MATH AND SCIENCE @ WORK

AP* PHYSICS Educator Edition

WEIGHTLESS WONDER – Reduced Gravity Flight

Instructional Objectives

Students will

- use trigonometric ratios to find vertical and horizontal components of a velocity vector;
- derive a formula describing height of a parabola in terms of time;
- determine vertical and horizontal displacement of trajectory motion; and
- analyze data to derive a solution to a real life problem.

Degree of Difficulty

This problem is a straightforward application of parabolic motion.

- For the average AP Physics student, the problem may be moderately difficult.

Background

This problem is part of a series of problems that apply math and science principles to human space exploration at NASA.

Exploration provides the foundation of our knowledge, technology, resources, and inspiration. It seeks answers to fundamental questions about our existence, responds to recent discoveries and puts in place revolutionary techniques and capabilities to inspire our nation, the world, and the next generation. Through NASA, we touch the unknown, we learn and we understand. As we take our first steps toward sustaining a human presence in the solar system, we can look forward to far-off visions of the past becoming realities of the future.

In our quest to explore, humans will have to adapt to functioning in a variety of gravitational environments. Earth, Moon, Mars, and space all have different gravitational characteristics. Earth's gravitational force is referred to as one Earth gravity, or 1g. Since the Moon has less mass than the Earth, its gravitational force is only one sixth that of Earth, or 0.17g. The gravitational force on Mars is equivalent to about 38% of Earth's gravity, or 0.38g. The gravitational force in space is called microgravity and is very close to zero-g.

When astronauts are in orbit, either in the space shuttle or on the International Space Station (ISS), Earth's gravitational force is still

Grade Level
11-12

Key Topic
Equations of Motion

Degree of Difficulty
Physics B, C: Moderate

Teacher Prep Time
5 minutes

Problem Duration
15-30 minutes

Technology
TI-Nspire Handheld

Materials
Student Edition including:
- Background handout
- Problem worksheet
- WeightWond_AP.tns

AP Course Topics
Newtonian Mechanics:
- Kinematics

NSES
Science Standards
- Physical Science
- Science and Technology
- History and Nature of Science

working on them. However, astronauts maintain a feeling of weightlessness, since both the vehicle and crew members are in a constant state of free-fall. Even though they are falling towards the Earth, they are traveling fast enough around the Earth to stay in orbit. During orbit, the gravitational force on the astronauts relative to the vehicle is close to zero-g.

Figure 1: C-9 jet going into a parabolic maneuver.

Figure 2: Astronaut crew training onboard the C-9 aircraft in preparation for the Microgravity Science Laboratory missions flown on the Space Shuttle Columbia in April and July of 1997.

The C-9 jet is one of the tools utilized by NASA to simulate the gravity, or reduced gravity, astronauts feel once they leave Earth (Figure 1). The C-9 jet flies a special parabolic pattern that creates several brief periods of reduced gravity. A typical NASA C-9 flight travels over the Gulf of Mexico, lasts about two hours, and completes between 40 and 60 parabolas. These reduced gravity flights are performed so astronauts, as well as researchers and their experiments, can experience the gravitational forces of Moon and Mars and the microgravity of space.

By using the C-9 jet as a reduced gravity research laboratory, astronauts can simulate different stages of spaceflight. This can allow crew members to practice what might occur during a real mission. These reduced gravity flights provide the capability for the development and verification of space hardware, scientific experiments, and other types of research (Figure 2). NASA scientists can also use these flights for crew training, including exercising in reduced gravity, administering medical care, performing experiments, and many other aspects of spaceflight that will be necessary for an exploration mission. A flight on the C-9 jet is the next best thing to blasting into orbit!

AP Course Topics

Newtonian Mechanics

- Kinematics:
  - Motion in two dimensions, including projectile motion.

NSES Science Standards

Physical Science

Knowledge of Motion and Force

- Laws of motion are used to calculate precisely the effects of forces on the motion of objects.

Science and Technology

Abilities of Technological Design

- Implement a proposed solution.
- Evaluate the solution and its consequences.
- Communicate the problem, process, and solution.

History and Nature of Science

Science as a Human Endeavour

- Individuals and teams have contributed and will continue to contribute to the scientific enterprise.

Nature of Scientific Knowledge

- Scientific explanations must meet certain criteria. First and foremost, they must be consistent with experimental and observational evidence about nature, and must make accurate predictions, when appropriate, about systems being studied.

Problem

Note: Students are given the following problem information within the TI-Nspire document, WeightWond.tns. They are also provided Figure 3 on the student handout that they can reference while doing the problem. Students can submit their answers within the WeightWond.tns file, or they can write their answers on the student handout.

To prepare for an upcoming mission, an astronaut participated in a C-9 flight simulating microgravity. The pilot flew out over the Gulf of Mexico, dove down to increase to a maximum speed then climbed up until the nose was at a 45° angle with the ground. At this point the velocity of the plane was 444 kilometers per hour (about 275 mph) and the altitude was 9,144 meters (about 30,000 ft). To go into a parabolic maneuver, the pilot then cut the thrust of the engine letting the nose of the plane continue to rise then come back down at a -45° angle with the ground. Ending the parabolic maneuver, the pilot throttled the engine back up and began another dive to prepare for the next parabola. The pilot completed 50 parabolas during the 2 hour flight.

Figure 3 shows the movement of the plane during a typical flight. The parabolic maneuver, where microgravity is felt, is highlighted. On page 1.7 of the TI-Nspire document you will find a similar graphic. Grab the point that lies on the curve and observe the direction and magnitude of the arrows (velocity vectors) as you drag the point along the flight path.

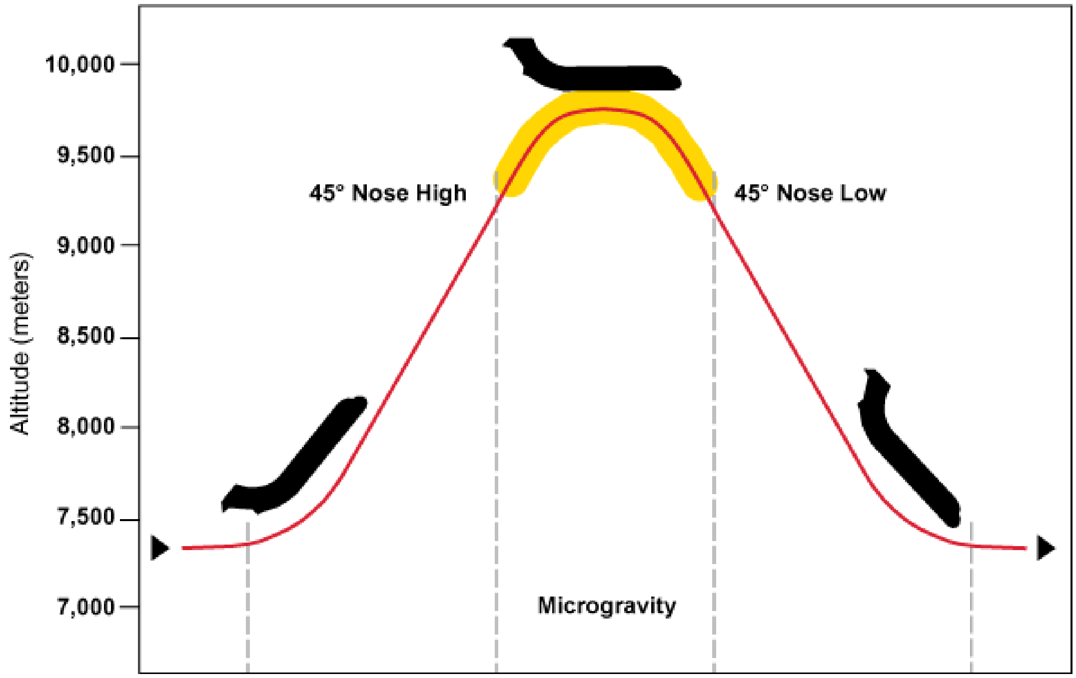


Figure 3: A typical microgravity maneuver. (not to scale)

Note: Acceleration due to gravity is approximately -9.8 m/s^2. *For this problem we will be ignoring other influences such as air resistance.*

Round all answers to three decimal places.

A. Find the initial and final vertical and horizontal velocities during one parabolic maneuver.

B. Find the time elapsed, in seconds, of one parabolic maneuver.

C. Find the maximum altitude, in meters, the plane reached. For students in Physics C, verify the maximum point using calculus.

D. Find the horizontal displacement, in meters, of the plane during one parabolic maneuver.

E. What percentage of the total flight was spent in microgravity?

F. How many parabolas would the pilot need to complete in order for the astronaut to have had at least 15% of his flight in microgravity?

Solution Key (One Approach)

Note: A TI-Nspire document with solutions shown (WeightWond_Solutions.tns) is provided for download with this problem.

A. Find the initial and final vertical and horizontal velocities during one parabolic maneuver.

Step 1: Convert initial velocity to meters per second.

$$\frac{444\text{ km}}{1\text{ hr}} \cdot \frac{1000\text{ m}}{1\text{ km}} \cdot \frac{1\text{ hr}}{3600\text{ s}} = 123.333\text{ m/s}$$

Step 2: Find the initial vertical and horizontal velocities.

Initial vertical velocity:

$$V_{iy} = V_i \sin\theta$$
$$V_{iy} = (123.333\text{ m/s})\cdot(\sin 45°) = 87.210\text{ m/s}$$

Initial horizontal velocity:

$$V_{ix} = V_i \cos\theta$$
$$V_{ix} = (123.333\text{ m/s})\cdot(\cos 45°) = 87.210\text{ m/s}$$

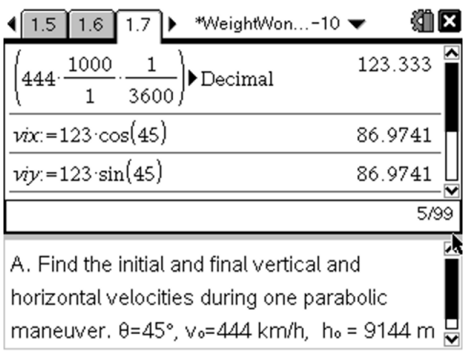
Step 3: Find the final vertical and horizontal velocities.

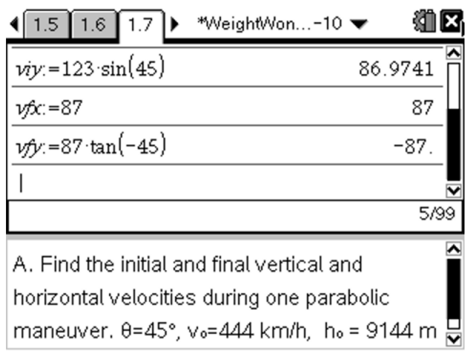
The horizontal velocity remains constant thus, the final horizontal velocity:

$$V_{fx} = 87.210\text{ m/s}$$

The final vertical velocity is found using the formula:

$$V_{fy} = V_{fx}\tan\theta$$
$$V_{fy} = (87.210\text{ m/s})\cdot(\tan(-45°)) = -87.210\text{ m/s}$$

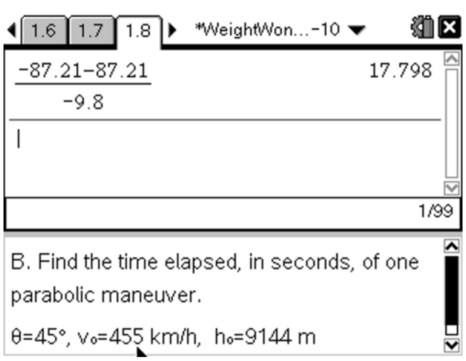




B. Find the time elapsed, in seconds, of one parabolic maneuver.

Using the formula for final vertical velocity, $V_{fy} = V_{iy} + at$, substitute your values in for final and initial vertical velocities and the acceleration due to gravity to solve for time.

$$-87.210\text{ m/s} = (87.210\text{ m/s}) + (-9.8\text{ m/s}^2)t$$
$$t = 17.798\text{ s}$$



C. Find the maximum altitude, in meters, the plane reached. For students in Physics C, verify the maximum point using calculus.

The vertical displacement is given by the formula $y = V_{iy}t + \frac{1}{2}at^2 + y_0$.

Substituting in velocity, acceleration, and initial altitude into the equation gives the quadratic equation: $y = -4.9t^2 + 87.2t + 9144$

To find the maximum altitude, find where the vertical velocity equals zero.

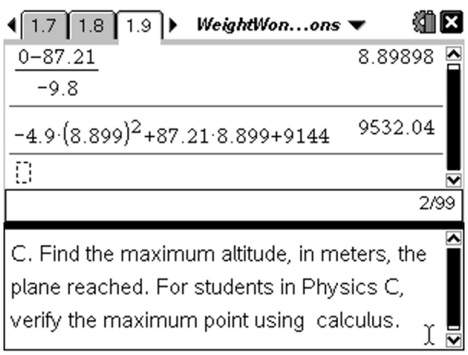
$$V_{fy} = V_{iy} + at$$
$$V_{fy} = 87.210\ \text{m/s} + \left(-9.8\ \text{m/s}^2\right)t$$

Substitute 0 in for the final vertical velocity and solve for *t*.

$$0 = 87.210\ \text{m/s} + \left(-9.8\ \text{m/s}^2\right)t$$
$$t = 8.899\ \text{s}$$

Substituting 8.899 into our altitude equation gives us the maximum altitude:

$$y = -4.9(8.899)^2 + 87.210(8.899) + 9144$$
$$y = 9532.040\ \text{m}$$



To verify the maximum altitude using calculus the students should find where the derivative of the altitude function is 0.

Our altitude function is $y = -4.9t^2 + 87.210t + 9144$.

$$\frac{dy}{dt} = -9.8t + 87.210$$
$$0 = -9.8t + 87.210$$
$$t = 8.899\ \text{s}$$

Now substitute 8.899 into the altitude function and solve for *y*.

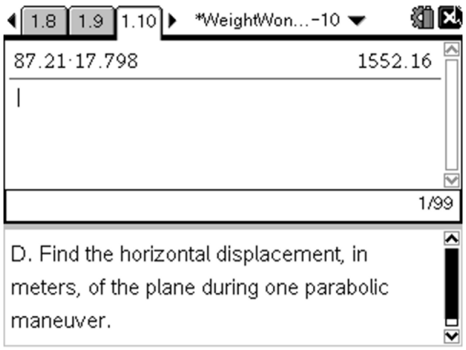
$$y = -4.9(8.899)^2 + 87.210(8.899) + 9144$$
$$y = 9532.04\ \text{m}$$

D. Find the horizontal displacement, in meters, of the plane during one parabolic maneuver.

Since horizontal velocity was constant, our equation is $x = 87.210t$.

Thus the horizontal displacement was:

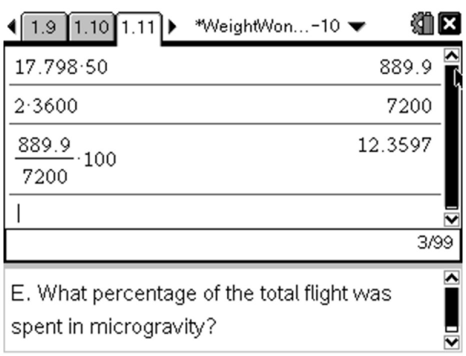
$$x = (87.210 \text{ m/s}) \cdot (17.798 \text{ s})$$
$$x = 1552.164 \text{ m}$$



E. What percentage of the total flight was spent in microgravity?

Each parabola lasted 17.798 seconds and the pilot performed 50 parabolas. The total time in microgravity was $17.798 \cdot 50 = 889.9$ seconds. The trip lasted for two hours, which is 7200 seconds.

$$\text{Percentage} = \frac{889.9}{7200} \cdot 100$$
$$\text{Percentage} = 12.360\%$$

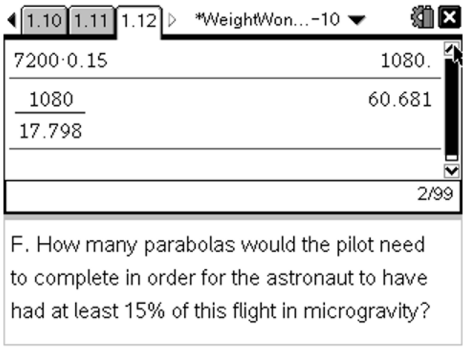


F. How many parabolas would the pilot need to complete in order for the astronaut to have had at least 15% of his flight in microgravity?

15% of 7200 seconds is 1080 seconds.

1080/17.798 = 60.681 parabolas

Thus the pilot would have needed to complete 61 parabolas.



Scoring Guide

Suggested 14-15 points total to be given.

Question		**Distribution of points**
A	*5 points*	1 point for correct conversion of initial velocity to meters per seconds.
		1 point for correct process used to find initial vertical velocity
		1 point for correct process used to find initial horizontal velocity
		1 point for recognizing horizontal velocity remains constant
		1 point for correct process used to find final vertical velocity
B	*2 points*	1 point for use of formula for final vertical velocity
		1 point for substitution and process used to solve for time
C	*4 - 5 points*	1 point for correct use of vertical displacement formula
		1 point for correct formula to find final velocity
		1 point for correct process used to find time of maximum altitude
		1 point for correct substitution to find maximum altitude
	Physics C	1 point for correct verification of maximum using calculus
D	*1 point*	1 point for correct process in finding horizontal displacement
E	*1 point*	1 point for correct substitution and process used to find percent
F	*1 point*	1 point for correct substitution and process used to find the number of parabolas

Contributors

Thanks to the subject matter experts for their contributions in developing this problem:

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