Simple Harmonic Motion – ID: 9461

By Peter Fox

Time required
45 minutes

Activity Overview

In this activity, students collect data on the motion of a simple pendulum. They then graph the acceleration of the pendulum vs. its displacement to show that the displacement of the pendulum is directly proportional to the force acting on it. They use this information to confirm that the motion of the pendulum fulfills the requirements of simple harmonic motion.

Concepts

- *Simple harmonic motion*
- *Force, acceleration, and displacement*

Materials

To complete this activity, each student will require the following:

- *TI-Nspire™ CAS technology*
- *Vernier CBR 2™ or Go!™Motion sensor*
- *copy of student worksheet*
- *pen or pencil*
- *pendulum*
- *balance*
- *safety goggles*
- *blank sheet of paper*

TI-Nspire Applications

Graphs & Geometry, Lists & Spreadsheet, Data & Statistics, Calculator, Notes

Teacher Preparation

Before carrying out this activity, review the concept of simple harmonic motion with students. Make sure they understand the requirements for motion to qualify as simple harmonic motion. A polystyrene fishing float and lightweight cord make an excellent pendulum. The "bob" of the pendulum should have a mass of at least 40–50 g, and the string should be at least 30 cm long.

- *The force calculated in the activity takes into consideration only the horizontal acceleration of the pendulum. To measure the entire net force acting on the pendulum, it would be necessary to include a second motion detector to measure the vertical motion. If the displacement of the pendulum is small relative to its length, the effect of ignoring the vertical motion is negligible. However, you should discuss this concept with students.*
- *The screenshots on pages 2–6 demonstrate expected student results. Refer to the screenshots on page 7 for a preview of the student TI-Nspire document (.tns file).*
- ***To download the .tns file and student worksheet, go to education.ti.com/exchange and enter "9461" in the search box.***

Classroom Management

- *This activity is designed to be* **student-centered**, *with the teacher acting as a facilitator while students work cooperatively. The student worksheet guides students through the main steps of the activity and includes questions to guide their exploration. Students should record their answers to the questions on blank paper.*
- *The ideas contained in the following pages are intended to provide a framework as to how the activity will progress. Suggestions are also provided to help ensure that the objectives for this activity are met.*
- *In some cases, these instructions are specific to those students using TI-Nspire handheld devices, but the activity can easily be done using TI-Nspire computer software.*

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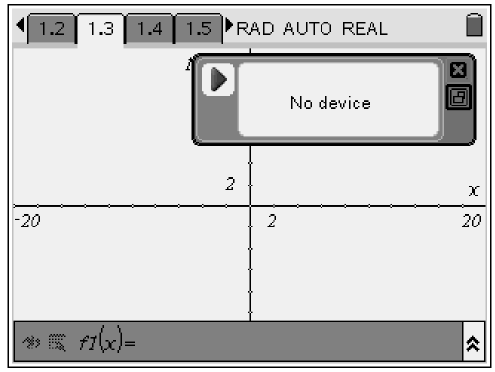
The following questions will guide student exploration in this activity:

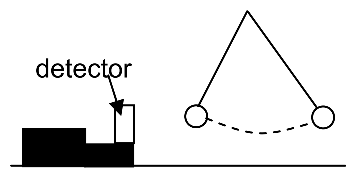
- Does the motion of a pendulum fulfill the requirements for simple harmonic motion?
- How can we make a mathematical model of simple harmonic motion?

Part 1 – Collecting displacement data

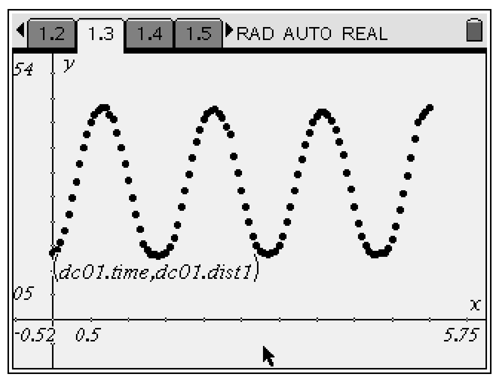
Step 1: Students will use a Vernier CBR2™ or Go!™Motion sensor to collect displacement data. When students reach page 1.3, they should insert a new data collection box and then connect their motion sensor to their handheld or computer. This should activate the motion sensor. The motion sensor should start clicking slowly, and the green light on the front should turn on. A distance display should appear in the data collection box.



Step 2: After making sure all students are wearing safety goggles, pass out pendulums to the students. They should set up their CBR 2 sensor by opening up the light gray part of the sensor so it is perpendicular to the floor, as shown in the figure to the right. Students should then practice swinging a pendulum so that it remains directly in front of the metal grid on the motion sensor at all times, and so that it does not hit the sensor as it swings.



Step 3: When students are able to swing the pendulum correctly, they will begin data collection. They should pull the pendulum back 20–30 cm, release it, and then press the ► button on the screen. After students have collected their data, they should examine the plot of the data. If the data plot is not smooth, or if there are gaps or large horizontal regions in the data plot, have students repeat the data collection. After students have collected a "clean" data set like the one shown, they should close the data collection box and disconnect the motion sensor.



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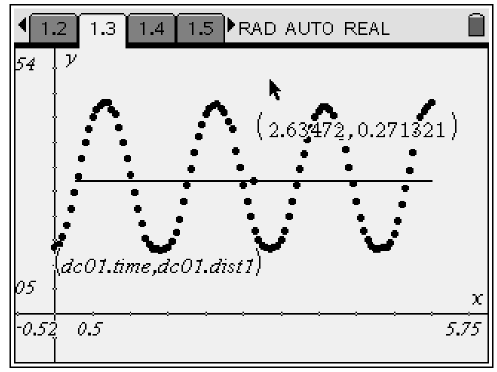
Part 2 – Calculating the force on the pendulum

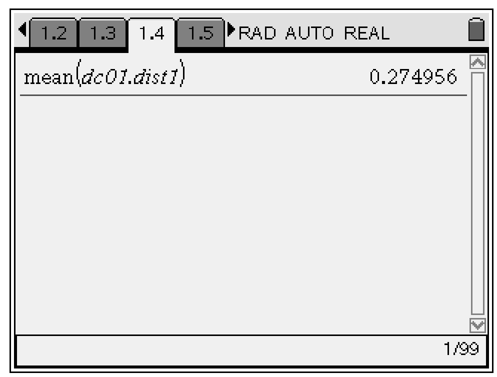
Step 1: Next, students will determine the rest position for the pendulum. They will first determine the rest position graphically. They should draw a line parallel to the *x*-axis that runs through the midpoint of the data set. They should then use the **Coordinates and Equations** tool to identify the *y*-coordinate of this line. Note: If you wish, you may have students measure the distance between the motion sensor and the pendulum bob when the pendulum is not swinging. They can then compare this distance to the rest position they determine below.



Q1. Based on your graph, what is the rest position of your pendulum?

A. *Student answers will vary. Check students' work to make sure they have correctly placed their lines.*

Step 2: Next, students calculate the rest position using the **mean** function and the *Calculator* application on page 1.4.



Q2. What is the calculated rest position of your pendulum?

A. *Student answers will vary. However, the calculated rest position should not be significantly different from the one determined graphically.*

Q3. Compare the rest position you obtained graphically with the one you calculated. Comment on any differences. Which value is more accurate? Explain your answer.

A. *Student answers will vary. The calculated value is probably more accurate because the graphically determined value depends on the students' ability to identify the central point of the data.*

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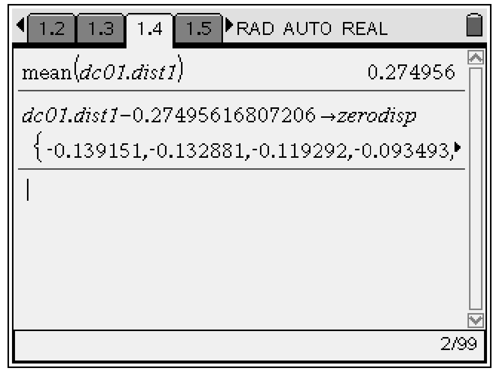
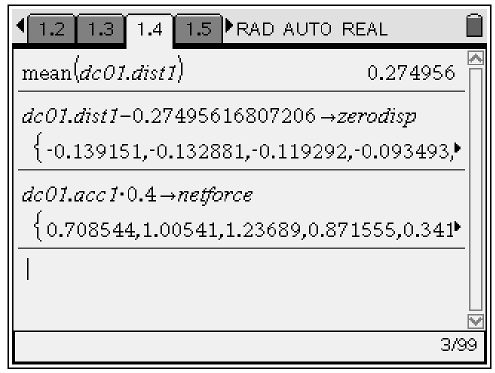
Step 3: Next, students re-center their displacement values by subtracting the rest position from all the collected data points.



Step 4: Next, students calculate the net force on the pendulum using the equation $F = ma$. They will need to measure the masses of their pendulums in order to use this equation.

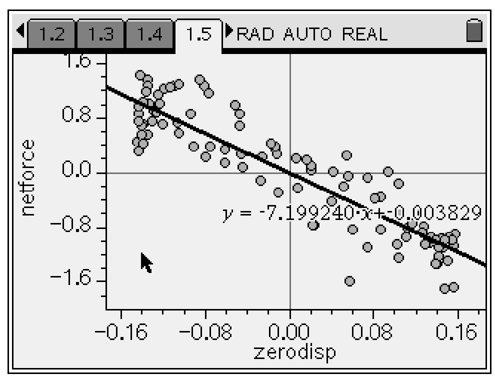
Q4. What is the mass of your pendulum in kilograms?

A. *Student answers will vary.*



Part 3 – Analyzing the pendulum's motion

Step 1: Next, students plot the net force on the pendulum vs. the displacement of the pendulum using the *Data & Statistics* application on page 1.5. They use a linear regression to find the best-fit equation for the data.



Q5. Is the displacement directly proportional to the net force?

A. *The displacement should be directly proportional to the net force (the data should lie on a straight line).*

Q6. What is the equation of the best-fit line you found?

A. *Student answers will vary. If you wish, you may encourage students to discuss the meaning of the slope of the best-fit line. Help them realize that the slope of this line is proportional to the frequency of the pendulum—if they used a higher frequency pendulum, the slope of this line would be greater.*

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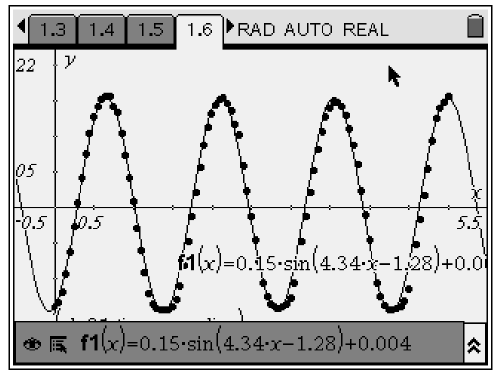
Q7. Based on these data, did your pendulum act as a simple harmonic oscillator (i.e., $F = -kx$) during this activity? Explain your answer.

A. *The displacement of the pendulum was directly proportional to the net force on the pendulum, which is a requirement for simple harmonic motion. Therefore, based on these data, the pendulum acted as a simple harmonic oscillator during this activity.*

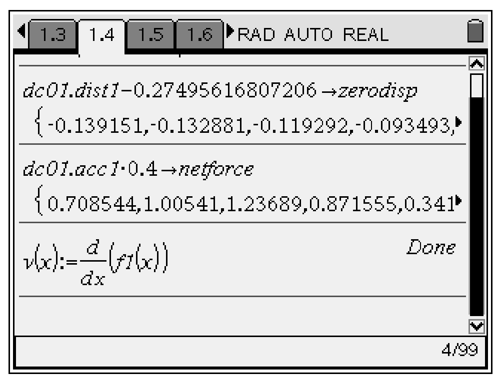
Step 2: Next, students attempt to fit a sine curve to their data. They plot the function $f1(x) = \sin(x)$ using the data they used on page 1.3. They should use the NavPad to drag the curve around to get it to fit the data as well as possible. If you wish, and time allows, you may have the students insert a *Data & Statistics* application, plot **zerodisp** vs. **run0.time_s,** and then use a sinusoidal regression to determine the best-fit equation for the collected data.



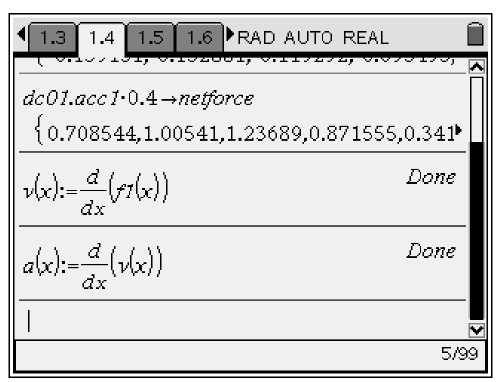
Q8. What is the best-fit equation for displacement vs. time for the data you collected?

A. *Student answers will vary.*

Step 3: Next, students calculate the function for the velocity of the pendulum, **v(x),** using the derivative function.



Step 4: Next, students calculate the function for the acceleration of the pendulum, **a(x),** using the derivative function.



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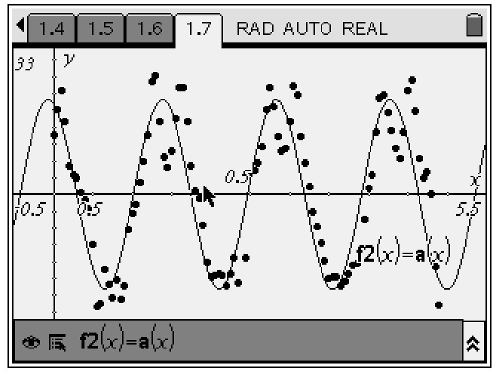

Step 5: Next, students graph **a(x)** on the same graph with the acceleration data the motion detector collected.

Q9. Does the second derivative seem to model the acceleration data well?

A. *The second derivative should give a fairly good fit to the acceleration data for regions in which* ***a(x)*** *is small. In regions in which* ***a(x)*** *is large, the errors are much greater. The amount of error in the fit will depend on how clean a sine curve students were able to generate when they collected their data. The more perfect the sine curve is, the better the fit of* ***a(x)*** *will be. However, because of the way the detector collects position data, the acceleration data will never produce a perfect fit to the acceleration function.*

Q10. Compare the displacement and acceleration equations (**f1(x)** and **a(x),** respectively). Based on these equations, did your pendulum exhibit simple harmonic motion? Explain your answer.

A. *The equations should show that* ***a(x)*** *≈ k •* ***f1(x)****. This type of relationship is consistent with simple harmonic motion.*

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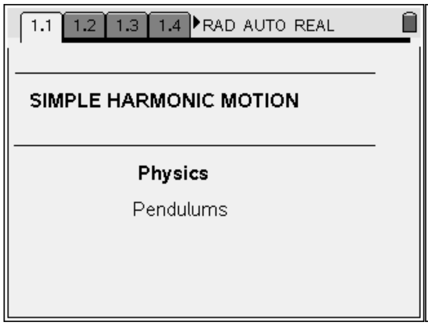
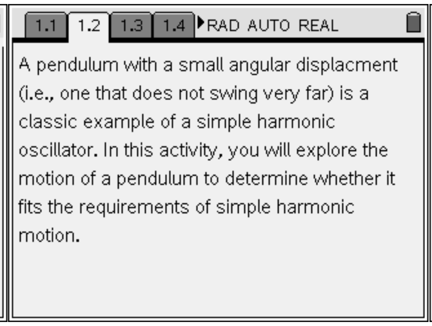
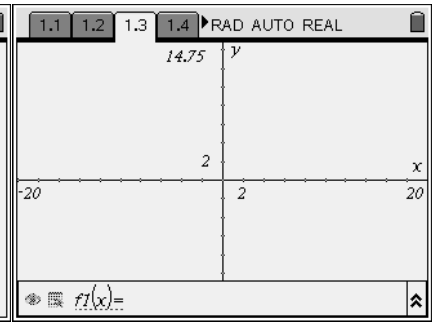
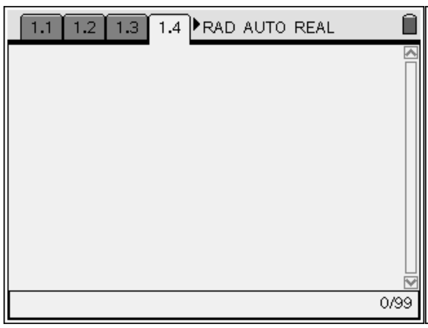
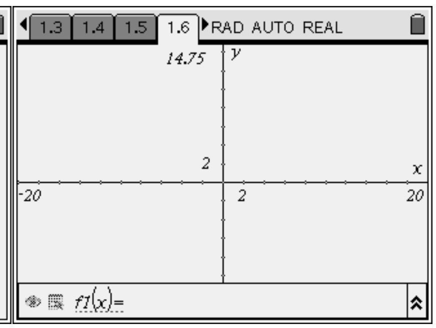
(Student)TI-Nspire File: *PhyAct17_Simple_Harmonic_Motion_EN.tns*

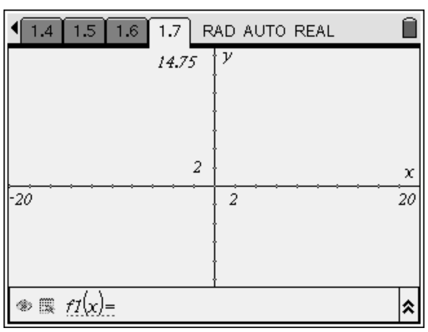












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