Exploring the Area Under a Curve

ID: 9735

Time required
45 minutes

Activity Overview

In this activity, students will explore Riemann sums to find the approximate area under the graph of $y = x^2$ on the interval [0, 1]. Additionally, students will learn to write formulas in expanded form and using summation notation. Finally, they will be asked to summarize their findings, including an analysis of when approximations produce overestimates or underestimates and the idea that an infinite number of approximating rectangles will yield the exact area under the curve.

Topic: The Fundamental Theorem of Calculus

- *Approximate the area under a curve by constructing a Riemann sum and calculating its sum.*

Teacher Preparation and Notes

- *It is essential to review the concept of summation notation with students beforehand.*
- *Students should know how to write an arithmetic sequence which will be part of the summation expression they will use to find approximate areas.*
- *The students will need to be able to compute the areas and heights on their own.*
- ***To download the student worksheet, go to education.ti.com/exchange and enter "9735" in the keyword search box.***

Associated Materials

- *AreaUnderCurve_Student.doc*

Suggested Related Activities

To download any activity listed, go to education.ti.com/exchange and enter the number in the keyword search box.

- *Riemann's Sums (TI-Nspire technology) — 9578*
- *Sums of Rectangle Areas: Approximate Integrals (TI-89 Titanium) — 3292*
- *Programming to Find Area Under a Curve (TI-84 Plus family) — 3878*

Problem 1 – Explore and discover

Two focus questions define this activity:

- How can you use rectangles to approximate the area under the curve $y = x^2$ and above the x-axis?
- Is there a way to use rectangles to find the exact area under the curve?

Some discussion should follow the posing of these questions, mainly related to how these rectangles should be used. Prior experience with limits will suggest that a large number of rectangles will produce a more accurate estimate. Tell the students that, for this activity, they will look at exactly five rectangles.

The student worksheet has two graphs for the students to try drawing rectangles.

Problem 2 – Using five right endpoint rectangles

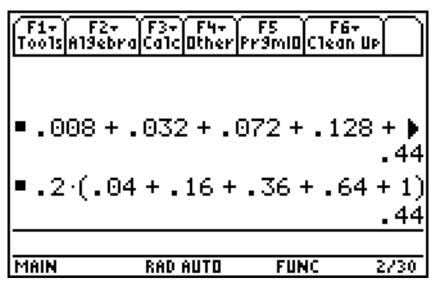
When the students divide the interval into five equal pieces, they should get the intervals [0, 0.2], [0.2, 0.4], [0.4, 0.6], [0.6, 0.8], and [0.8, 1.0]. The students are to draw the rectangles on their graph. They can use the Trace feature to determine the heights of the rectangles.

The area for each rectangle is 0.02 * height of the rectangle.

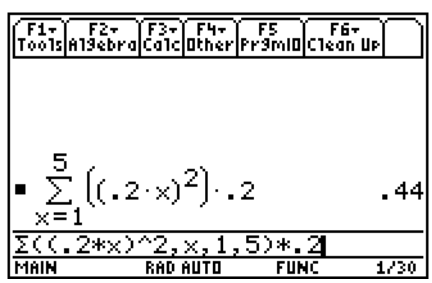
Interval	Right Endpoint	Height	Area
[0, 0.2]	0.2	0.04	0.008
[0.2, 0.4]	0.4	0.16	0.032
[0.4, 0.6]	0.6	0.36	0.072
[0.6,0.8]	0.8	0.64	0.128
[0.8,1.0]	1.0	1.0	0.2

To find the approximation of the area under the curve, students are to use the formula given on the worksheet and then also find the sum of the last column. They should get the same answer $R_5 = 0.44$.

Then, students are to write the sum of the rectangles in Sigma notation.



Since students need to sum from 1 to 5, they have to adjust the endpoints. Each endpoint is 0.2 times the index number. To factor in the width of the interval, they need to have the *0.2 at the end of the summation. The sigma command can be found in the Calc menu.



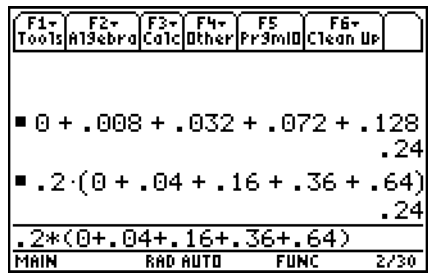
Problem 3 – Using left endpoint rectangles

Students are to draw the rectangles on their graph using left-endpoints and then complete the table.

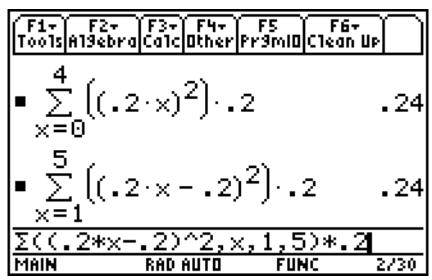
The area for each rectangle is 0.02 * height of the rectangle.

Intervals	Left endpoints	Height	Area
[0, 0.2]	0	0	0
[0.2,0.4]	0.2	0.04	0.008
[0.4, 0.6]	0.4	0.16	0.032
[0.6, 0.8]	0.6	0.36	0.072
[0.8, 1.0]	0.8	0.64	0.128

To find the approximation of the area under the curve, students are to use the formula given on the worksheet and then also find the sum of the last column. They should get the same answer, $L_5 = 0.24$.



When students use the sigma notation, they have the option of summing from 0 to 4 as in the top screen or adjusting the number and keeping the limits the same. Both appear at the right.



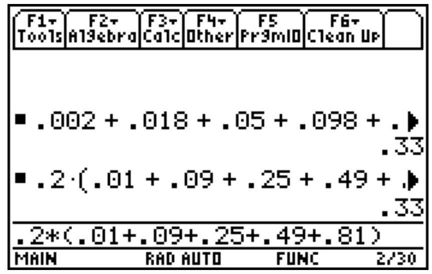
Problem 4 – Using five midpoint rectangles

Students are to draw the rectangles on their graph using the midpoints and then complete the table

The area for each rectangle is 0.02 * height of the rectangle.

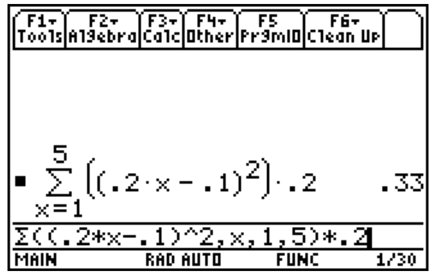
Interval	Midpoint	Height	Area
[0, 0.2]	0.1	0.01	0.002
[0.2, 0.4]	0.3	0.09	0.018
[0.4, 0.6]	0.5	0.25	0.05
[0.6, 0.8]	0.7	0.49	0.098
[0.8, 1.0]	0.9	0.81	0.162

To find the approximation of the area under the curve, students are to use the formula given on the worksheet and then also find the sum of the last column. They should get the same answer, $M_5 = 0.33$.



The sigma notation is $\sum_{x=1}^{5}\left(0.2\cdot x-1\right)^{2}\cdot 0.2$.

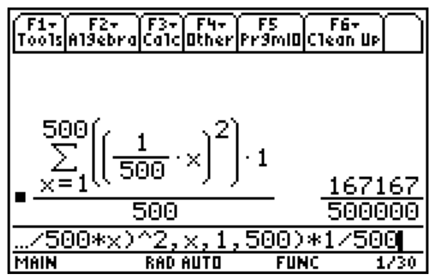
Discuss with students about whether they think that this is an overestimation or underestimation of the true area.



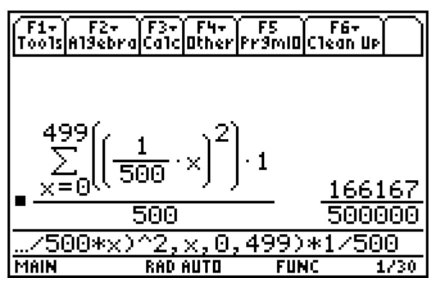
Emphasize the similarities and difference between these calculations and the previous ones in Problems 2 and 3.

Problem 5 – Summarize your findings

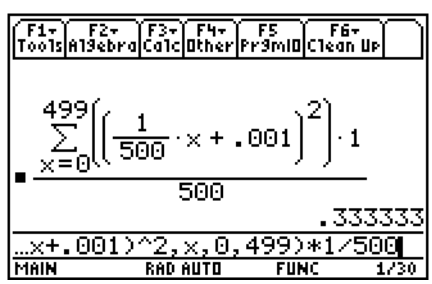
Students should readily agree that the midpoint approximation produces the best answer.



Furthermore, students should respond that the concavity of the function and whether it is increasing or decreasing will determine which estimates are *overestimates* versus *underestimates*. In this activity, students were working with a region that was determined by a function that was concave up and increasing. In this type of situation, right-endpoint and midpoint approximations will always give *overestimates* and left-endpoint approximations will always yield *underestimates*.



Finally, students will observe that a large number of approximating rectangles will improve upon their answers. Additionally they will see that summation notation offers a convenient method of quickly evaluating a large number of rectangles. The screens to the right show the summation for each method with 500 approximating rectangles.



As an extension to this activity, you may decide to use the limit and evaluate these sums for an infinite number of rectangles.