Exploring Space Through MATH
Applications in Algebra 1

EDUCATOR EDITION

Space Shuttle Ascent: Mass vs. Time

Note: This problem is related to the Algebra 1 problem, *Space Shuttle Ascent: Altitude vs. Time*, in the Exploring Space Through Math series.

Instructional Objectives

The 5-E's Instructional Model (Engage, Explore, Explain, Extend, Evaluate) will be used to accomplish the following objectives.

Students will

- create scatter plots from a data table;
- determine correlation and interpret its meaning;
- find linear regression equations;
- find the slope and *y*-intercept from a linear equation; and
- communicate the meanings of slope and *y*-intercept as they relate to a real-world problem.

Prerequisites

Students should have prior knowledge of scatter plots, types of correlations, linear equations (slope and *y*-intercept) and linear graphs. Students should also have experience using a TI-Nspire graphing technology to create scatter plots and to find linear regression equations.

Background

This problem is part of a series that applies algebraic principles in NASA's human spaceflight.

The Space Shuttle Mission Control Center (MCC) and the International Space Station (ISS) Control Center use some of the most sophisticated technology and communication equipment in the world. Teams of highly qualified engineers, scientists, doctors, and technicians, known as flight controllers, monitor the systems and activities aboard the space shuttle and the ISS. They work together as a powerful team, spending many hours performing critical simulations as they prepare to support each mission and crew during normal operations and any unexpected events.

Since its first flight in 1981, the space shuttle has been used to extend research, repair satellites, and help with building the ISS. NASA plans to retire the space shuttle, but space exploration has made major advances due to the success of space shuttle missions. Critical to any space shuttle mission is the ascent into space.

Key Concept
Modeling data with linear regression equations

Problem Duration
80 minutes

Technology
TI-Nspire Handhelds, computer with projector

Materials
- Student Edition
- TI-Nspire File
- Space Shuttle Launch Video

Degree of Difficulty
Moderate to Difficult

Skills
Use TI-Nspire technology to create scatter plots, find regression equations; interpret graphs and equations of linear data

NCTM Standards
- Number and Operations
- Algebra
- Data Analysis and Probability
- Problem Solving
- Communication
- Connections

The ascent phase begins at liftoff and ends when the space shuttle reaches Earth's orbit. The space shuttle must accelerate from zero to approximately 7,850 meters per second (which is approximately 17,500 miles per hour) in eight and a half minutes to reach the minimum altitude required to orbit Earth. It takes a very unique vehicle to accomplish this task.

There are three main components of the space shuttle that enable the launch into orbit (Figure 1). The main component is the orbiter. It not only serves as the crew's home in space and is equipped to dock with the ISS, but it also contains maneuvering engines for finalizing the orbital trajectory, or flight path. The External Tank (ET), the largest component of the space shuttle, supplies the propellant (liquid oxygen and liquid hydrogen) to the Space Shuttle Main Engines (SSMEs) which are liquid propellant rocket engines. The third component is a pair of Solid Rocket Boosters (SRBs) which are reusable. They are attached to the sides of the ET and provide the main thrust at launch (Figure 2).

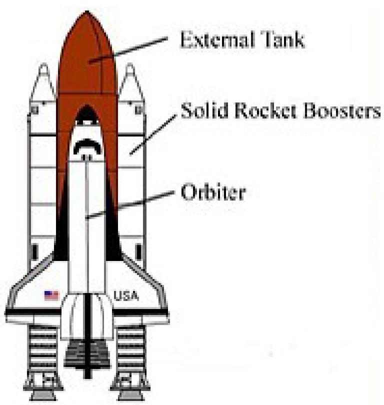


Figure 1: Main components of the space shuttle

Figure 2: Space Shuttle Discovery at liftoff

One of the flight controllers in the Space Shuttle Mission Control Center is the Booster Engineer. This position, or console, known as Booster, is in charge of monitoring the SSMEs, the SRBs, and the ET during the countdown and the ascent phase, until all of those systems are safe. The components of the space shuttle experience changes in position, velocity, and acceleration during the ascent into space. These changes can be seen by taking a closer look at the entire ascent process (Figure 3).

The ascent process begins with the liftoff from the launch pad. Propellant burns from the SRBs and the ET causing the space shuttle to accelerate very quickly. This high-rate of acceleration causes a rapid increase in dynamic pressure, known as *Q* in aeronautics (sometimes called velocity pressure). As the space shuttle breaks the sound barrier, its structure can only withstand a certain level of dynamic pressure before it suffers damage. Before this critical level is reached, the engines of the space shuttle are throttled down to about 67% of full power to avoid damage. About 50 seconds after liftoff, the dynamic pressure reaches its maximum aerodynamic load (Max *Q*). The air density then drops rapidly due to the thinning atmosphere, and the space shuttle can be throttled to full power without fear of structural damage. The command is given, "Go at throttle up!"

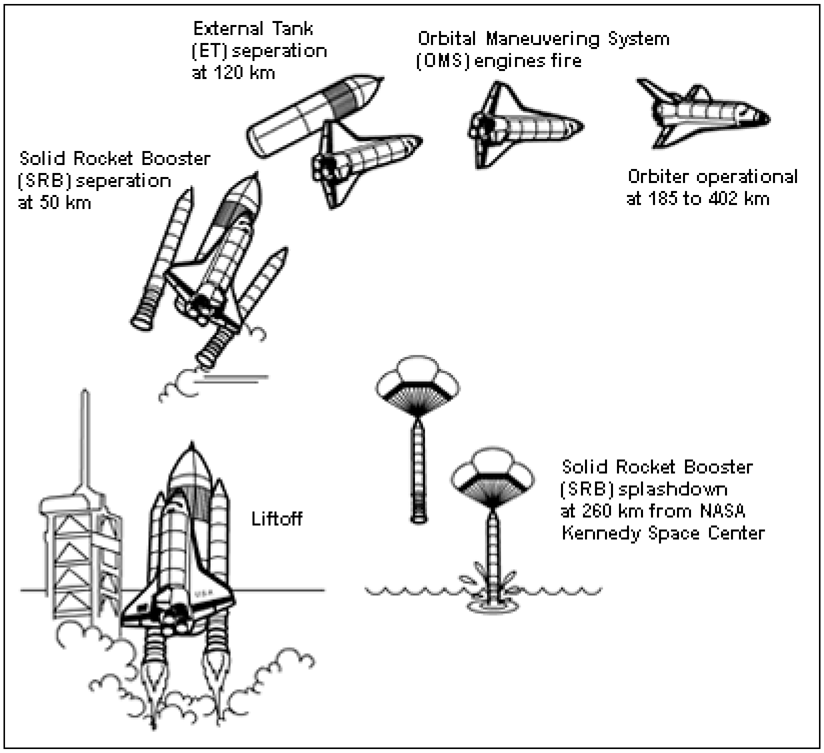


Figure 3: Space shuttle ascent process

As the space shuttle climbs, the velocity is increasing and the density of the air is decreasing. About two minutes after liftoff the atmosphere is so thin that the dynamic pressure drops to near zero. The SRBs, having used their propellant, are commanded by the space shuttle's onboard computer to separate from the ET. The jettisoning of these booster rockets marks the end of the first ascent stage and the beginning of the second. The spent SRBs fall into the ocean and are recovered, refurbished, reloaded with propellant, and reused for several missions. The second stage of ascent lasts about six and a half minutes, during which time the Booster flight controller continues to monitor the ET and the SSMEs until Main Engine Cut Off (MECO) and ET Separation. The ET re-enters the Earth's atmosphere, breaking up before impact in the ocean, and the space shuttle maneuvers into orbit. This lesson focuses on the first ascent stage, which occurs during the first two minutes after liftoff.

NCTM Principles and Standards

Number and Operations

- Develop a deeper understanding of very large and very small numbers and of various representations of them.

Algebra

- Understand relations and functions and select, convert flexibly among, and use various representations for them.
- Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior.
- Use symbolic algebra to represent and explain mathematical relationships.
- Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships.
- Approximate and interpret rates of change from graphical and numerical data.

Data Analysis and Probability

- For bivariate measurement data, be able to display a scatter plot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools.
- Identify trends in bivariate data and find functions that model the data or transform the data so that they can be modeled.

Problem Solving

- Solve problems that arise in mathematics and in other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.

Communication

- Use the language of mathematics to express mathematical ideas precisely.

Connections

- Recognize and apply mathematics in contexts outside of mathematics.

Lesson Development

Following are the phases of the 5-E's model in which students can construct new learning based on prior knowledge and experiences. The time allotted for each activity is approximate. Depending on class length, the lesson may be broken into multiple class periods.

Note: TI-Nspire embedded questions begin in the Explain section of the activity. Download file: NASA_Shuttle_Ascent-Mass_Time.tns

1 – Engage (20 minutes)

- Either assign or let students choose small groups of 3-4 in order to work through this activity. Each group should review the background section for several minutes to be sure that they understand the material. Ask if any group needs clarification.
- This video shows the launch of Space Shuttle Discovery STS 121 mission on July 4, 2006. To access the video, follow the link provided and choose the video titled: "The Rocket's Red Glare." http://www.nasa.gov/mission_pages/shuttle/shuttlemissions/sts121/launch/sts121-allvideos.html.
- Instruct students to complete questions 1-9.
- In their small groups, students discuss and share their answers to the questions. Circulate to facilitate discussion in small groups.

2 – Explore (10 minutes)

- Ask students to work through questions 10-11 in their groups.
- Call on students to give their answers and discuss.

3 – Explain (15 minutes)

- Have students open the file, *NASA_Shuttle_Ascent-Mass_Time*, on their TI-Nspire handhelds. They will then need to read through pages 1.1-1.4 and follow the directions to create and interpret a scatter plot on page 1.5.
- Have students work with their team to answer questions 12-15 that are embedded in the TI-Nspire file (pages 1.6 -1.9).

4 – Extend (15 minutes)

- Have students read the information on page 2.1 and complete question 16 (located on Nspire page 2.2) in the TI-Nspire file as directed.
- Have the students work with their group to answer question 17 (located on Nspire page 2.3).
- Call on students to give their answers and discuss.

5 – Evaluate (20 minutes)

- Have students read through Nspire pages 3.1- 3.3 and follow the directions to create and interpret a scatterplot on Nspire page 3.4.
- Have students complete questions 18-22 (located on Nspire pages 3.5 - 3.9) individually. Questions 18-22 can also be given as homework or an assessment by printing the page rather than having students answer with TI-Nspire handhelds.

ENGAGE

Video – Space Shuttle Ascent: Mass vs. Time

Solution Key

1. In the launch of the Space Shuttle Discovery what does “auto sequence start” mean?

   In the video, right after the statement, “And we have a go for auto sequence start”, the next statement we hear is, “Discovery's onboard computers have primary control of all the vehicle's critical functions.”

2. What are the 3 main components of the space shuttle?

   The orbiter, the external tank (ET), and the solid rocket boosters (SRBs).

3. Which component is the first to ignite? Which component ignites next and what is the result?

   The orbiter’s main engines are the first to ignite. Then the solid rocket boosters ignite causing liftoff.

4. At about 40 seconds after launch, due to the velocity of Discovery, what occurs?

   Discovery breaks the sound barrier.

5. About how many seconds after liftoff do the main engines throttle back to about 67% of rated performance to reduce the structural stress that occurs as the space shuttle breaks the sound barrier?

   40 seconds

6. “Discovery Houston, Go at throttle up!” is the command given when the atmosphere is so thin that there is no danger of structural damage due to dynamic pressure. About how many seconds after liftoff does this occur?

   60 seconds

7. What occurs at approximately 2 minutes into flight?

   SRB separation.

8. What happens to the depleted SRBs?

   The SRBs are recovered from the ocean, refurbished, reloaded with propellant, and reused for several missions.

9. What powers Discovery after SRB separation?

The orbiter's 3 main engines burning propellant from the external tank.

EXPLORE

Solution Key

Problem

On July 4, 2006, Space Shuttle Discovery was launched from Kennedy Space Center on mission STS-121, to begin a rendezvous with the International Space Station (ISS). Before each mission the projected data is compiled to assist in the launch of the space shuttle and to ensure safety and success during the ascent. To complete this data flight design, specialists take into consideration a multitude of factors, such as space shuttle mass, propellant used, mass of payload being carried to space, and mass of payload returning. They must also factor in atmospheric density, which is changing throughout the year. After running multiple tests, information is compiled in a table showing exactly what should happen each second of the ascent.

Table 1: STS-121 Discovery Ascent data (total mass)

Time (s)	**Space Shuttle Total Mass** (kg)
0	2,051,113
10	1,935,155
20	1,799,290
30	1,681,120
40	1,567,611
50	1,475,282
60	1,376,301
70	1,277,921
80	1,177,704
90	1,075,683
100	991,872
110	913,254
120	880,377

Table 1 shows the total mass of Discovery for mission STS-121 every 10 seconds from liftoff to SRB separation. Total mass includes the orbiter, SRBs, ET, propellant, and payload. It is during the first stage of the ascent, that the space shuttle is burning the greatest amount of propellant. You can see in the table that the space shuttle has a total mass of 2,051,113 kg at $t = 0$. After 2 minutes its total mass is only 880,377 kg, or 43 % of the original mass. The burning of this vast amount of propellant is needed to get the space shuttle through Earth's atmosphere and into orbit.

10. What happens to the mass as time progresses?

    It decreases.

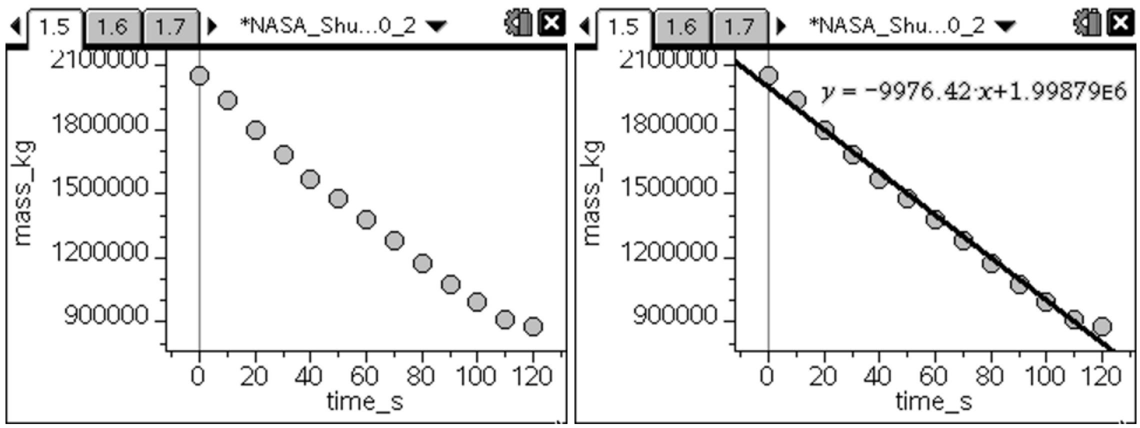
11. If this data were to be graphed with time on the *x*-axis and mass on the *y*-axis, which of the following would be characteristic of the graph?

    The graph slopes down and it has a positive *y*-intercept.

EXPLAIN

Solution Key

Create scatter plot on page 1.5 in the TI-Nspire file and model data using linear regression.



12. Does the line fit the data? Explain.

    Yes, because the points of the scatter plot appear to be on the line or are very close to it on either side.

13. What is the correlation of the data?

    Negative

14. How much propellant does Discovery burn per second? Explain what this represents with regard to the equation.

    The space shuttle burns 9,976 kg of propellant per second. This represents the slope of the line which is -9,976.

15. What is the *y*-intercept of the equation of the line that models the mass versus time data? Explain what this represents in relation to the space shuttle.

    The *y*-intercept is 1,998,791. This represents the mass of the shuttle at liftoff, or when $t = 0$.

EXTEND

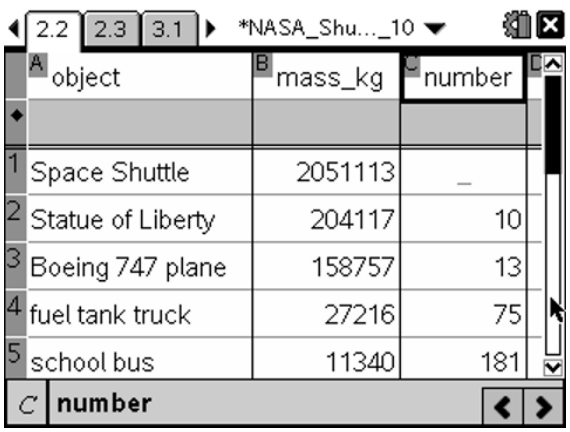
Solution Key

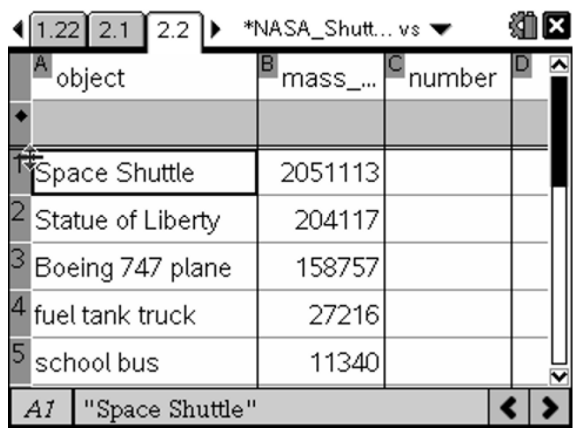
16. To gain perspective regarding the magnitude of the propellant consumption of the space shuttle, find the approximate number of each type of object that it would take to equal the mass of the entire space shuttle system at launch. Round to the nearest whole number.

Table 2: Mass of various Objects (also located on Nspire page 2.2)

Object	Approximate mass (kg)	Approximate number of objects to equal the mass of the space shuttle at launch
Statue of Liberty	204,117	
Boeing 747 airplane	158,757	
Fuel tank truck	27,216	
School bus	11,340	

Complete the table on page 2.2 in the TI-Nspire file.





17. A Boeing 767 airplane burns about 24,500 kg of fuel on a 6 hour flight from New York to Los Angeles. Approximately how much time would it take the space shuttle to burn an equivalent amount of propellant?

$$\frac{24500}{9976} = 2.5 \text{ seconds}$$

EVALUATE

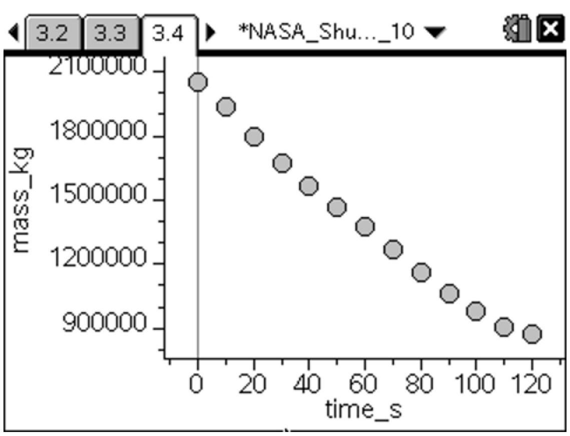
Solution Key

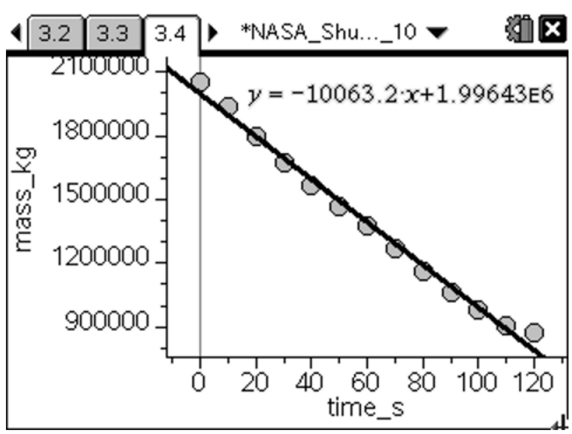
On May 12, 2009, Space Shuttle Atlantis was launched from Kennedy Space Center on mission STS-125 to repair the Hubble Space Telescope. Astronauts installed two new instruments, repaired two inactive ones, and performed the component replacements to keep the telescope functioning until 2014. Table 3 shows the total mass of Atlantis for mission STS-125 every 10 seconds from liftoff to SRB separation.

Table 3: STS-125 Atlantis Ascent data (total mass)

Time (s)	**Space Shuttle Total Mass** (kg)
0	2,049,780
10	1,932,475
20	1,795,086
30	1,676,053
40	1,562,508
50	1,468,886
60	1,374,449
70	1,264,663
80	1,163,639
90	1,061,679
100	978,131
110	902,427
120	874,457

Complete the scatter plot on page 3.4 in the TI-Nspire document.





18. Does the data appear to be linear? Explain.

    Yes, because the points of the scatter plot appear to be on the line or are very close to it on either side.

19. What is the correlation of the data?

    Negative

20. What is the slope of the equation? Explain what this represents with regard to the space shuttle.

    The slope is -10,063. This means that the space shuttle is burning about 10,063 kg of propellant each second.

21. What is the *y*-intercept of the equation of the line that models the mass versus time data? Explain what this represents in relation to the space shuttle.

    The *y*-intercept is 1,996,427. This represents the mass in kg of the space shuttle at liftoff, or when $t = 0$.

22. Compare the propellant burn rate (slope) for each mission. Which space shuttle has the fastest burn rate and what might be the reason.

    Atlantis. Possibly because the payload is heavier.

Contributors

This problem was developed by the Human Research Program Education and Outreach (HRPEO) team with the help of NASA subject matter experts and high school mathematics educators.

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