. The increase in the number of protons produces greater electrical forces of repulsion. The additional neutrons increase the strong nuclear forces, which counteract the increased electrical repulsion of the protons. As the number of protons increases, a higher ratio of neutrons to protons is needed to offset the electrostatic repulsion between the protons. This is why the slope of the line increases with atomic number.*

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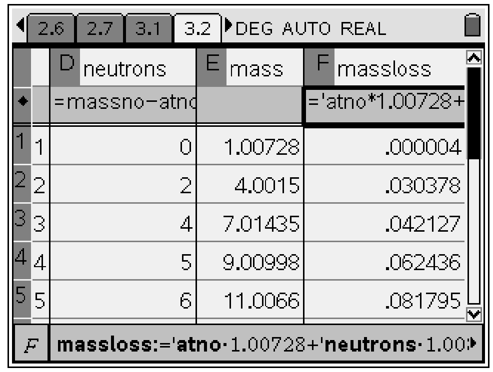
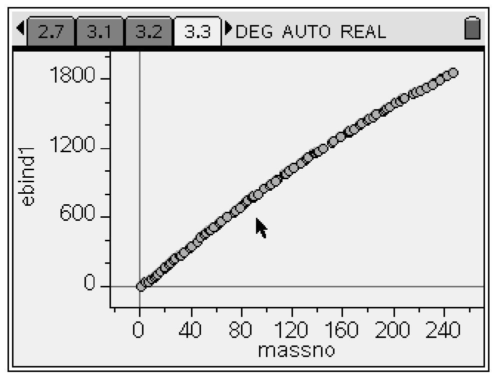
Problem 3 – Fission and fusion

Step 1: Next, students should read the text on page 3.1 before moving to page 3.2. The *Lists & Spreadsheet* application on page 3.2 is similar to the one on page 2.4, but it includes radioactive nuclei in addition to stable nuclei. Students should enter equations into the spreadsheet to calculate the mass loss and binding energy for all the listed nuclei. In column F, students should enter the equation **massloss** = **atno** · 1.00728 + **neutrons** · 1.00866 – **mass**. In column G, students should enter **ebind1** = **massloss** · 931.49.



Step 2: Next, students should make a scatter plot of **ebind1** vs. **massno** on the *Data & Statistics* application on page 3.3. After examining the graph, students should answer questions 11 and 12.



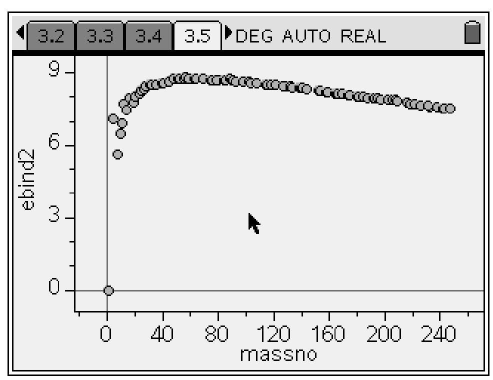
Q11. Make a general statement about the relationship between binding energy and mass number.

A. *Binding energy increases as mass number increases.*

Q12. A nucleon is a proton or neutron. What is the relationship between number of nucleons and mass number?

A. *The number of nucleons equals the mass number.*

Step 3: Next, students should go back to the *Lists & Spreadsheet* application on page 3.2. They should enter the equation for the binding energy per nucleon, **ebind2** = **ebind1**/**massno**, into the spreadsheet in column H. Students should then use the *Data & Statistics* application on page 3.5 to make a scatter plot of **ebind2** vs. **massno**. Students should use this scatter plot to answer questions 13–16.



Q13. Which nucleus has the greatest binding energy per nucleon?

A. *Fe-56*

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Q14. Which light nucleus has an unusually high binding energy per nucleon?

A. *He-4*

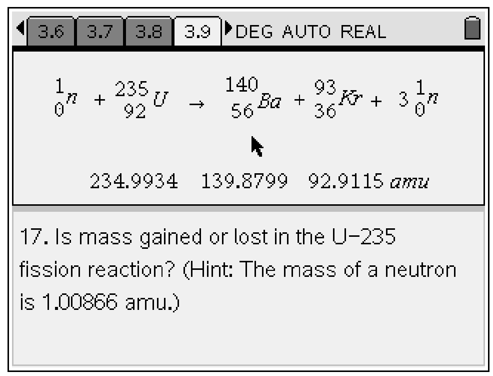
Q15. Make a general statement about the relationship between binding energy per nucleon and mass number for light nuclei.

A. *The binding energy per nucleon increases with increasing mass number for light nuclei.*

Q16. Make a general statement about the relationship between binding energy per nucleon and mass number for heavy nuclei.

A. *The binding energy per nucleon decreases with increasing mass number for heavy nuclei.*

Step 4: Next, students should move to page 3.8 and read the text there. They should then examine the information on page 3.9, which shows a nuclear fission reaction for the fission of U-235. Students should use the information on page 3.9 to answer questions 17–19.



Q17. Is mass gained or lost in the U-235 fission reaction? (Hint: The mass of a neutron is 1.00866 amu.)

A. *Mass is lost. The mass of one neutron plus one U-235 nucleus is greater than the mass of three neutrons plus one Ba-140 nucleus plus one Kr-93 nucleus.*

Q18. What is the change in mass and energy in MeV for the U-235 fission reaction?

A. *mass loss = 236.0021 − 235.8174 = 0.1847 amu*
energy loss = 0.1847 · 931.49 = 172.0 MeV

Q19. Is energy absorbed or released in the fission reaction?

A. *Because mass is lost, energy must be released.*

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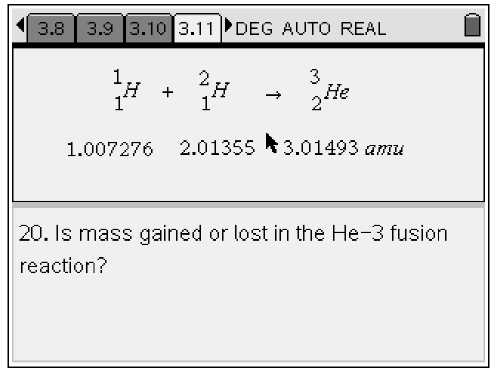
Step 5: Next, students should move to page 3.11, which shows a nuclear fusion reaction. They should use the information on page 3.11 to answer questions 20–23.



Q20. Is mass gained or lost in the He-3 fusion reaction?

A. *Mass is lost. The mass of one He-3 nucleus is less than the total mass of one H-1 nucleus and one H-2 nucleus.*

Q21. What is the change in mass and energy in MeV for the He-3 fusion reaction?

A. *mass loss = 3.02083 − 3.01493 = 0.0059 amu*
energy loss = 0.0059 · 931.49 = 5.5 MeV

Q22. Is energy absorbed or released in the fusion reaction?

A. *Because mass is lost, energy must be released.*

Q23. Would energy be released in the fusion of heavy nuclei? Explain your answer.

A. *No, energy would not be released. Fusing heavy nuclei would produce a nucleus with a lower binding energy per nucleon than those of the nuclei that fused. Therefore, the process would require energy. The mass would be greater than the sum of the masses of the original nuclei.*

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Nuclear Binding Energy – ID: 9521

(Student)TI-Nspire File: *PhyAct23_nuclearbinding_EN.tns*

1.1

NUCLEAR BINDING ENERGIES

Physics

Nuclear Physics

1.2

The nuclei of atoms contain protons and neutrons. The next page illustrates the formation of a helium−4 nucleus from two neutrons and two protons. Compare the masses of the nucleons (protons and neutrons) and the He−4 nucleus.

1.3

$$2\,{}^{1}_{0}n + 2\,{}^{1}_{1}p \rightarrow {}^{4}_{2}He$$

1.4

1. What is surprising about the mass of the He−4 nucleus compared with the masses of the neutrons and protons that comprise the nucleus?

2. Is mass conserved in this nuclear reaction?

1.5

Masses of atoms and nuclei are often expressed in atomic mass units, amu.

1 amu = 1.66054×10^{-27} kg

The following table gives the masses in amu of a neutron, a proton, and a He−4 nucleus.

1.6

	A	B	C	D	E
1	neutron	1.00866			
2	proton	1.00728			
3	He−4	4.0015			
4					
5					

A1 neutron

1.7

3. How much mass is lost in the formation of a He−4 nucleus from two neutrons and two protons?

1.8

Accompanying the loss of mass in a nuclear reaction is the release of energy. One of the most famous equations is Einstein's relationship between mass and energy, $E = mc^2$. On the next page, determine the energy equivalent in MeV of 1 amu of mass. (Hint: 1 J = 1 kg·m^2/s^2)

1.9

©1 *amu* = 1.66054E−27 *kg* — Done

©*c* = 2.997925E8 $\frac{m}{s}$ — Done

©1 *MeV* = 1.60218E−13 *J* — Done

3/99

1.10

4. What is the energy equivalent in MeV of 1 amu?

5. The amount of energy released in the formation of a He−4 nucleus is 28.3 MeV. Does this value agree with the mass loss you calculated in question 3?

1.11

The energy released in the formation of nuclei from protons and neutrons is known as the nuclear binding energy. It is due to the strong nuclear force between protons and neutrons that are within 10^{-15} m.

2.1

$${}^{A}_{Z}X \quad {}^{4}_{2}He \quad {}^{19}_{9}F$$

Z = **atomic number** = number of protons

A = **mass number** = number of neutrons + protons

1.10 1.11 2.1 2.2 DEG AUTO REAL

6. What is the mass number of F−19?

7. How many neutrons are in an F−19 nucleus?

8. Write an equation for the number of neutrons in terms of *A* and *Z*.

1.11 2.1 2.2 2.3 DEG AUTO REAL

The following page contains a representative list of 81 of the 266 stable (non−radioactive) nuclei. Enter an equation in column D for calculating the number of neutrons for each nucleus in the table.

2.1 2.2 2.3 2.4 DEG AUTO REAL

	A symbol	B massno	C atno	D neut
◆				
1	H−1	1	1	
2	He−4	4	2	
3	Li−7	7	3	
4	Be−9	9	4	
5	B−11	11	5	

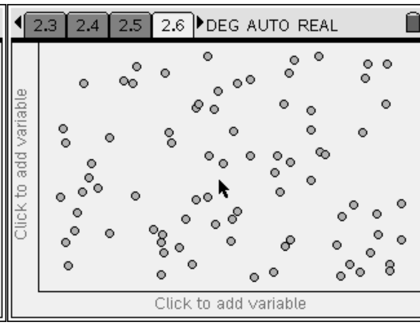
A1 "H−1"

2.2 2.3 2.4 2.5 DEG AUTO REAL

On the next page, make a plot of **neutrons** vs. **atno**. Then, use the **Movable Line** tool to find the slope of the graph in two regions:

(1) light nuclei Z < 20

(2) heavy nuclei Z > 60



2.4 2.5 2.6 2.7 DEG AUTO REAL

9. For the stable nuclei, what can you say about the ratio of neutrons to protons as *Z* increases?

10. Explain this trend by taking into account the strong nuclear force and the electric force.

2.5 2.6 2.7 3.1 DEG AUTO REAL

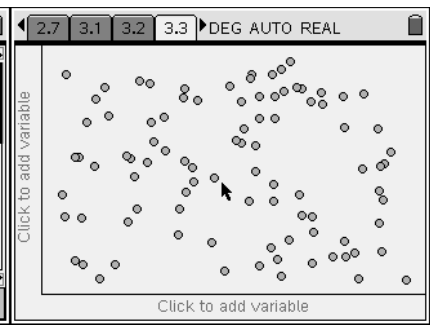
The spreadsheet on the next page lists the nuclear masses for a number of nuclei. This list includes some radioactive nuclei with *Z* > 83. In column F, enter an equation to calculate the mass loss of each nucleus in the table. In column G, enter an equation for binding energy in MeV.

(1 amu = 931.49 MeV)

2.6 2.7 3.1 3.2 DEG AUTO REAL

	A symbol	B massno	C atno	D neut
◆				=mass
1	H−1	1	1	
2	He−4	4	2	
3	Li−7	7	3	
4	Be−9	9	4	
5	B−11	11	5	

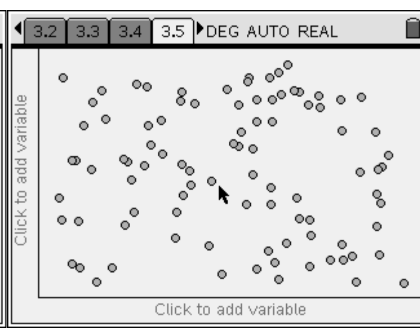
A1 "H−1"



3.1 3.2 3.3 3.4 DEG AUTO REAL

11. Make a general statement about the relationship between binding energy and mass number.

12. A nucleon is a proton or neutron. What is the relationship between number of nucleons and mass number?



3.3 3.4 3.5 3.6 DEG AUTO REAL

13. Which nucleus has the greatest binding energy per nucleon?

14. Which light nucleus has an unusually high binding energy per nucleon?

15. Make a general statement about the relationship between binding energy per nucleon and mass number for light nuclei.

16. Make a general statement about the relationship between binding energy per nucleon and mass number for heavy nuclei.

The graph on page 3.5 provides a basis for understanding why fission (splitting nuclei) releases energy for heavy nuclei and why fusion (joining nuclei) releases energy for light nuclei.

$$ {}_{0}^{1}n + {}_{92}^{235}U \rightarrow {}_{56}^{140}Ba + {}_{36}^{93}Kr + 3\,{}_{0}^{1}n $$

234.9934 139.8799 92.9115 *amu*

17. Is mass gained or lost in the U-235 fission reaction? (Hint: The mass of a neutron is 1.00866 amu.)

18. What is the change in mass and energy in MeV for the U-235 fission reaction?

19. Is energy absorbed or released in the fission reaction?

$$ {}_{1}^{1}H + {}_{1}^{2}H \rightarrow {}_{2}^{3}He $$

1.007276 2.01355 3.01493 *amu*

20. Is mass gained or lost in the He-3 fusion reaction?

21. What is the change in mass and energy in MeV for the fusion reaction of He-3?

22. Is energy absorbed or released in the fusion reaction?

23. Would energy be released in the fusion of heavy nuclei? Explain your answer.

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