

Curve Ball

Math Objectives:

Identify and understand the vertex form of a quadratic equation,
Graph height as a function of time

Science Objectives:

Represent motion and changes in motion graphically, Use a quadratic equation to describe a ball's motion



Materials:

TI-73 Explorer™, Calculator-Based Ranger™ (CBR 2™), CBL/CBR™ Application,
A bouncing ball (Racquetballs, kickballs, volleyballs, and basketballs work well.)

Time:
1 class period

Suggested grade level: 8

OVERVIEW

Travelers can view many sights and works of architecture that are either shaped like parabolas, or that have parabolas as part of their structure. Examples include the St. Louis Gateway Arch, the Golden Gate Bridge, the Arch de Triumph, and the Trevi fountains. Parabolas are graphs of quadratic equations.

In this adventure, students will make a Height–Time plot of a bouncing ball using the CBR 2 and the Ranger program with the Ball Bounce application. A quadratic equation will be generated to describe the ball's motion.

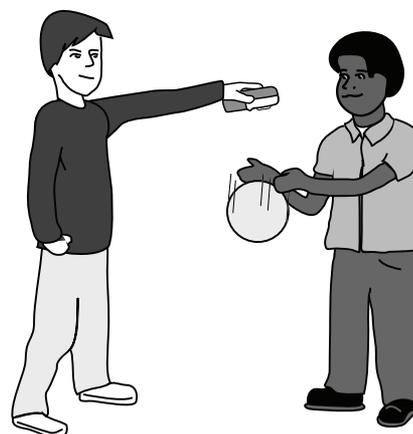


- NOTE** Demo the activity using the overhead calculator so the entire class can see the process. If you have only one CBR 2, have each group individually perform this activity. If you have enough CBR 2 units, have students work in small groups.



SET UP

1. Connect the CBR 2 to the calculator using the I/O unit-to-unit cable.
2. Set up the activity as shown in the picture. Students should hold the CBR 2 at shoulder height and hold the ball 0.5 meter under it.
3. Avoid using a soft or felt-covered ball such as a tennis ball, since the pulses from the CBR 2 tend to be absorbed by these surfaces.



- To expedite the data collection, use three students: one to hold the CBR 2, one to release the ball, and one to run the calculator.
- If you have trouble obtaining good results, try using a bigger ball.
- With the Ball Bounce application, the CBR 2 can be detached from your calculator during data collection.

When reattached following the data collection, the CBR 2 will send the data to your calculator.

- To launch the CBL/CBR App, press the **[APPS]** key, select **2:CBL/CBR** and then press **[ENTER]** to view the CBL/CBR screen. **See Figures 1a-b.**
- Press **[ENTER]** to access the application, then select **3:RANGER**. The Ranger start up screen will launch. Press **[ENTER]** to access the application. **See Figures 2a-b.**

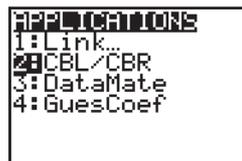


Figure 1a



Figure 1b

- From the **MAIN MENU** select **3:APPLICATIONS**. Press **1:METERS** or **[ENTER]** to select meters for the unit of measurement. Press **3:BALL BOUNCE**. **See Figures 3a-c.**



Figure 2a



Figure 2b



Figure 3a



Figure 3b



Figure 3c



DATA COLLECTION

- Time will be plotted on the horizontal axis. The distance from the CBR 2 to the ball (in meters) will be plotted on the vertical axis.
- Press **[ENTER]**. These last two screens are instruction screens. Data collection will not begin during these screens. If desired, detach the CBR 2 while collecting data. This will make following the ball with the CBR 2 sensor easier. You may resample as often as needed by pressing the CBR 2 trigger. **See Figures 4a-b.**

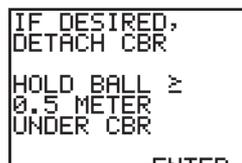


Figure 4a

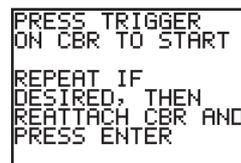


Figure 4b

- Position the CBR 2 at shoulder height and hold the ball 0.5 meters below it. Get ready to release the ball when the CBR 2 begins clicking.
- For best results, hold the sides of the ball, and then quickly move your hands outward to release the ball. Be sure to move your hands out of the way after the ball is released.
- When you are ready, press the trigger on the CBR 2 to begin data collection. To resample, press the trigger on the CBR 2 again.
- When you are satisfied with the data collection, reattach the CBR 2 to your calculator and press **[ENTER]**. The graph will appear. Your plot only needs to have two complete bounces. **See Figure 5.**
- If you are satisfied with your plot, press **[ENTER]**, sketch your plot on the axes provided on your worksheet, and go on to the next section. If not press **5:REPEAT SAMPLE** from the **PLOT MENU** and try again. **See Figure 6.**
- For any one bounce, a plot of the Height–Time has a *parabolic* shape. The equation that describes this motion is *quadratic*:

$$y = A(x - H)^2 + K$$

where A affects the width of the parabola and (H, K) is the *vertex* of the parabola. In this activity, the control variable, x, represents time and y represents height.

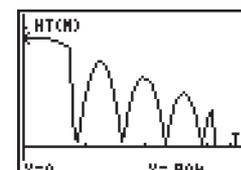


Figure 5



Figure 6

9. This equation is called the *vertex form* of the quadratic equation. Use the arrow keys (\rightarrow and \leftarrow) to move the cursor along the Height–Time plot. Identify the vertex of the first complete bounce and record the x - and y -coordinates as H and K on your worksheet. **See Figure 7.**

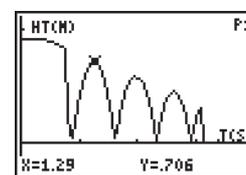


Figure 7

10. When you have finished tracing the data, press ENTER . From the **PLOT MENU**, select **7:QUIT**. Press ENTER , then CLEAR . **See Figure 8.**



Figure 8

11. Press Y= and enter the expression $A(x - H)^2 + K$, substituting your values of H and K from step 9, and leaving A as a variable. From the sample data, $H = 1.29$ and $K = 0.706$, so the equation is $A(x - 1.29)^2 + 0.706$. To enter your equation into Y= , first press 2nd [TEXT] . Highlight **A**, press ENTER , then key in the rest of the equation. Be sure to enter *your* H and K values, not the values from the sample data. Use the arrow keys to highlight **Done**.

Press ENTER . **See Figures 9a-c.**

12. Press 2nd [QUIT] to return to the home screen. To find the equation of the parabola, use a guess and check method to find the value of **A**.

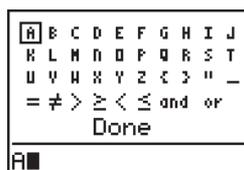


Figure 9a

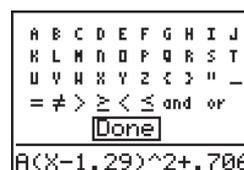


Figure 9b

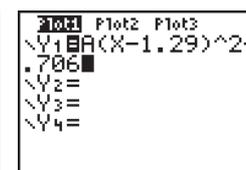


Figure 9c

13. Start with an initial guess of **A = 1**. Store this value by pressing 1 STO 2nd [TEXT] , highlight **A**, press ENTER , then highlight **Done** and press ENTER . This will substitute the value 1 in for **A** in the quadratic equation that you typed in to Y= .

14. Press GRAPH to see the equation with the data. This sample screen shows the parabola with a value of 1 for **A** using the sample H and K values.

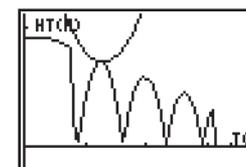


Figure 10

See Figure 10.

15. If the equation doesn't fit the data well, press 2nd [QUIT] to return to the home screen and store a different value to **A** using the method described in step 13. **Hint:** Try negative as well as positive values.

16. For each new value of **A** that you test, press GRAPH to view your adjusted equation. Experiment until you find one that provides a good fit for the data. Record the value of **A** that works best in the space provided on your worksheet.



EXTENSION ACTIVITY

- Using what you discovered about the value of A in a quadratic equation for a bouncing ball, write the equation of a parabolic ball bounce with a vertex of $(7, 0.48)$.
- Find the equation of a second parabola of your bounce plot. Then, type it in Y1 of the Y= editor and graph it to see how well it matches.
- If a more or less bouncy ball was used instead, would it affect the value of A in the equation? If so, describe how.

NOTE For fun practice, students can play the Guess My Coefficients App. Have them choose **2: Quadratic** under Select a Game, and **2: $y = (x - h)^2 + k$** under Select a Quadratic.

★ NOTE For help downloading and installing applications, see Appendix B. For information and help using the Guess My Coefficients App, see Appendix E.

DISCUSSION NOTES

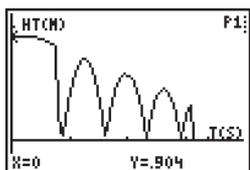
You use the formula $A = x^2$ to find the area of a square whose side is x . This is a *quadratic relationship* between the side length and the area of a square since the variable x is raised to the second power. There are many examples of quadratic relationships in the real world, including height and time variations for a bouncing ball. An object's increase in speed is due to Earth's gravitational pull. Since the gravitational pull is constant, the value of A is constant, -4.9 when working in meters, -16 when working in feet.

WORKSHEET ANSWERS

NOTE: The data in these tables is simulated. Actual data will be slightly different.

1. Possible answer:

List L ₁	List L ₂
0	0.90
0.43	0.90
0.51	0.88
0.60	0.86
0.77	0.82
0.81	0.24
0.90	0.05
0.94	0.20
1.03	0.43
1.08	0.52
1.11	0.59
1.24	0.70
1.29	0.71
1.41	0.60
2.96	1.46
1.5	0.44
1.63	0.06
1.76	0.34
1.97	0.58
2.23	0.26



- Sample data answer: $H = 1.29, K = 0.706$
- Sample data answer: -5
- Sample data answer: $y = -5(x - 1.29)^2 + 0.706$
- A positive A value makes the parabola open upward, and a negative A value makes the parabola open downward.
- Increasing the size of A makes the parabola narrower. Decreasing the size of A widens the parabola.
- The second equation would have the same value of A but different vertex coordinates.
- The other students would have the same value of A for their trials.
- The A values should all be close.
- The value of A for a quadratic equation of a bouncing ball is constant.

EXTENSION

- $y = -5(x - 7)^2 + 0.48$
- Sample data answer: $y = -5(x - 1.6)^2 + 0.5$
- The value of A would be the same as in the other trials.

**TI-NAVIGATOR™ EXTENSION ACTIVITY**

1. Load the **Arch.act** activity settings file. Students will see a photo of the St. Louis Gateway Arch in Activity Center. The activity settings files can be found on the CD located in the back of the book.
2. Challenge students to submit quadratic equations that best fit the arch. They can resubmit their equations as many times as you or they wish. See who can come closest!

ADVENTURE

12

Curve Ball Worksheet

Math Objectives:

Identify and understand the vertex form of a quadratic equation,
Graph height as a function of time

Science Objectives:

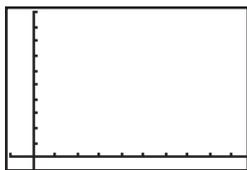
Represent motion and changes in motion graphically, Use a quadratic equation to describe a ball's motion

Materials:

TI-73 Explorer™, Calculator-Based Ranger™ (CBR 2™), CBL/CBR™ Application,
A bouncing ball (Racquetballs, kickballs, volleyballs, and basketballs work well.)

In this adventure, you will bounce a ball and find a quadratic equation to describe the ball's motion.

1. Sketch the plot of the ball bounce data.



2. What were the values of H and K of the vertex of your ball bounce?
3. What was the value of A that best fit your ball bounce parabola?
4. Using the value of A and the H and K values, write the vertex form of the quadratic equation below.
5. What effect does the sign (positive or negative) of A have on the parabola?
6. What effect does increasing the size of A have on the shape of the parabola? What effect does decreasing the size of A have on the shape of the parabola?
7. How do you think the equation of the second bounce parabola would change, if at all?

8. Would you expect your classmates to have the same value of A for their trials or do you think the A value would vary?

9. Find out the value of A from the other groups of students in your class. How do these values compare to your value of A ?

10. What conclusion can you make about the value of A for a quadratic equation for a bouncing ball?

EXTENSION

1. Using what you discovered about the value of A in a quadratic equation for a bouncing ball, write the equation of a parabolic ball bounce with a vertex of $(7, 0.48)$.

2. Find the equation of a second parabola of your bounce plot. Then, type it in **Y1** of the $\boxed{Y=}$ editor and graph it to see how well it matches.

3. If a ball that was more or less bouncy was used instead, would it affect the value of A in the equation? If so, describe how.