Mathematical Methods (CAS) 2003 Examination 2

Note: To use Derive efficiently, students should be familiar with the 'tick plus equals' and 'tick plus approximately equals' evaluation buttons. These simultaneously 'author' and 'evaluate' expressions exactly and numerically. Students should also be familiar withthe use of defined functions in the form f(x):= rule of function.

Question 1

a. If X = length of a randomly selected metal rod. We need Pr(X> 141.5) = 1 - Pr(X<141.5), which we can easily get using the cumulative distribution function NORMAL.

#1: 1 - NORMAL(141.5, 140, 1.2)

#2: 0.1056497736

The answer is 0.106, correct to three decimal places.

b. Find d, where Pr(X<140-d)=0.075. This will require us to solve this equation numerically for d.

#3: NORMAL(140 - d, 140, 1.2) = 0.075

#4: NSOLVE(NORMAL(140 - d, 140, 1.2) = 0.075, d, Real)

#5: d = 1.727437768

Hence d=1.7, correct to one decimal place.

c. Y = number of rods in a sample of 12 with a size fault is a binomial random variable with n=12 and p=0.15. We want Pr(Y = 2), so can use the BINOMIAL_DENSITY function.

#6: BINOMIAL_DENSITY(2, 12, 0.15)

#7: 0.2923584904

The answer is 0.292, correct to three decimal places.

d. V = no of rods with a size fault in a sample of 12 taken from a box of 25 is a random variable with a hypergeometric distribution. We want Pr(V≥2) = 1-Pr(V≤1), and now we can use the HYPERGEOMETRIC_DISTRIBUTION function. Note that you can look up the Help menu to see the order of the parameters, but you need to know the form of a hypergeometric probability calculation to be able to interpret this correctly.

#8: 1 - HYPERGEOMETRIC_DISTRIBUTION(1, 12, 4, 25)

#9: 0.672173913

The answer is 0.672, correct to three decimal places.

e. i. The sum of the probabilities must equal 1.

#10: $k + 0.15 + 0.17 = 1$

#11: SOLVE(k + 0.15 + 0.17 = 1, k, Real)

#12: $$k = \frac{17}{25}$$

#13: $$k = 0.68$$

e. ii. The mean is found by multiplying the profit by probability and summing. If you then use 'tick plus equals', you can expect an answer in fractional form.

#14: $(x - 5)\cdot 0.68 + (x - 8)\cdot 0.17$

#15: $$\frac{17\cdot(5\cdot x - 28)}{100}$$

e. iii. Set the mean of Y equal to 0, and solve for x.

#16: $(x - 5)\cdot 0.68 + (x - 8)\cdot 0.17 = 0$

#17: SOLVE((x - 5)·0.68 + (x - 8)·0.17 = 0, x, Real)

#18: $$x = \frac{28}{5}$$

#19: $$x = 5.6$$

The answer is $5.60.

e. iv. The rods that are sold are the good rods and the rods with other faults. Hence, the proportion of rods sold that are good rods = proportion of good rods/proportion of rods sold.

#20: $\frac{0.68}{0.68 + 0.17}$

#21: $$0.8$$

Question 2

It may be useful to declare a function definition for this example.

#22: $f(x) :\!= 2 - 2\cdot COS\left(\frac{x}{2}\right)$

a. The ramp is highest when x=4. Hence evaluate f(4), numerically since answer is to be given to two decimal places and we are dealing with a trigonometric function.

#23: f(4)

#24: 2.832293673

Hence the answer is 2.83 metres.

b. First find the gradient of the ramp, by determining f'(x).

#25: f'(x)

#26: $SIN\left(\frac{x}{2}\right)$

Since the gradient is a sin function, with amplitude 1, it must always be less than or equal to 1.

c. i. The area will be the integral of f from -4 to +4, or by symmetry, twice the integral of f from 0 to 4. Note that we have defined f, so our answer should refer back to the original function given.

#27: $\int_{-4}^{4} \left(2 - 2\cdot COS\left(\frac{x}{2}\right)\right) dx$

c. ii. Now numerically evaluate this, since an answer correct to two decimal places is required.

#28: 8.725620585

d. i. The y-coordinate of A is 1. Solve (exactly) f(x)=1 to get the x-coordinate of A.

#29: f(x) = 1

#30: SOLVE(f(x) = 1, x, Real)

#31: $x = \frac{10\cdot\pi}{3} \vee x = -\frac{2\cdot\pi}{3} \vee x = \frac{2\cdot\pi}{3}$

The x–coordinate of A is 2π/3.

d. ii. The gradient of the tangent to the curve at A will be f'(2π/3), so the gradient of the normal will be –1/f'(2π/3).

#32: $-\frac{1}{f'\left(\frac{2\cdot\pi}{3}\right)}$

#33: $-\frac{2\cdot\sqrt{3}}{3}$

d. iii. First write down the equation of the normal to the curve at A. Then set y=0 and solve to find the x–coordinate of B. (B has y–coordinate 0). Use the distance formula to find the length of AB.

#34: $y - 1 = -\frac{2\cdot\sqrt{3}}{3}\cdot\left(x - \frac{2\cdot\pi}{3}\right)$

#35: $-1 = -\frac{2\cdot\sqrt{3}}{3}\cdot\left(x - \frac{2\cdot\pi}{3}\right)$

#36: $\mathrm{SOLVE}\left(-1 = -\frac{2\cdot\sqrt{3}}{3}\cdot\left(x - \frac{2\cdot\pi}{3}\right), x, \mathrm{Real}\right)$

#37: $x = \frac{4\cdot\pi + 3\cdot\sqrt{3}}{6}$

#38: $\sqrt{\left(\left(\frac{2\cdot\pi}{3} - \frac{4\cdot\pi + 3\cdot\sqrt{3}}{6}\right)^2 + (1 - 0)^2\right)}$

#39: $\frac{\sqrt{7}}{2}$

Hence the exact length of AB is √7/2.